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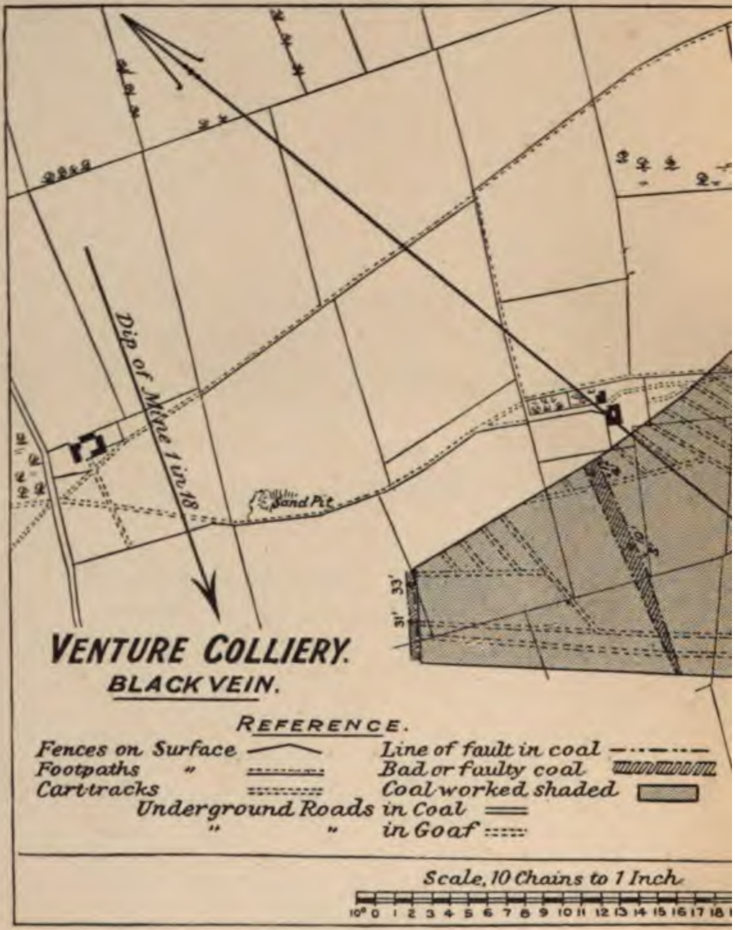

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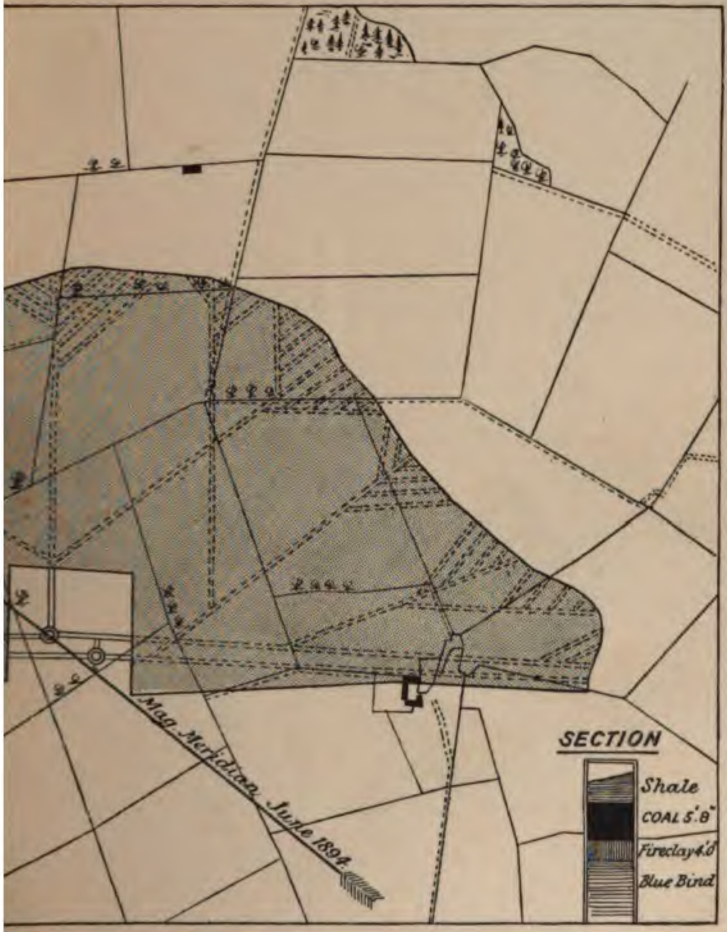

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A PRACTICAL TREATISE
ON
MINE SURVEYING

BY

ARNOLD LUPTON

MINING ENGINEER, CERTIFICATED COLLIERY MANAGER, SURVEYOR,
MEMBER OF THE INSTITUTION OF CIVIL ENGINEERS,
MEMBER OF THE INSTITUTION OF MECHANICAL ENGINEERS,
MEMBER OF THE INSTITUTE OF MINING ENGINEERS,
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LATELY PROFESSOR OF COAL MINING AT THE VICTORIA UNIVERSITY
(YORKSHIRE COLLEGE, LEEDS), AND SOMETIME
EXAMINER IN MINE SURVEYING TO THE CITY AND GUILDS OF LONDON INSTITUTE

WITH ILLUSTRATIONS

LONGMANS, GREEN, AND CO.
39 PATERNOSTER ROW, LONDON
NEW YORK AND BOMBAY
1902

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PREFACE

This book has been prepared with the intention of assisting students in learning the art of Surveying. The author, during the twenty-one years of his Professorship in the Mining Department of the Yorkshire College, had to teach a great many students the elements of this art, and for that purpose put together various notes. As a former Examiner in Mine Surveying to the City and Guilds of London Institute also, the author gained a considerable insight into the needs of students. He has added to his own experience as a practical surveyor by reading a number of books on surveying and papers published in the transactions of various scientific societies both in this and other countries.

Where it has been thought advisable to reproduce extracts or drawings from these, acknowledgment will be found in the text.

Whilst primarily the object the author has had in view has been the preparation of an elementary text-book, he has endeavoured to make the book of value as a reference book to the more advanced parts of the subject, and the chapters dealing with Trigonometrical Plotting, Hypsometry, Method of finding the True North, Metalliferous Mine Surveying, Photographic Surveying, Prospecting with the Magnetic Needle, etc., have been included with this purpose in view.

The reader should endeavour, as far as possible, to get practical experience of the instruments and in the method of using them, and the author would recommend such of his readers as have not done so to view the collection of surveying instruments at the South Kensington Museum, London.

The author would like to acknowledge the uniform courtesy shown to him by those members of the Government Departments (Royal Observatories, Greenwich and Kew, the Ordnance Survey Office, the Meteorological Office, etc.) who have supplied him with various information, and also his thanks to the various makers of surveying instruments herein described.

The tables of Logarithms, Antilogarithms, Squares, Sines, Cosines, Tangents, etc., which form a portion of the appendix, are taken from a work on Elementary Physics by Mr. John Henderson, D.Sc. (Edin.), A.I.E.E., F.R.S.E., to whom the author is indebted for permission to reproduce them.

In conclusion, the author wishes to state that professional engagements might have entirely prevented him from completing this work had it not been that among his assistants he numbered some experienced surveyors, and he thinks it fair to acknowledge the valuable assistance he has had from them, especially from Mr. Herbert Perkin. He would also like to thank those of his friends who have undertaken the revision of various parts of the work.

Any corrections or additions which suggest themselves to the reader will be gratefully acknowledged.

ARNOLD LUPTON.

6, De Grey Road, Leeds,
July, 1901.

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MINE SURVEYING

CHAPTER I.

NEED AND ADVANTAGES OF ACCURATE PLANS, ETC.

MINE surveying is necessary for two reasons: In the first place, a map or plan, and section, are necessary to guide the miner in his daily work, so that when the workings have extended over a considerable area, it may be seen at a glance which parts of the mineral have been got and which remain to get; in what direction the roads go, how far apart they are one from another; how machinery can be best arranged for underground haulage; how the ventilation of the mine may be most economically conducted; and how the drainage may be effected. The plan should also show the direction of faults, and where the mineral has been found good, or where inferior or unworkable. The section will show the inclination of the bed or vein, and the height above or depth below any given datum-line. Contour-lines on the plan give the same information for the whole mine. In the second place, the plan of the mine (see Frontispiece) is required to show the position of the underground workings with regard to objects and boundary-lines on the surface. To take mineral from underneath the land without the previous sanction of the landowner may be treated as felony, and, if it is done through accident or inadvertence, may be punished with a heavy fine. It is, therefore, of the highest importance that the owner or tenant of the mine should not only have an accurate plan of the boundary of the estate under which he has a licence to work, but an equally accurate plan of the underground workings, drawn upon the same paper (or other drawing material) that is used for the plan showing the boundaries, fences, buildings, roads, streams, and other notable objects above ground. A mining plan is therefore, generally speaking, incomplete unless it is also a plan of the land above; and a mining surveyor is therefore not competent for the entire production of a mining plan unless he understands land surveying as well as mine surveying. It frequently happens, however, that the plan of the surface is made by a land surveyor, and the plan of the mine by a mine surveyor; and this combination often produces very accurate results. In some respects it is better that the whole of the plan should be made by one surveyor, who is responsible for the accuracy of the combination of underground and surface work, and in this case that person should be the mine surveyor, as he is the man who possesses the additional knowledge of the mine which is necessary for a proper survey.

Meridian Line.—Even in case the mine surveyor is relieved from the work of land surveying by having an accurate map of the estate put into his hands, he cannot delineate upon it the workings of the mine unless he has some knowledge of land surveying, because he will require to mark upon the plan a meridian line to which his underground survey must be referred. This meridian line may be drawn north and south in the geographical meridian, or line of longitude; or it may be drawn in the direction of the magnetic pole, or it may be some other line which is marked out both on the surface and in the mine below, in the same vertical plane. None of these lines can be correctly marked upon the surface plan without some knowledge of land surveying. It is, therefore, necessary that the mine surveyor should be instructed in the art of land surveying.

Every art in which it is possible to achieve perfection has a fascination for the human mind, and surveying is one of these arts.

Degree of Accuracy attained.—The accuracy with which the survey may be made is only limited by the skill and care of the surveyor, provided he has the opportunity of using the most suitable instruments which are made; and, as a general rule, the surveyor obtains the accuracy necessary for his purpose. It is, however, perhaps also true that, as a general rule, he is not much more accurate than is necessary. Thus, in a mine of large extent, the workings of which are neither near a boundary nor near to some important building which must not be disturbed, an error of half a chain in the position of any part of the workings is by no means uncommon.

Reasons for Great Accuracy.—On the other hand, when approaching some important building, or when approaching a boundary which must not be *passed* under a heavy penalty, and which must yet be *reached* because the owner of the mine does not wish to sacrifice any portion of the mineral which is his, then minute accuracy is often attained. In some metalliferous mines great value attaches, perhaps sometimes reaching £1000, to a single square yard of ground, and in such a case it is necessary that the plan should be so accurate that no rival skill can detect an error.

If the owner of a mine inadvertently crosses the boundary, and gets mineral to which he has no right, he may be obliged to pay in damages nearly the whole market price of the mineral, possibly ten times the royalty ordinarily payable, so that in the case of a seam of coal, he might be fined to the extent of two or three shillings per square yard for every yard in thickness.

In order to avoid crossing the boundary, there are only two courses—one is to leave a considerable margin of the mineral inside the boundary, and the other is to have a plan of extreme accuracy, and to mark out the limits of workings underground upon this plan from day to day. To leave a wide margin of coal or other mineral ungot, unless it is required for the purposes of a permanent barrier, involves a corresponding loss and waste of mineral.

An accurate plan is also necessary for engineering reasons. It may be necessary to drive an underground road or tunnel from one pit to some other pit, and a serious loss may result if the mark aimed at is not hit in the centre.

For reasons of safety an accurate plan is much to be desired. Abandoned workings may be full of water, and if the plan of these abandoned workings does not show them all and in a correct position, the workings from some new mine may inadvertently break in upon accumulations of water, and thus lead to fatal, and financially disastrous results. It is, therefore, in the highest degree desirable that mine surveyors should habituate themselves to the making of accurate plans, because a habit of carelessness, once acquired, is difficult to throw off when minute accuracy is necessary.

It is, however, not the surveyor who requires to be impressed with the importance of an accurate plan, it is rather those who have to pay for his services, and they do not always see where they get any return for an expenditure on carefully made maps and plans. It thus happens at some collieries that hundreds of pounds are annually wasted which would be saved by the employment of a careful surveyor, not merely to make a plan of the roads after they are driven, but to set out the roads in the right direction. The cause of this waste is easily explained: without an accurate plan, showing the existing workings, faults, and inclination of the seam, it is impossible to lay out the roads so that the shortest length of road may suffice; hence an unnecessary number of roads, and these roads crooked, are often made. Also, even if the roads are correctly schemed, they will not be made in the direction intended unless the workmen are guided by marks carefully fixed by the surveyor. Each yard of road in the mine costs so much to make, varying according to circumstances—in coal-mines from 2*s*. to 20*s*., and in metalliferous mines and cross-measure drifts from 10*s*. to £10; it also costs so much to maintain, and then there is the cost of transit. Thus in a mine raising 300,000 tons of coal a year, the cost of making and maintaining roadways of all kinds, and of haulage, may, combined, easily amount to £20,000 a year. If the length of the roadways is 5 per cent. longer than necessary, the cost will be increased in a corresponding degree, or to the extent of £1000 a year. In many cases the costs are on a higher scale, and, of course, the loss from unnecessary lengths of road is correspondingly increased.

It is absolutely certain that the money spent on the production of accurate plans and contours, and sections giving every engineering and geological detail, is repaid many times over (tenfold to a hundredfold) every year in the ordinary course of working.

CHAPTER II.

THE MEASUREMENT OF DISTANCES.

CHAINS, TAPES, POLES, MEASURING-WHEELS.

THE instruments generally used by the mine surveyor are as follows:—

Measuring-chains.—Gunter's chain is that usually employed for land surveying and in coal-mines. This chain (see Figs. 1 and 2) is 66 feet long, or the eightieth part of a mile. It is divided into 100 parts, called "links." 100,000 square "links," or 10 square chains, equal 1 acre. The chain is constructed either of iron, steel, or brass wire. If made of steel wire, it is about $\frac{1}{10}$ inch in diameter. A chain-length is composed of a hundred pieces of wire, which have a loop at each end, and are 6 inches in length. These pieces are united by three short links, about $\frac{5}{8}$ inch, internal measurement, made of flat wire.

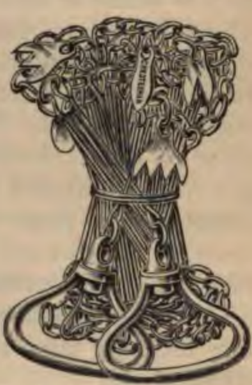
FIG. 1.—Gunter's measuring-chain.

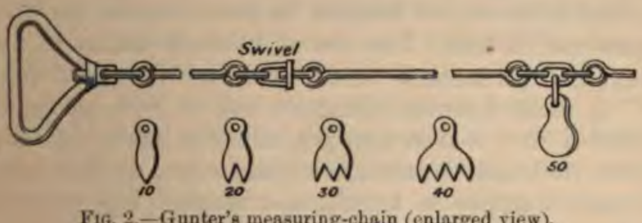

FIG. 2.—Gunter's measuring-chain (enlarged view).

These three short pieces and the long pieces make up a length of nearly 8 inches, or exactly 7·92 inches. At each end of the chain the 6-inch piece is shortened to about 4 inches;

then comes a small link, and then a brass handle, making up the total length of 7·92 inches. Measuring from the outside of the handle for a length of 10 links, the end of the tenth link is in the centre one of the three small loops connecting two 6-inch pieces. Attached to the centre loop is a small brass tag, with one prong, which indicates a length of 10 links from the end of the chain. Measuring 10 links further, another brass tag is similarly attached to the chain; but this second tag has two prongs. At the end of the next 10 links is another brass tag, which has three prongs; at the end of the next 10 links is a similar brass tag, with four prongs; the end of the next 10 links is the centre of the chain, and has a simple round-ended brass tag. Each end of the chain is constructed in the same way, measuring from the outside of the handle to the centre, so that the same tag may count 40 or 60, according as it is before or after the centre, 30 or 70, 20 or 80, 10 or 90. At 25 links from each end of the chain, instead of the three simple loops connecting two 6-inch pieces, there is one loop and two swivel-jointed loops, so that if the chain has got twisted it may be untwisted. The swivel-joint also marks the length of 25 or 75 links. At the centre of the chain is another swivel link; this is marked by the round-ended tag above mentioned. Sometimes 10 links at each end of the chain are made of brass, so that the end of the chain may be held near a magnetic compass without attracting the needle. If the chain is made of brass or iron wire instead of steel wire, it is about $\frac{1}{8}$ inch thick. For ordinary mine surveying it is desirable to have a good strong chain.

Engineers often use a chain 100 feet long, divided into links of 1 foot in length. Where a section is being levelled, it is convenient to have the lengths in feet, because the altitudes are measured in feet. The use of 100-foot chains is making headway, and has much to recommend it. Whenever the term "link" is used, however, Gunter's link of 7·92 inches is the one referred to. In the Cornish mines a chain 10 fathoms, or 60 feet, in length is used, the chain being divided into 120 parts, each 6 inches in length, and marked with a tag every 6 feet (*i.e.* every fathom).

Tapes.—A 66-foot painted tape, divided on one side into feet and inches, and on the other into links, is very convenient for measuring offsets, and the width and height of roads. The best kind of tape is the "metallic" tape, made with fine brass wires interwoven with vegetable fibre.

Steel Tapes.—Where great accuracy is desired, steel tapes may be used. The steel tape, being one continuous ribbon of metal, is less liable to stretch than the chain. One side is marked with feet and inches, and the other with links. Steel tapes have to be carefully used, in order to avoid breaking, and must be cleaned after use, or the marking will become obliterated by rust.

Sometimes a tape much longer than 100 links is used. Mr. Eckley B. Cox, of Drifton, Pennsylvania, showed the writer a steel tape 500 feet in length. This tape was very light, about $\frac{1}{12}$ inch broad and $\frac{1}{70}$ inch thick. Every tenth foot was marked with a piece of brass wire soldered on with white solder, the number of each mark being shown by figures on the solder. The tape is carried on a reel, from which the required length may be unwound. One end of the tape is held at one station, and the distance to the other is read off upon the tape to the nearest 10-foot mark; from this mark to the station the length is measured by a 6-foot staff marked in feet and decimals of a foot. By the use of this long tape, the entire length of a line can be measured at one operation to the hundredth part of a foot, and the errors due to marking off chain-lengths on rough and uneven ground are thus avoided.

When measuring large tracts of outlying country, where portability and lightness are of great importance, what is known as a compound steel band chain is often used. It consists of two or more separate steel bands, each one chain long. These can be joined together by swivels and hooks, and used in lengths of one, two, or more chains.

The first chain of each set is divided into links in the usual manner; but the other chains are not subdivided. The bands are wound up on a steel cross.

Measuring-poles.—For measuring short lengths poles are often used, divided into links by painting alternate lengths of one link black and white. The divisions of the pole are sometimes in feet for architectural purposes; and for measurements of extreme accuracy, the divisions are subdivided into tenths. As a general rule, poles are only used for measuring offsets to the line measured by the chain. For this purpose a 10-link (or, in the alternative, a 10-foot) pole is most convenient. In some cases the base-line for a trigonometrical survey has been measured along a line, carefully levelled for the purpose, by means of poles laid end to end, so as to avoid the errors due to the inaccuracy of chains or tapes.

Measuring-rods have been so constructed that the length is uniform for all temperatures. These are made by using a rod compounded of two side by side, one brass and the other iron, which have an unequal expansion. At each end is a cross-piece, projecting on one side, with a centre-mark so placed that the centre-marks maintain an equal distance during variations of temperature.

Pacing.—Distances are sometimes measured by pacing. With a little practice a surveyor may learn to step a yard, and in this way to measure distances with an error not greatly exceeding 5 per cent. The ordinary pace is much shorter, being, say, 30 to 33 inches. There is a great difficulty, however, in counting the paces, as it is difficult to maintain concentrated attention. Paces may be counted by means of a pedometer, an instrument which registers the movements of the body made in walking, thus counting the paces.

A man may educate himself to take a pace of even length uphill and downhill, the natural tendency being to take a long pace downhill and a short pace uphill. To maintain, however, uniformity of pace, a man of average height should adopt a pace not exceeding 2 feet 9 inches; and then, with practice, he may maintain this for the whole day both uphill and downhill.

Measuring-wheel.—A measuring-wheel may also be used, with a counter to record the number of revolutions. The wheels of any carriage, whether propelled by steam, horse, or hand-power, or an ordinary bicycle or tricycle fitted with a counter, will do. The circumference of the wheel being known, say 10 feet, the distance traversed will be the number of revolutions multiplied by 10 feet. Of course, this will only give the distance with approximate accuracy, but for many purposes, such as a preliminary geological survey, this accuracy might be quite sufficient. For still less accurate measurements, there are other means, such as the speed of a steamer on a river or lake.

Accuracy of Steel Tape.—For any purposes required by the mining engineer, a steel tape is sufficiently accurate. The expansion of steel between the temperature of freezing and boiling water is rather more than 1 in 1000, say 1·2; and the expansion in length for 1° is about 6·4 parts in a million, and for 50° is about 3·2 parts in 10,000, or, say, one part in 3125. In temperate regions a variation of 50° is as much as is to be expected; in England this is an extreme variation. Suppose the steel tape to be tested and found correct at a temperature of 50°,[1] then for a variation of 10° either higher or lower, the variation would be about 6·4 parts in 100,000, or, more correctly, 1 in 15,625. Where extreme accuracy is required, this correction should be made. To enable it to be done more readily, Mr. W. F. Stanley of London makes a patent band chain handle adjustment, in which, by means of a screw, the chain or band can be lengthened or shortened as desired. A scale on the handle also enables adjustment to be made for variation in temperature during the performance of the work.

A steel tape ·37 inch wide and ·01 inch thick, 66 feet long, when laid out on a pavement, requires a pull of about 4 lbs. to draw it straight over the slight inequalities of the pavement. A total pull of 8 lbs. will stretch it $\frac{1}{16}$ beyond the mark made at the 4-lb. pull. A total pull of 12 lbs. gives a total stretch of a bare eighth; a total pull of 16 lbs. gives a stretch of a good eighth; and a total pull of 20 lbs. stretches the chain $\frac{3}{16}$ beyond the mark made with the 4-lb. pull. The steel tape is not suitable for rough usage, and is therefore only used for the main lines of an important survey, and for those details which it is necessary to mark on the plan with extreme accuracy, or for measuring the base-line of a trigonometrical survey on the surface.

For the ordinary work of a mining survey a strong chain is the best measuring instrument.

Testing a Chain.—Before beginning a survey, and frequently during the survey, it is necessary to test the chain, to see that it is the right length. The importance of this will be understood when the reader considers that if the links of a chain are joined by three rings, then there are eight wearing surfaces for each link, or 800 in a chain-length. If each should wear the $\frac{1}{100}$ part of an inch, this means that the chain is lengthened by 8 inches. For the purpose of testing, a flat piece of pavement or piece of level ground beside a straight wall should be carefully measured with a pole or foot rule, and a chisel-mark put on every tenth link; the chain is then drawn tight over or against these marks. If any section of the chain is too long, it is shortened by taking out a loop; if any section is too short, it is lengthened by putting in a loop; the two ends of the piece of wire forming the loop are not welded together, so that the link can be easily opened with a chisel and closed with a hammer. A few hours' work with the chain over rough ground, where the chain has to be pulled tight to draw it into a straight line, or to set it free from some obstruction against which it has caught, may be sufficient to stretch the chain an inch or more. A carefully tested steel tape is a very convenient instrument for testing the accuracy of a chain in the absence of any more certain fixed measure.

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[1] Messrs. Chesterman claim that their steel tapes are practically accurate at 62° Fahr., and say the expansion is about ·008 inch in 100 feet for each degree.

Method of Chaining.—When measuring on the surface with a chain, the method is as follows: The line to be measured having been marked out with poles, the chain is managed by two men—the leader and the follower. The leader takes one end of the chain, and draws it in the direction of the pole towards which he is steering; the follower holds the other end of the chain at the peg or mark where the line begins. The leader carries ten arrows; these arrows (see Fig. 3) are pins made of iron wire about $\frac{3}{16}$ inch in diameter, pointed at one end, and formed into a ring 2 inches in diameter at the other end, and may be any convenient length from 13 inches to 20 inches; to render them more conspicuous, a piece of coloured ribbon is tied at the top of each. The follower directs the leader to the right or left until the chain is drawn tight in an absolutely straight line for the next pole; the leader then places an arrow at the end of the chain, and lets the chain lie upon the ground until directed to drag it forward. In case there are two marks in the requisite line behind the leader, he can put himself in direction by turning round so as to face the follower, and then moving the chain till he has placed it in a line with the guiding poles or pegs. Whilst the chain is lying on the ground, offsets can be taken to any building or other object to the right or left, or the distance of any fence, ditch, pathway, etc., that is crossed by the chain may be exactly noted. On receiving a signal, the leader drags the chain forward another length, putting a second arrow in the ground. When signalled forward again, the follower takes up

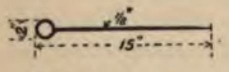


FIG. 3.—Arrow.

the first arrow and advances to the second arrow, and so on; thus the number of chains measured is always the same as the number of arrows in the hands of the follower. When the tenth arrow has been placed in the ground, the leader drags the chain forward and lets it lie upon the ground in its proper position until the follower has picked up the tenth arrow and handed the whole ten to the leader, who must never receive from the follower less than ten arrows. Any breach of this rule will probably lead to confusion. In order to mark the end of the chain when the leader has no arrow in his hand, he must make a mark with a wooden peg. After receiving from the follower the ten arrows, he puts one down beside this peg, thus marking the end of the eleventh chain.

Measuring Rough Ground.—In measuring over hillocky ground and through fences, copses, etc., it is necessary to draw the chain straight between the arrows, otherwise the length will measure greater than it really is. In order to make it straight (that is, nearly straight), it is necessary to pull tight, though violent pulling is unnecessary and injurious.

In measuring up or down a bank, the length of the slope being greater than the horizontal distance, the measured length will be too great for a plan. In order to measure the correct horizontal length for a plan, it is usual, when measuring downhill, for the follower to hold his end of the chain on the ground, and for the leader to fix a pole vertically in the ground at some convenient length, and then to hold the chain on a level with the starting-point against this staff, and read the length; the horizontal distance is thus measured in steps (see Fig. 4). This method is only adopted for very short slopes, or in case the surveyor has no instrument for measuring the inclination.

In the case of a long uniform slope, the length of the slope is measured by drawing the chain straight down it, the angle of the slope is taken with a suitable instrument, and the length of the slope as measured is reduced by calculation to the true horizontal distance before putting the length on the plan.

It is sometimes a good practice to put a wooden peg into the ground at the end of every tenth chain, from which measurements can be taken at some later period of the survey.

Taking a Line through Obstructions.—The measurement of a line is often hindered by some obstruction, such as a stone wall. In this case it is necessary to measure up to the stone wall, which is say 48 links distant from the last peg, the thickness of the wall is found by measuring on to the top to be say 3 links, making the distance through the wall 51 links. The follower then, taking hold of the fifty-first link, holds it against the foot of the wall, and directs the leader as before where to fix the end of the chain.

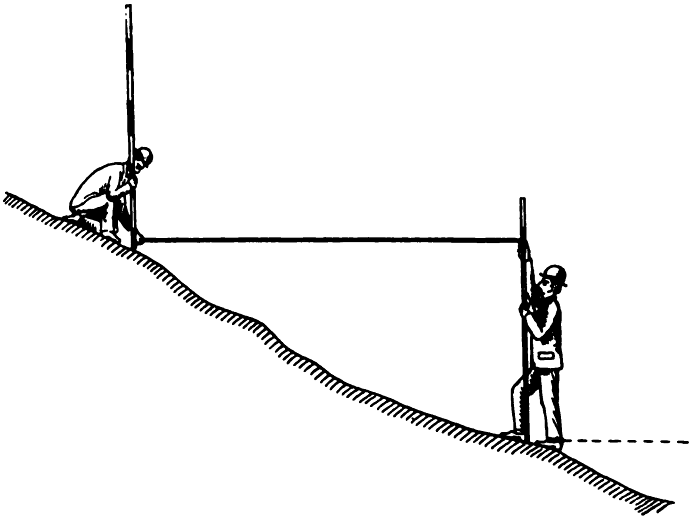


Fig. 4.—Measuring in steps.

In a country containing many trees, it is often difficult to set out a line which may not lead into the trunk of a tree. In such a case there are three courses to be adopted: the first is to cut down the tree; the second is to end the line at the tree; and the third (see Fig. 5) is as follows: Measure at right angles to the line an offset (**A** to **B**) longer than the width of the obstruction, and at 2 chains back measure another offset (**C** to **D**) of equal length, and at 4 chains back a similar offset (**E** to **F**); three poles set up at the end of these offsets will be in

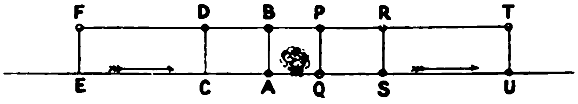


Fig. 5.—Obstruction to survey-line.

a straight line parallel with the main line. This line is then continued for a distance of 6 chains past the obstruction, and three offsets, **PQ**, **RS**, **TU**, set out from this parallel line in the opposite direction to the three original offsets; three poles set up at the end of these three offsets at **Q**, **S**, **U**, will be in a straight line, and a continuation of the original line. The same course may be adopted if the original line runs into a building.

Chaining Underground.—When chaining in the mine, arrows are not usually employed, because the ground is too hard for them to pierce; the end of the chain is usually marked on stone or rail with a piece of chalk, and the number of the chain written by the side of the mark; the leader chalks on a piece of stone the figures 1, 2, 3, 4, 5, etc., up to 10, or marks (*/*, *//*, *///*, *////*, */////*, *//////*, *///////*, etc.), and then begins again. It is, however, seldom that the lines in an underground survey reach a length of 10 chains.

This system of marking the length leads to many errors; the attention of the leader and follower and surveyor may be called off, and the number of chain-lengths forgotten; and it would save many errors in measurement if the system of arrows adopted by surface surveyors was copied in the mine. Instead of an arrow, a simple ring of metal would suffice; the end of the chain would be marked by the chalk as usual, and the ring of metal laid down beside it would form an automatic counter of the number of chain-lengths, the leader starting with 10 rings in his possession, and the follower, taking the rings up, will know the number of chain-lengths by the number of rings he holds. At the end of each line the follower would give up his rings to the leader, who would always start with ten rings. So many of the lines measured in mining surveys are, however, less than 1 chain in length, and the length so seldom exceeds 5 chains, that mining surveyors as a rule have not thought it worth while to adopt such a system; but the writer's experience leads him to think that it would lead to a considerable saving of time. It rarely happens that the end of any line to be measured corresponds exactly with the end of the chain; therefore, except in these rare cases, the chain should be drawn forward past the dial or mark indicating the end of the line, and then the exact distance to the mark read off upon the chain. If this rule is always observed, it will be conducive to accuracy of measurement, as the chain will always be read from the follower's end.

Surveying-poles.—These are used for marking out the line to be measured, and generally vary in length from 10 to 15 links; a 12-link pole is a very convenient length. It is generally made of pine (see Fig. 6), about 1⅛ inch diameter at the base, gradually tapering to ¾ inch at the top; the base is shod with iron, about 9 inches in length, ending in a point; with this iron point a hole can be made, even in hard ground, in which the pole can be fixed.

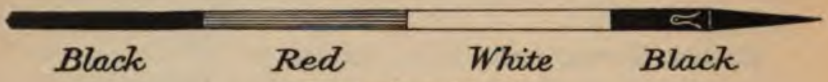


FIG. 6.—Surveying-pole.

It is necessary that the pole should be perfectly vertical, as it frequently happens that only the top of it can be seen over hedges or other obstructions; therefore, if the top is not over the point, the line will not be set out straight. Fig. 7 shows a surveyor in the act of fixing a pole in line with two other poles. The surveyor, desiring to mark out a line, fixes two poles in the desired direction, at a convenient distance apart, say 20 to 50 yards; he then fixes a third pole in the same line at a further distance of say 20 to 50 yards; if these poles are in a straight line, when standing behind one pole at a distance of say 10 yards, and closing one eye, the other two poles should be invisible. A fourth pole is now fixed in the same line. The first pole can now be taken out and placed in advance, forming the fifth mark; then the second can be taken up and placed in front, forming the sixth mark, and so on; by means of these four poles a straight line of any length can be marked out across the country. If three poles are always in the ground, it will be at once evident if one of them has got moved.

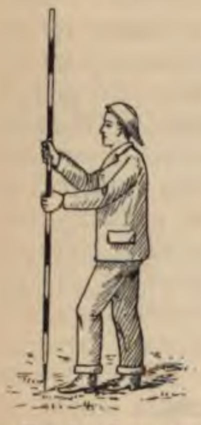


FIG. 7.—Fixing a pole.

In practice a good deal of care is required to keep the line quite straight, as it is not always easy to fix the poles perfectly plumb, or they may be blown on one side by the wind, or may be inaccurately fixed to the extent of half an inch. If the third pole is 60 yards in advance of the first pole, and half an inch out of its correct position, that is a deflection of 1 in 60 × 36 × 2, or 1 in 4320. This deflection in a small survey might not be very serious, but the deflected line may be deflected still further in the same direction, and the error of 1 in 4000 may soon be increased to 1 in 1000.

For setting out long and important lines, the eye of the surveyor is often assisted by the telescope mounted on a theodolite stand. With a good instrument and great care almost perfect accuracy may be maintained in poling out a line. Sometimes small flags about a foot square are fastened to the top of the poles to make them more conspicuous. The poles are all painted black, red, and white in alternate lengths of 1 link (or 1 foot), so as to make them more easily visible, and this also fits them for use as measuring-poles.

For special purposes, as, for instance, for use in a large trigonometrical survey, poles of extra length and strength are used; these are maintained in a vertical position by means of guy ropes (see Fig. 8) fastened to pegs in the ground, or to weights. Sometimes a pole is fixed on a wooden frame.

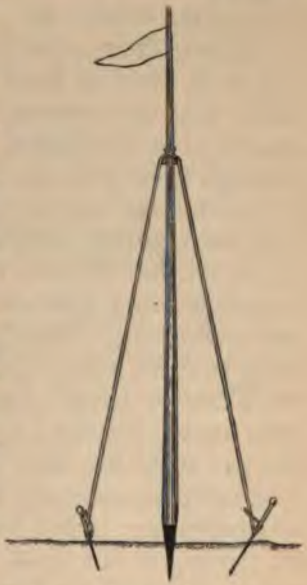


FIG. 8.—Pole fixed by guy ropes.

It happens very frequently that it is necessary to range a straight line between two fixed points, neither of which is visible from the other, or, if visible one from the other, it is inconvenient to go to either of them so as to range out the line from the beginning; but whilst one of these points is invisible from the other, they are both visible from an elevated piece of ground between them. The surveyor and his assistant proceed to this intermediate position, and, each holding a pole and standing about 50 yards apart, face each other, placing themselves as nearly as they can guess in a line between the two fixed points, **A** and **B** (Fig. 9). The surveyor at **D′**, looking towards **A**, motions the assistant at **C′** into the line **AC′D′**; his assistant at **C′** looking towards **B**, motions the surveyor into the line **BD″C′**. As the surveyor is moved towards the line **BDC**, the assistant has to be moved at the same time towards the line **ACD**, and this movement is continued until the two lines exactly coincide, then **ACDB** form one straight line. With a little practice this operation can be performed in two or three minutes.

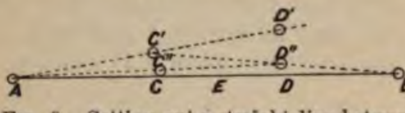


FIG. 9.—Setting out a straight line between two points not visible from each other.

Where great accuracy is required, a theodolite may be used to check the positions **C** and **D**, first erecting it at **D** to ascertain if **C** is in the straight line **ACD**, and then erecting it at **C** to ascertain if **D** is in the straight line **BDC**; a central position **E** may be marked out with a peg, and a centre line accurately fixed; a transit theodolite may then be fixed over this and directed towards **A**; when the telescope is reversed the cross-hairs should be upon the station **B**.

Although four poles are sufficient with which to mark out a line, it is usual to have more, perhaps seven or eight, in one line. With six or seven poles standing in a line there is less chance of a divergence from the original direction, because although three poles are sufficient if there are no accidents, still if two of these should be accidentally knocked a little on one side, the direction would be lost. As each pole is pulled up, a peg is put into the hole to mark the place, so that the line may be easily found another day. The kind of peg that is used varies according to the circumstances of the case; sometimes a small twig cut from a hedge is the best kind of mark, as it is not likely to attract attention; on other occasions a piece of wood about 18 inches long and 1½ inch square, pointed at one end and flat at the top, may be driven in. For a permanent station it is, however, necessary to have a stake which cannot be easily withdrawn, say 3 feet long and 4 inches square, driven down till the top is but little above the ground, with a cross-mark nicked in the top to show the line of survey. Pegs of this kind, however, should not be put down in a place where they will interfere with agricultural work, such as mowing-machines, but should be placed by the side of a hedge or ditch, where they will be no impediment and attract no notice.

TABLE SHOWING THE EQUIVALENT VALUES OF VARIOUS MEASUREMENTS.

LINEAL MEASURE (LENGTH).

Mile.	Chains.	Yards.	Feet.	Links.	Inches.
1	80	1760	5280	8000	63,360
·0125	1	22	66	100	792
·000568	·04545	1	3	4·545	36
·000189	·01515	·333	1	1·515	12
·000125	·01	·22	·66	1	7·92
·0000158	·00126	·0278	·0833	·126	1

SQUARE MEASURE (AREA).

Acres.	Roods.	Perches.	Sq. yards.	Sq. feet.	Sq. inches.
1	4	160	4840	43,560	6,272,640
·25	1	40	1210	10,890	1,568,160
·00625	·025	1	30¼	272¼	39,204
·0002066	·000826	·0331	1	9	1,296
·000023	·0000918	·00367	·111	1	144
·000000159	·00000064	·0000255	·000772	·00694	1

1 square mile = { 27,878,400 sq. feet.
3,097,600 sq. yards.
640 acres.

Acres × ·0015625 = sq. miles.
Sq. yards × ·000000323 = sq. miles.
10 sq. chains = 100000 sq. links = 1 acre.
46,656 cub. inches = 27 cub. feet = 1 cub. yard.

NOTE.—The above tables will be found to comprise many of the data needed by the surveyor. To use the tables: Suppose it is required to convert yards into links. On referring to the table we find 1 yard is equal to 4·545 links, so by multiplying yards by 4·545 we get the equivalent distance in links. Other units of measurement may be converted in a similar manner.

CHAPTER III.

METHOD OF SURVEYING ON THE SURFACE BY MEANS OF CHAIN AND POLES.

For the purpose of making a survey on the surface of an estate of moderate size, say 1000 acres, it is not necessary to have expensive instruments. A score of straight poles, a good Gunter's chain, ten arrows, an off-set staff or tape, and some pegs to mark the stations, are all the instruments required; a compass and theodolite may be very useful and advantageous, but they are not absolutely necessary.

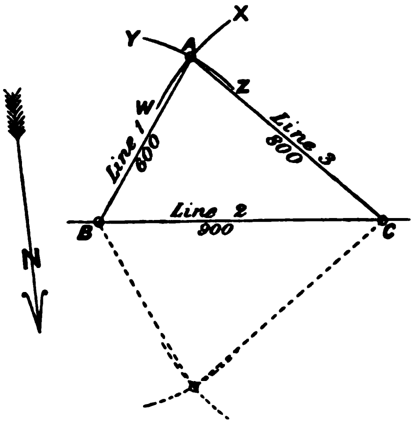


Fig. 10.—Simple surface survey.

The method of making a survey with chain and poles may be explained in the following manner: Let Fig. 10 be the plan of an estate on level ground, of triangular form **ABC**. From the point **A**, **B** is visible; fix a pole at **A** and another at **B**, and measure the distance **A** to **B**. From the point **B**, **C** is visible; fix a pole at **C**, and measure the distance **B** to **C**. From the point **C**, **A** is visible; measure the distance **C** to **A**. The measurements are entered in a book, thus—

Line No. 1	...	**A** to **B**,	length	600	links
„ No. 2	...	**B** to **C**,	„	900	„
„ No. 3	...	**C** to **A**,	„	800	„

The survey is now complete, and, if the lines have been measured accurately, these three measurements are sufficient for the production of an accurate plan.

It will be seen that **A**, **B**, and **C** are angles, and that the figure measured is a triangle (from Lat. *tres, tria*, three, and *angulus*, an angle, meaning a figure with three angles). The length of each side has been measured, but not the angles; so that if only two of the sides had been measured the lines could not be drawn on paper in their correct position as regards each other. Having got the lengths of the three sides, however, they can be plotted with the aid of an elementary knowledge of geometry.

Plotting a Triangular Survey.—The method is explained by reference to Fig. 10. The line No. 2, being the longest, is drawn on the paper, and the length marked by means of the scale (Fig. 54). The scale is 12 inches in length; each inch represents a length of one chain, or 100 links. The scale may be made of cardboard, boxwood, ivory, or metal. It is generally made of boxwood or ivory, the length of which is not appreciably affected by temperature; each inch is divided into ten parts, each part representing 10 links. A needle-pricker is used to prick out the ends of the line, the prick-hole being surrounded with a little ring sketched with the pencil. The compasses (Fig. 55) are now opened, and by means of the scale set to the length of line No. 3. (800), and with their aid the arc **WX** is drawn from the point **C**. The compasses are now adjusted to the length of line No. 1 (600), and from the point **B** the arc **YZ** is drawn; this intersects the arc **WX** at the point **A**. By means of a straight-edge the lines **CA** and **AB** are now drawn, and the plan is complete.

To test the accuracy of the drawing, the scale should be laid on the line **AB**, which should measure 600, and on the line **AC**, which should measure 800. If the lines, when measured, are found incorrect, it shows that the compasses have not been set to the right length.

Booking a Surface Survey.—If the survey is plotted by the person who measured it on the ground, there is no difficulty; if, however, it is measured by one person and plotted by another, the notes of the survey as above given are not sufficient, because the point **A**, instead of being as shown, might be on the other side of the line **BC**, as shown by the dotted lines. It is therefore necessary that the surveyor should make some sign in his note-book of the direction in which he turns, and the way of booking the above survey would be as shown in Fig. 11. In this the angles are sketched by the surveyor looking the way he is going. The longer side of the angle represents the direction in which he is going, the shorter side represents the direction of the line which he is leaving, and, in plotting the lines **AB** and **CA** with the compasses, they must be drawn on that side of the base-line which will give the angles corresponding to those sketched in the field-book. If the bearing of each line is noted, this would do instead of sketching the angle.

For the purpose of plotting lines 1 and 3 on the proper side of the base-line No. 2, it is not necessary that the bearing noted should be accurately observed; it would be sufficiently near if the note is made north-west or north-east, south-west or south-east, as the case may be. This approximate observation of the bearing can, of course, be made immediately by a glance towards the sun if it is shining, or, failing that, by the aid of a magnetic pocket-compass. *The bearing noted in the pocket-book is that of the point towards which the surveyor is moving when measuring the line.*

The survey above described is the simplest possible kind of land survey, and at the same time it is a type of every species of surface survey. No piece of ground can be enclosed in less than three straight lines. No line can be measured on a curve; therefore every piece of ground must be measured by straight lines.

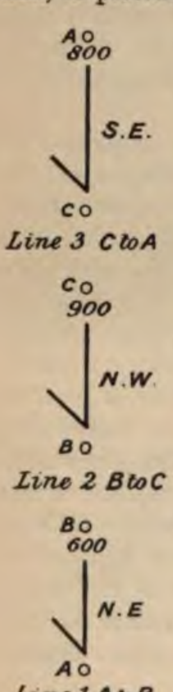


FIG. 11.—Surveyor's notes of Fig. 10.

Solution of Triangles.—Plane triangles are composed of six parts, namely, three angles and three sides, and when three parts (*one of which must be a side*) are given, the other parts can be determined. There are four cases: (1) Measuring the three sides; (2) measuring two sides and the angle enclosed; (3) measuring one side and the angles at each end; (4) measuring two sides and the angle opposite one of them.

In the system of surveying now under consideration, the surveyor has no instruments for measuring angles, and therefore the first method—that of measuring the three sides—is the only one open to him, and every part of the estate he measures must be divided into triangles, of which each side is measured, and, if it is carefully done, a plan may be made of almost perfect accuracy.

It is, however, inevitable that mistakes should be occasionally made in the following ways: (1) by the lines of poles not being set out perfectly straight; (2) by the measurements made with the chain having some accidental error; (3) by accidental mistakes in the position of pegs or other marks, and in booking or plotting the survey.

Tie-lines.—In order to detect such mistakes, it is necessary to measure tie-lines. With a proper system of tie-lines it is impossible for any error to escape detection (except where details are filled in by unchecked offsets). Referring to Fig. 12, the triangle **ABC** is the plan of an estate, the sides of which have been measured and found to be as follows: **AB**, 550; **BC**, 620; and **AC**, 600. A tie-line is then run from **A** to **D**, measuring 485. The length **BD** is also measured, 298. By this means the large triangle **ABC** is divided up into two smaller triangles, **ABD** and **ACD**. In plotting the survey, the large triangle **ABC** is first plotted, then the distance **BD** is marked off on the line **BC**; the distance **AD** should then measure 485. If it does not measure exactly 485, it shows that there is some mistake in the measurements, and they must be measured over again until the error is discovered. If, on the other hand, the line **AD** does measure exactly 485, it is strong evidence that the survey has been accurately made.

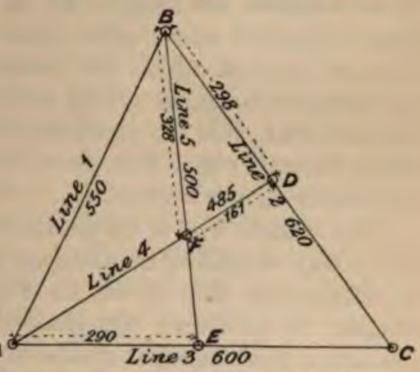


FIG. 12.—Surface survey, showing the use of tie-lines.

It is, however, just within the bounds of possibility that, by a combination of errors of measurement, a plan might be produced that was not correct, notwithstanding the tie made by the line **AD**. Such a combination of errors is in the highest degree improbable; but if absolute proof is required, another tie-line must be measured. It is also better to have two tie-lines for another reason: in case an error should have been made, it is useful to know which of the measurements is most probably wrong; and the more tie-lines that are made, the more quickly and certainly can the exact position of the mistake be detected. To make a second tie-line, measure on the line **AC** the length **A** to **E**, 290; then measure the length **B** to **E**, 500. When the lines 1, 2, 3, and 4 have been plotted, then on the line **AC** measure the length **A** to **E**. If the distance **BE** measures 500 on the plan, it is conclusive evidence that the survey and plan are correct. There is, however, other evidence. The lines **AD** and **BE** intersect at the point **F**, and the distances **BF** 328 and **DF** 161 should be noted in the survey-book. By subtraction, the lengths **EF** and **AF** are then known. The large triangle is now divided into seven triangles and one trapezium, of each of which the measurements are known. If all the measurements, as scaled on the plan, agree with those in the note-book, it is incontestable evidence that the plan is absolutely accurate; and this proof of accuracy is obtained merely by measuring two tie-lines.

The student who is unfamiliar with the practical difficulties in the way of making an accurate plan may possibly be inclined to think that the sketch in Fig. 12 shows an unnecessary number of tie-lines; but that is not the case. In ordinary practice a good surveyor makes so many tie-lines that the possibility of accidental error is entirely eliminated; and if the plan is wrong, the fact that an error existed *somewhere* must have been discovered when plotting. Most of these tie-lines are measured, not merely as tie-lines, but as lines of survey necessary for the location of fences, buildings, rivers, pits, or other objects, the position of which must be correctly marked upon the plan.

Offsets.—In the preceding examples it has been assumed that the boundary-lines to the estate, in the form of a triangle, were perfectly straight lines, and coincided with the lines measured. In practice, however, it seldom happens that a fence or wall continues straight for any considerable distance, and, in order to survey this fence or wall, it is necessary to set out and measure a straight line beside it; this line proceeds to the end of the fence, or until it turns away in another direction. For the second fence, or for the altered direction of the first fence, another line has to be set out and measured; this is shown in Fig. 13, which is a copy of a portion of a field note-book. The thick lines here indicate the fences, and the thin lines those which are marked out by the surveyor's poles. Line 1 is shown 980 links long. At the beginning it is seen that the fence is 6 links to the left hand; at 400 links along the line the fence is 7 links to the left hand, and it is observed that between these two points the fence is perfectly straight. These two measurements to the left, 6 and 7 links respectively, are called "offsets." An offset is always measured at right angles to the survey-line from which it starts, and in order that they may be correctly measured, it is essential that the surveyor should be able to mark out a line at right angles by pointing a pole from the survey-line to that part of the fence which is at right angles to the place where he is holding the pole. If the offsets are merely intended to show gentle curves in a hedge, it will not be of any appreciable importance whether the line of the offset is measured at an angle of 70° or 90° from the survey-line. If, however, the offset is intended to denote the correct position of the corner of some building or other landmark, it is, of course, of the utmost importance that it should be correctly set out at an angle of 90°.

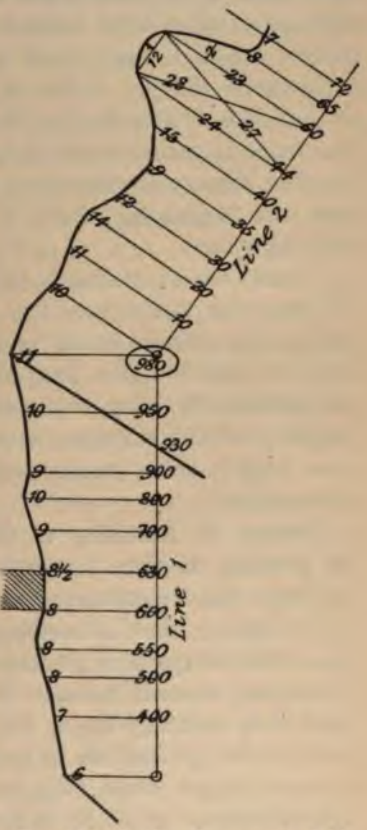


FIG. 13.—Portion of surveyor's field note-book.

A competent surveyor cannot fail to mark out a short offset, not exceeding, say 15 links, with sufficient accuracy for the ordinary landmarks in a survey. Should the offset, however, be longer, every important mark should be fixed by triangulation.

On referring to Fig. 13, it will be seen that the fence ceases to be perfectly straight beyond 400 links, therefore offsets have to be taken at 500 and at 550 links; at 600 links the line passes a building, the end of which forms a portion of the boundary. The position of this is ascertained by the offsets. At 930 the line crosses another fence; at 950 there is an offset of 10 links to the left to the first fence; at 980 the line ends; there is an offset of 11 links to the first fence and to the fence corner. No. 1 line comes to an end because the boundary fence turns sharply to the right. It is therefore necessary to set out No. 2 line in the direction the boundary now takes. The fence is rather crooked, and the offsets are therefore closer together than was necessary on No. 1 line; and at 40 links from the beginning of the line the fence bends to the left, forming a bay.

Unless the exact line of this fence is a matter of importance, the surveyor would take the inner points of the bay by measuring the offsets at 44 and 60 links, and these offsets are only 24 and 23 links long respectively; and, indeed, it is not an uncommon thing to measure an offset 40 or 50 links in length; but the surveyor cannot expect to set out an offset of that length with precise accuracy without the aid of some instrument.

Degree of Accuracy of Offset depends on Scale of Plan.—In plotting a plan, however, on a scale of say 2 chains to an inch, the thickness of an offset line represents a distance of 2 links; and therefore precise accuracy in measuring the exact shape and position of every little twist in a hedge would be wasted, because it would be impossible to reproduce this accuracy on a 2-chain plan. If, however, the plan were to be plotted on a larger scale, say 2 inches to 1 chain, or on a larger scale still, as for building purposes, then the measurements must be taken with extreme accuracy, because any defect will appear on the plan. For such a purpose the position of the corners of the bay must be accurately found out by triangulation, and the tie-lines shown in the figure; 27 and 28 links long respectively must be measured (for such measurements a tape is generally used), and from the main offsets branch offsets are measured, as shown by the figures 1 and 2, so that the exact shape of the fence may be ascertained.

Note-book not to Scale.—It will be observed that in Fig. 13 the line 2 is only shown for a length of 72 links, and line 1, 980 links long, occupies little more space on the paper. That is because the figure does not represent a plotted plan, but a sketch in the surveyor's book. It is necessary that every measurement should be clearly shown; thus where the offsets are few, the measurement on a line of great length only occupies a small space in the field-book; on the other hand, where the offsets are many, a line of short length may occupy several pages.

It will also be observed that the number of offsets to be taken will not only depend on the nature of the fences, but on the scale of the plan on which they are to be marked.

Survey-lines.—The student will now understand that one of the chief purposes of a survey-line is that of a base from which to measure offsets to the fences, buildings, etc., and that this base must be sufficiently near to the objects which he desires to put upon the plan to enable him to set out right-angle offsets to them by the eye. This being so, his knowledge of the system by which the land in Great Britain and Ireland is divided into small fields by hedges and walls, which are generally crooked, will suggest to him that, as a general rule, it will be necessary to run a survey-line by the side of every fence, whether this fence be merely a division betweeen two fields or the boundary of a road, railway, or estate. It is also necessary to run a line past every building, and frequently two, three, or four lines have to be run to fix with the necessary precision and certainty the position, shape, and dimensions of some building of only moderate importance.

How to survey an Estate.—Fig. 14 shows the plan of a small estate divided into ten fields by walls, hedges, and fences; it contains two pit-shafts, colliery buildings, a chapel, and a school. The surveyor is instructed to make an accurate plan of the surface. The owner of the property or his agent points out to him the boundary-lines, indicating in each case whether his boundary is the centre or side of a ditch, or the centre of a hedge, or the side of a wall. The surveyor makes a rapid hand-sketch, similar to the plan in Fig. 14 (but necessarily rough and disproportionate), and is now in a position to begin the survey. His first step will be to mark on the sketch the lines which he intends to measure. The longest line which

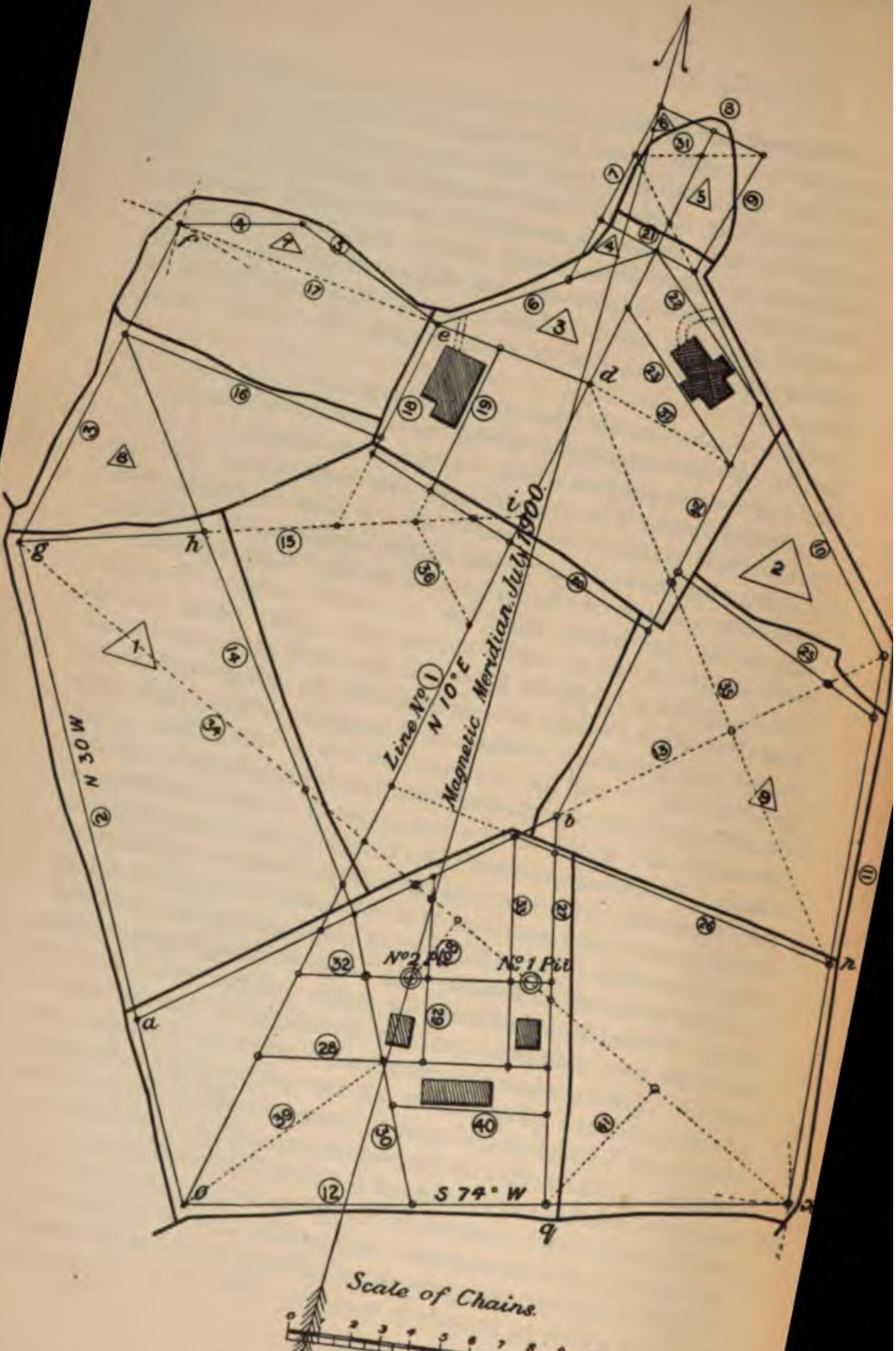


FIG. 14.—Plan of a small estate.

can be measured forms, as a rule, the best base-line; this, he finds, is from south-west to north-east, and is marked No. 1 on the plan; No. 2 line is set out beside a boundary fence, also Nos. 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. These twelve lines suffice for the delineation of the boundaries, but additional lines are necessary for the internal fences and buildings, and lines 13 to 33 are set out. These lines are set out, not with special regard to the construction of triangles, but to their convenience as lines from which to take offsets to the fences, etc. It is, however, found that a complete system of triangulation is made if some of the internal lines, as first sketched out, are prolonged a little. Thus line 13, starting at the point *a*, and proceeding to *b*, may be prolonged as indicated by the dotted line to the point *c*; line 17, beginning at the point *d* and proceeding to *e*, may be prolonged, as shown by the dotted line, to the point *f*; line 15, beginning at *g* and ending at *h*, is prolonged to *i*; lines 10, 18, 19, and 26 are also prolonged, as shown by their dotted extensions. If all these lines are correctly measured, an accurate plan of them may now be produced, and, by means of the offsets, the fences and buildings may be correctly put down. Out of all the 33 lines above mentioned, No. 1 is the only one which has not been set out along a fence, and which serves no other purposes than those of a base-line and a tie-line.

Estate divided into Triangles.—At first sight the student will fail to see that most of the estate has been divided into triangles, but on careful examination he will detect a good many, which are marked on the plan $\triangle_1$, $\triangle_2$, $\triangle_3$, etc.; there are, in fact, nine triangles, of which, however, only Nos. 1 and 2 are of large dimensions.

Hypothetical Triangles.—In addition, however, to these nine triangles that have really been laid out across the fields, there are a number of other triangles which may be legitimately completed by a hypothetical line which can be measured off the plan formed by plotting the former triangles. Thus take the corner *x* formed by the junction of lines 11 and 12; it is the apex of a hypothetical triangle, the other two corners of which are at *o* and *c*. *o* is the starting-point of all the measurements, and is therefore the point first marked upon the plan; *c* is fixed on the plan as the apex of the triangle $\triangle_2$, formed of portions of lines 1, 10, and 13. These two points being fixed, it is easy with a scale to measure the distance *o* to *c*, which is the base of the hypothetical triangle of which *x* is the apex. In order to put *x* on the plan, the compasses would be set to the length *ox* (line 12), and from the centre *o* an arc would be described; the compasses would then be set to the length *cx* (line 11), and another arc described, intersecting the previous arc at the point *x*.

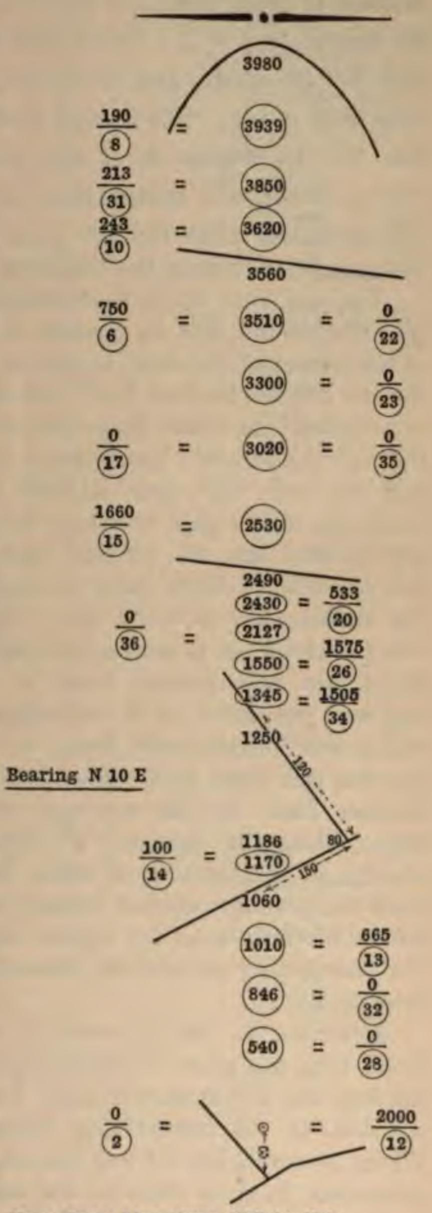
To take another instance, point *f* is the apex of a hypothetical triangle of which the other corners are at *g* and *d*; the point *d* is on the base-line; the point *g* has been fixed on the plan by means of the triangle ⚠, formed by the lines 1, 2, and 15. With the compasses set to the length *gf* (line 3), and from the centre *g*, an arc is described; then with the compasses set to the length *df* (line 17), and from the centre *d*, another arc is described, cutting the previous arc at the point *f*. In this way the hypothetical triangle *gdf* is completed. In a similar manner smaller hypothetical triangles at the north-eastern corner of the estate may be set out.

The whole of the boundary-lines are now fixed by these triangles, and the accuracy of the work is checked by the internal lines, which can now be drawn in their right places without any further use of the compasses.

Tie-lines.—It will, however, be advisable to run some other lines across the estate, merely as check-lines; thus, from *g* to *x* a tie-line (No. 34) might be run passing through the No. 1 pit; this not only gives additional certainty to the points *g* and *x*, but to the position of the pit, which is of the utmost importance in a mineral plan. The advantages of this tie-line do not end there, the point *x* being fixed by it independently of point *c*; the position of point *c* can be fixed from the point *x*, and its accuracy thereby confirmed. Other tie-lines should be measured; for instance, on line 11 the position of the fence corner *p* is only fixed by the measurements at the meeting of lines 26 and 11. On line 12 the position of the fence corner *q* is only fixed by the measurements at the intersection of line 27, and therefore neither of these points has been ascertained with absolute certainty. A tie-line may therefore be run from *d* to *p* (line 35); this gives additional security, not merely to the point *p*, but to line 24 and to point *d* itself. A short tie-line (No. 41) from *q* to the line 34 will check that position. Other short tie-lines may also be measured, as 36 and 37 near the school-house and chapel, 38 to fix the position of No. 2 pit, 39 near the southern boundary, and 40 at the back of the colliery workshops. By these forty-one lines and their offsets the survey is completed, and it is impossible that an important error can escape detection. The figure now shows a total of thirty-four triangles.

Intersection of Lines.—It is in the highest degree important that the crossing or intersection of two lines should be carefully noted, and the careful performance of this work is characteristic of a good surveyor; it frequently involves a good deal of trouble, and therefore it is frequently neglected by the hasty or careless surveyor. The base-line is touched and crossed by the following lines: Nos. 2, 12, 39, 28, 32, 13, 14, 34, 26, 36, 20, 15, 17, 35, 37, 23, 22, 6, 21, 10, 31, and 8. It is not sufficient, however, to mark on the base-line the distance at which these lines cross or intersect; but it must also be noted at what length on each of the crossing or touching lines the intersection or meeting takes place. This is shown on Fig. 15, which is copied from the surveyor's notebook.

Entry of Intersections and



No. 1 Base Line from S.W. to N.E.

FIG. 15.—Surveyor's field notes, from which Fig. 14 has been plotted.

Stations in Note-book.—In this note-book the student will read the figures (540) $= \frac{0}{28}$; this means that at 540 on the base-line, line No. 28 starts, and is on the *right hand* of the base-line. Also (1010) $= \frac{665}{13}$; this means that at 1010 on the base-line, line No. 13 crosses it at 665 links from its starting-point. $\frac{1660}{15} =$ (2530); this means that at 2530 on the base-line, line No. 15, which is on the *left hand* of the base, reaches it at a distance of 1660 from the beginning of line 15.

The peg from which the measurements start is driven firmly into the ground, and its position is also fixed by a measurement to the corner of the field, shown in Fig. 15. The surveyor notes that he intends to start No. 2 line on the left-hand side from the same point; he notes that another line will join this peg from the right-hand side; but he has not yet given the line a number, and the note $\frac{2000}{12}$ (that is, 2000 in line No. 12) on the right-hand side of the page has been added at a later period of the survey after line No. 12 had been set out and measured, and was found to be 2000 links in length from its beginning up to the starting-peg of No. 1 line, where it ended. At 540 a peg was put down as a convenient place for starting another line, which was subsequently found to be No. 28; at 846 a second peg was put down as a convenient place for starting a line, which was subsequently found to be No. 32; at 1010 a third peg was put down as a convenient place for the intersection of another line. As the surveyor measures the line, he leaves pegs (noting the distance) at suitable places for the starting, ending, or intersection of other lines; the numbers of these lines he, perhaps, cannot foresee at the time. Space must be left in the note-book for figures which have to be added after the intersecting or touching lines to which they refer have been measured.

Referring to the intersection of line 34, it must be understood that the place of intersection, 1345, was not noted when the line was first measured, and for this reason—that the exact position of the intersecting tie-line could not be foreseen. When, however, line 34 was run and came across the base-line, poles were fixed in some of the stations previously left on the line, so that the exact point of the intersection of the two lines could be established; then a measurement was taken from a station previously measured on the base-line, as, for instance,

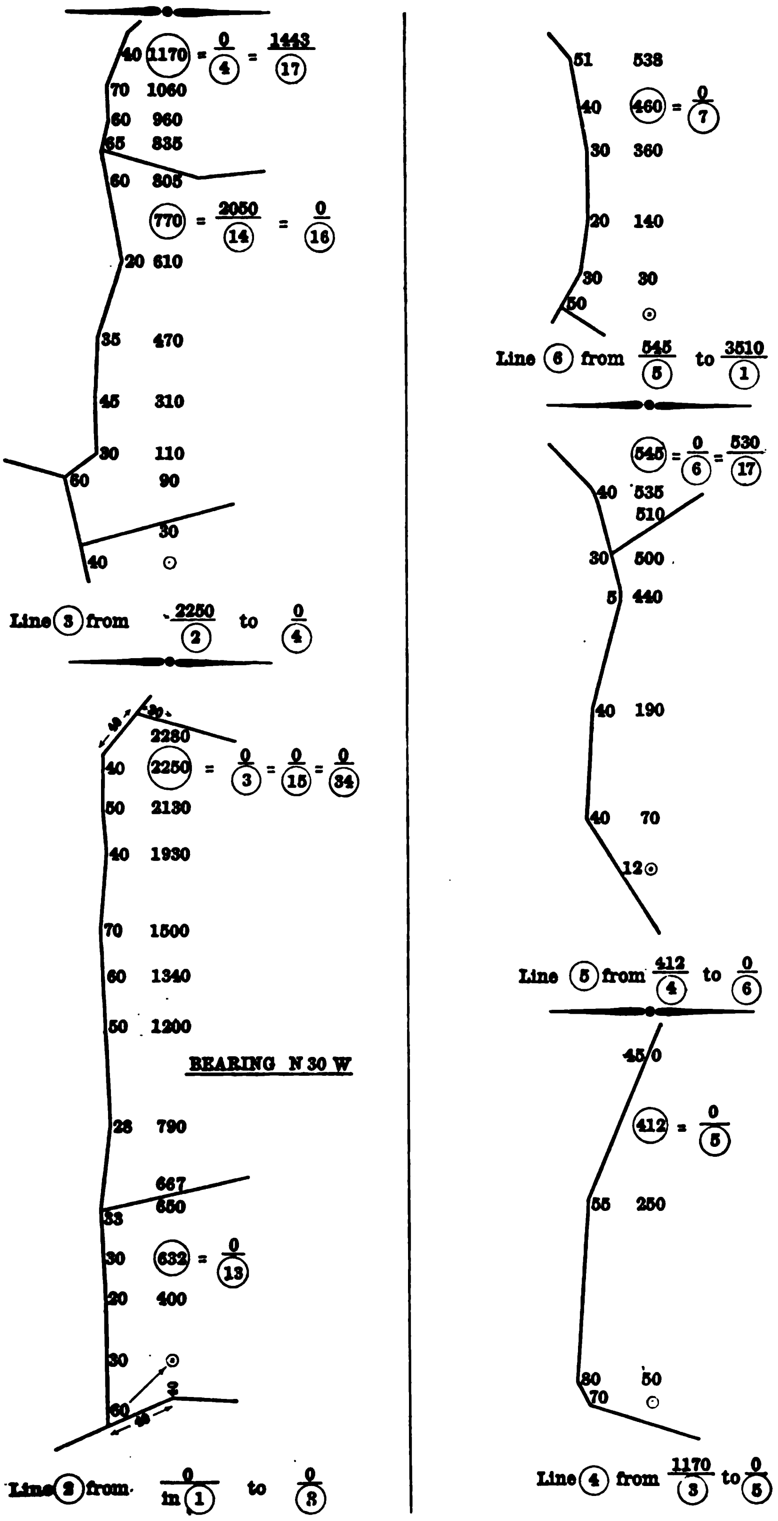
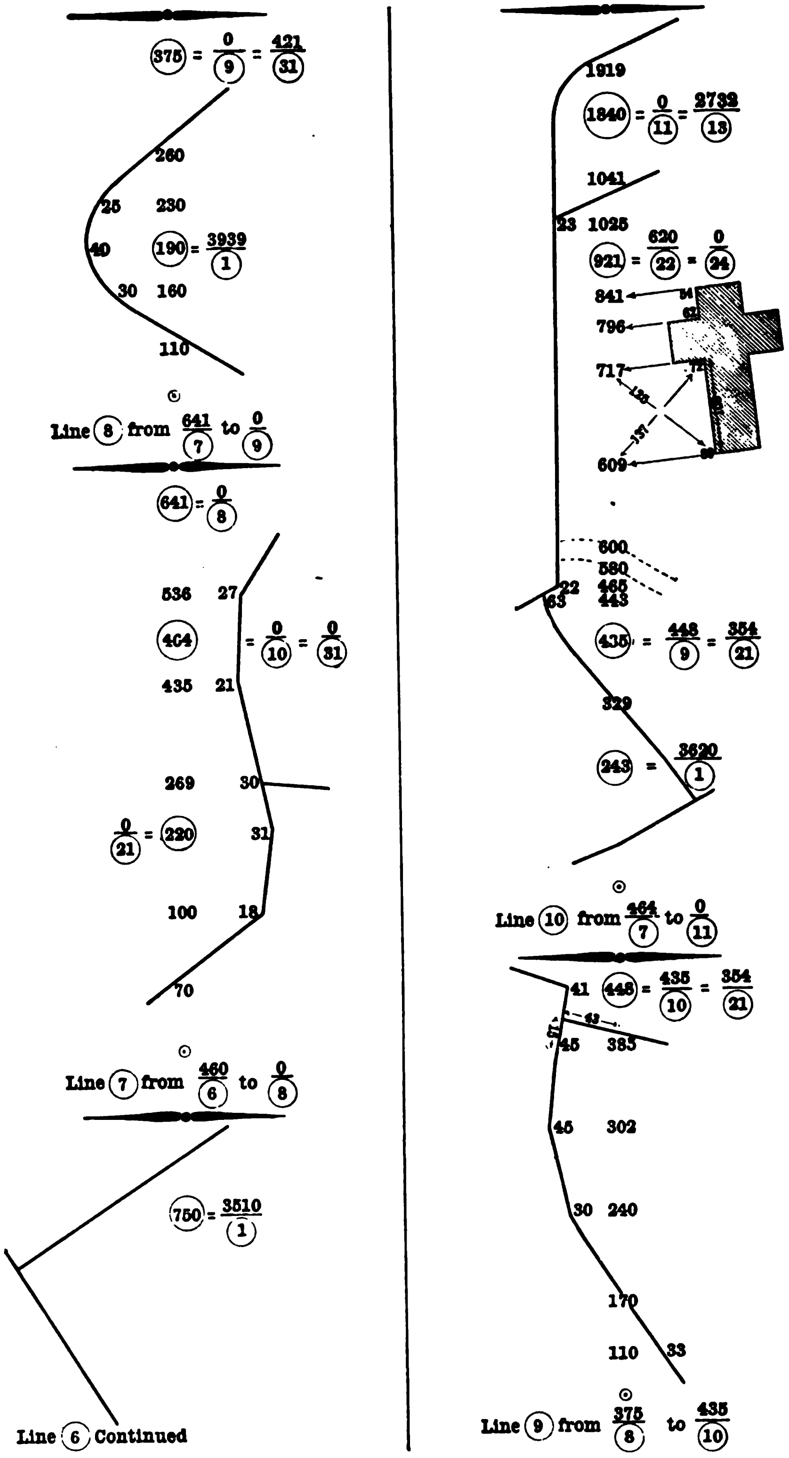


Fig. 16 (1).—Surveyor's field notes, from which Fig. 14 has been plotted. Lines 2 to 41.

FIG. 16 (2).—*continued.*



Line ⑧ from 641/⑦ to 0/⑨

Line ⑦ from 460/⑥ to 0/⑧

Line ⑥ Continued

Line ⑩ from 464/⑦ to 0/⑪

Line ⑨ from 375/⑧ to 435/⑩

Fig. 16 (3).—*continued.*

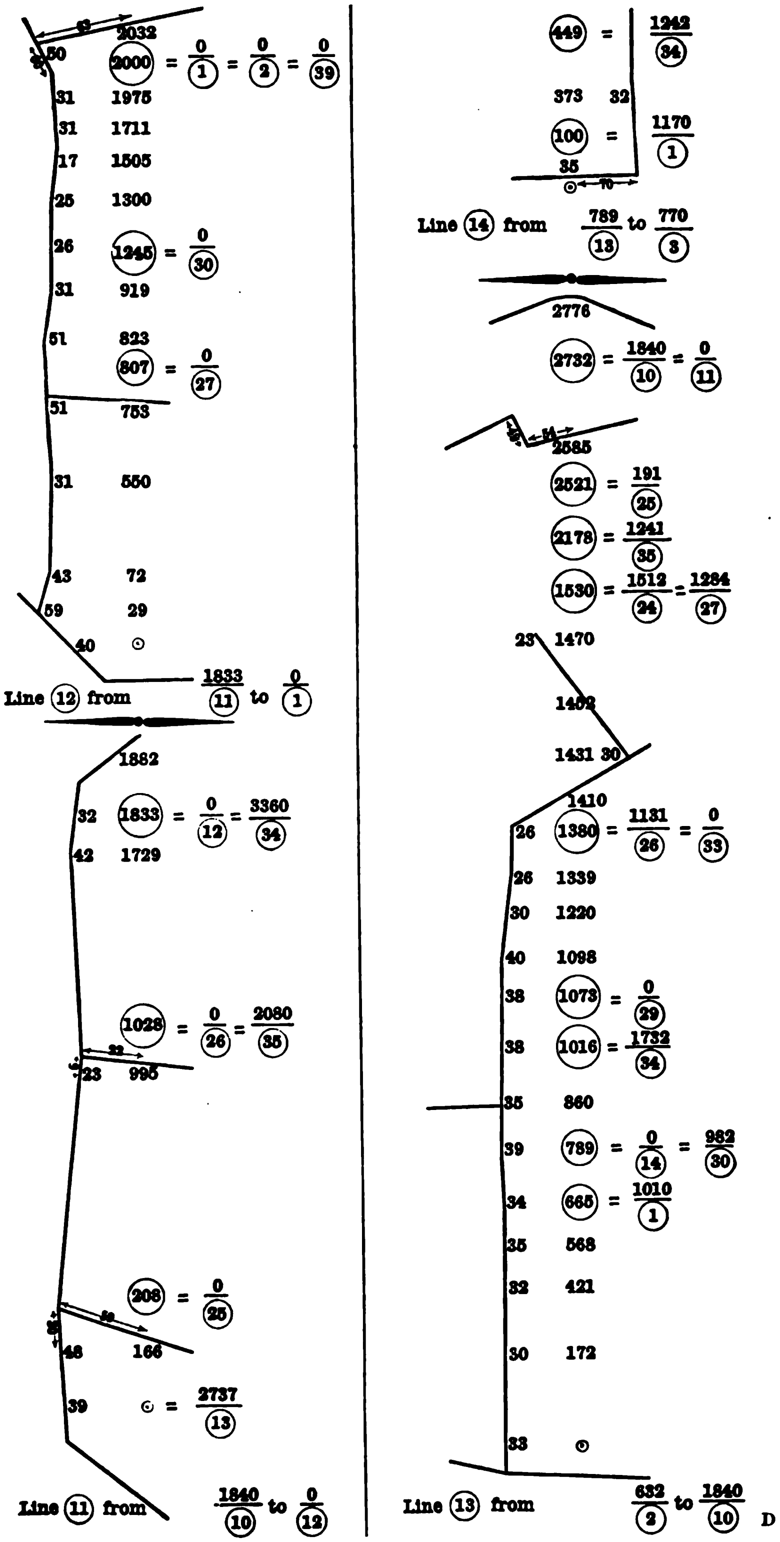
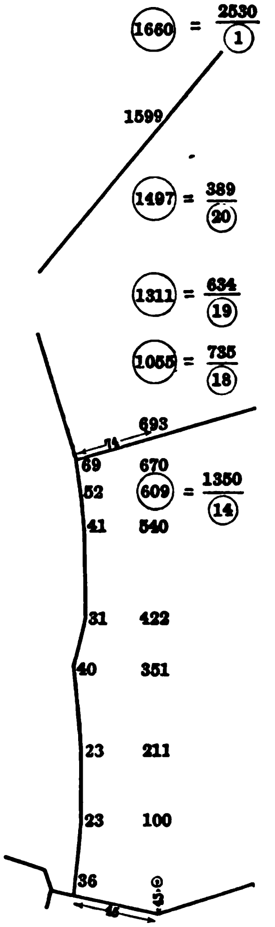
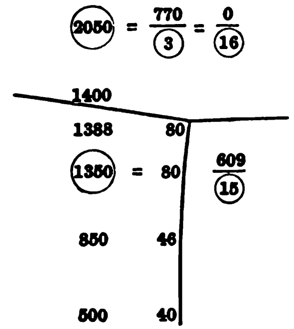


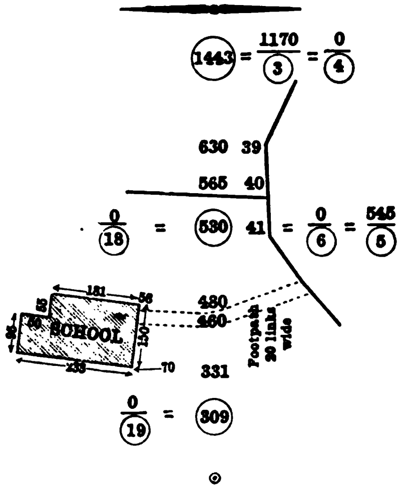
Fig. 16 (4).—*continued.*



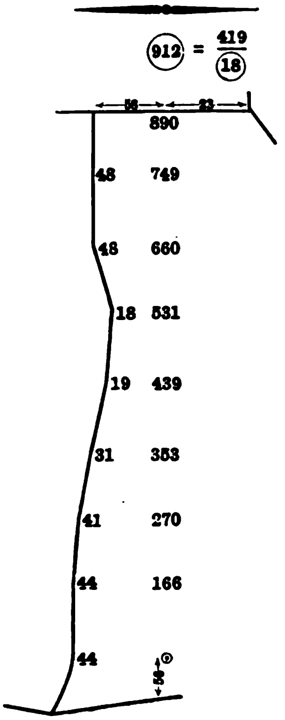
Line ⑮ from $\frac{2250}{②}$ to $\frac{2530}{①}$



Line ⑭ Continued



Line ⑰ from $\frac{3020}{①}$ to $\frac{1170}{③}$



Line ⑯ from $\frac{770}{③}$ to $\frac{419}{⑱}$

FIG. 16 (5).—*continued.*

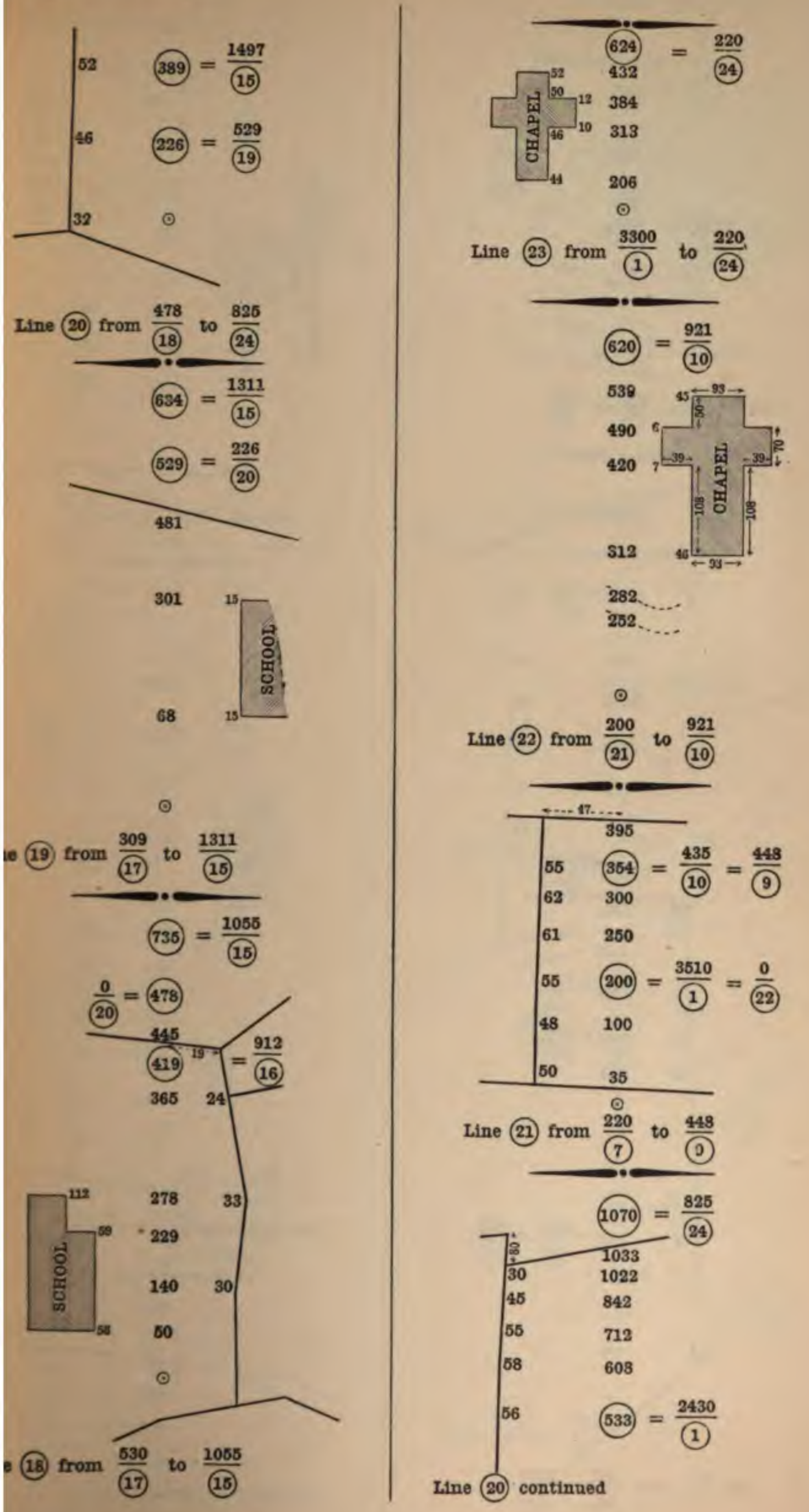
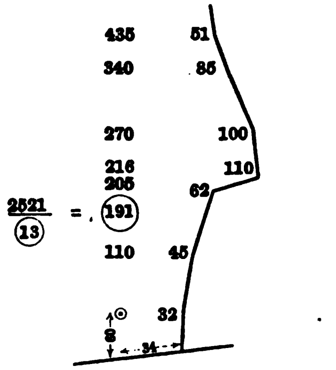
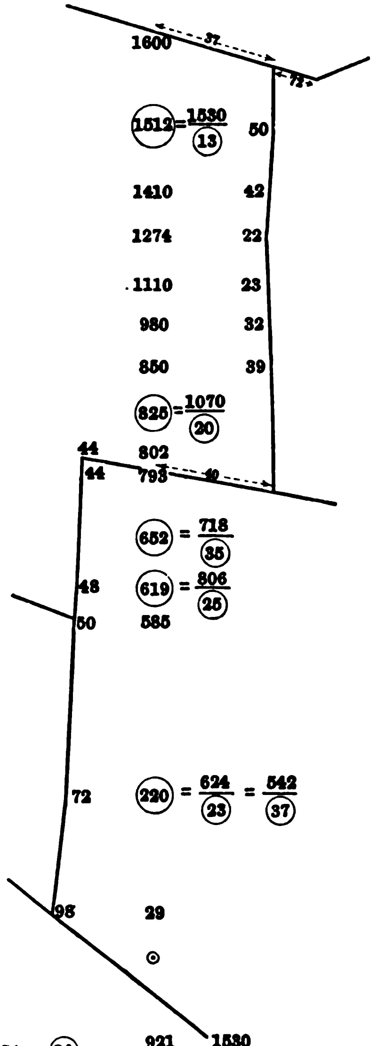


FIG. 16 (6).—*continued.*

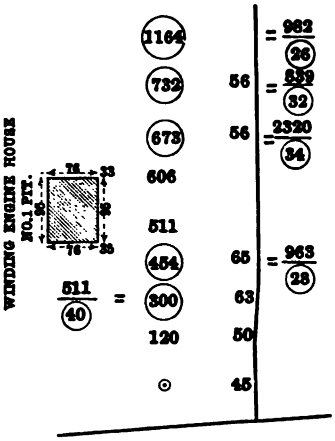


Line ㉕ from 208/⑪ to 619/㉔

* * *

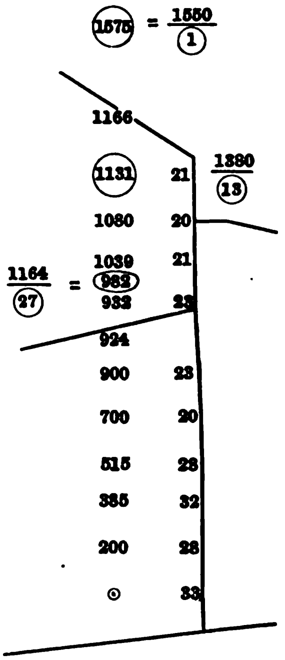


Line ㉔ from 921/⑩ to 1530/⑬



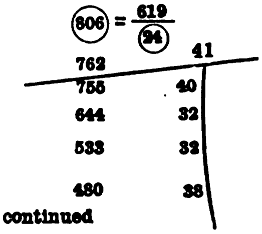
Line ㉗ from 807/⑫ to 1530/⑬

* * *



Line ㉖ from 1028/⑪ to 1550/①

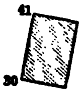
* * *



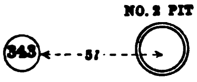
Line ㉕ continued

$(618) = \frac{545}{㉘}$

570



467



$(73) = \frac{1799}{㉞}$

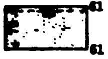
⊙

e ㉙ from $\frac{1073}{⑬}$ to $\frac{545}{㉘}$

1032

$(963) = \frac{454}{㉗}$

G ENGINE HOUSE
NO.1 PIT.

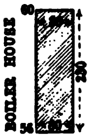


928

852

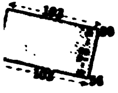
$(838) = \frac{756}{㉝}$

780



550

G ENGINE HOUSE
NO.2 PIT.



$(545) = \frac{618}{㉙}$

498

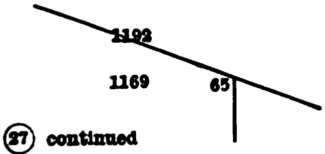
422

$(420) = \frac{482}{㉚}$

⊙

Line ㉘ from $\frac{540}{①}$ to $\frac{454}{㉗}$

$(1284) = \frac{1530}{⑬}$



ine ㉗ continued

$(839) = \frac{732}{㉗}$

(772) NO.1 PIT

$(709) = \frac{480}{㉝}$

$(427) = \frac{343}{㉙}$

(371) NO.2 PIT

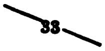
$(230) = \frac{768}{㉚}$

⊙

Line ㉜ from $\frac{646}{①}$ to $\frac{.732}{㉗}$

$(421) = \frac{375}{⑧}$

$(213) = \frac{3850}{①}$



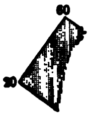
⊙

Line ㉛ from $\frac{464}{⑦}$ to $\frac{375}{⑧}$

$\frac{230}{㉜} = (962) = \frac{789}{⑬}$

(768)

632

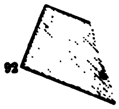


535

$(482) = \frac{420}{㉘}$

$(333) = \frac{0}{㊵}$

319



Line ㉚ from $\frac{1245}{⑫}$ to $\frac{789}{⑬}$

FIG. 16 (8).—*continued.*

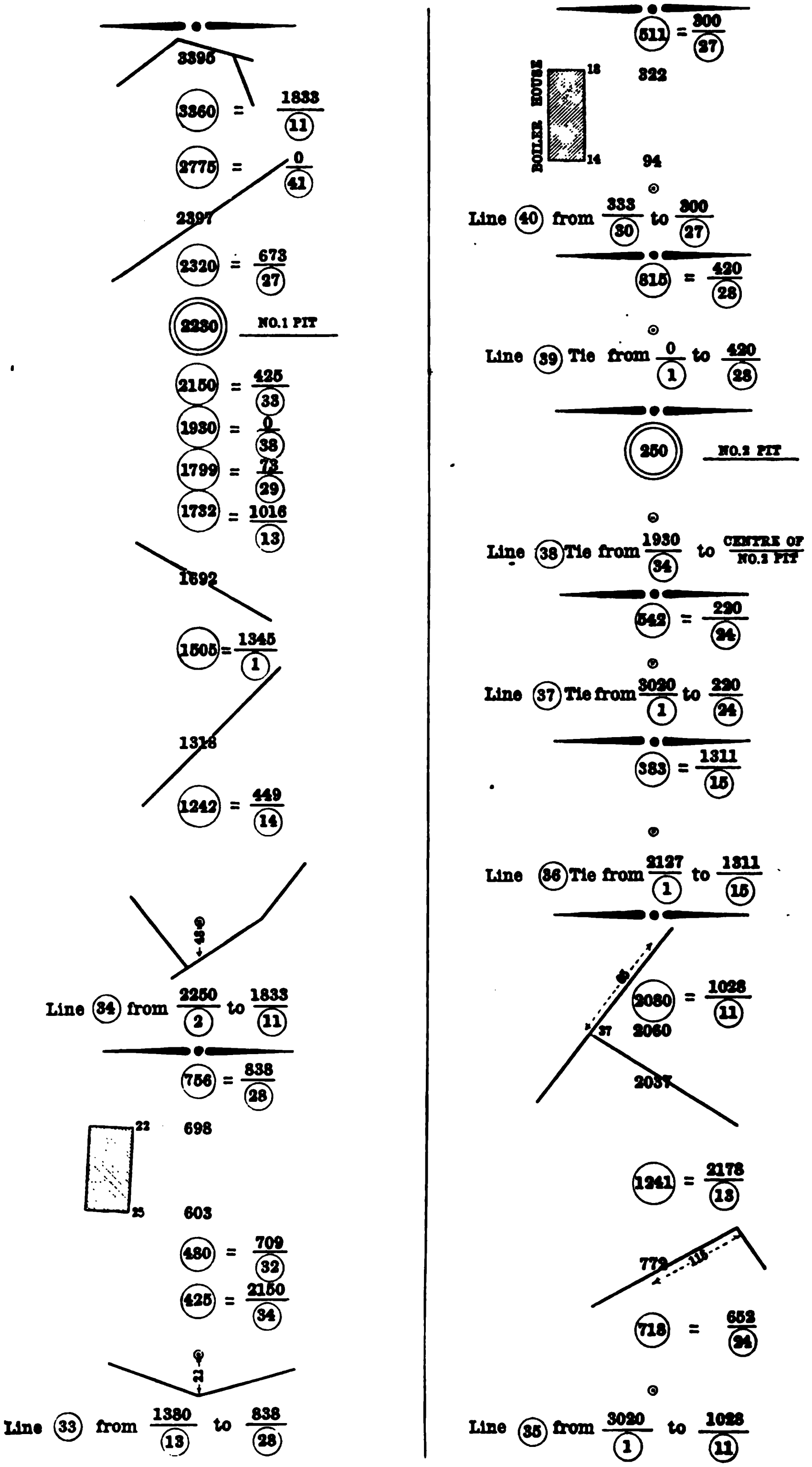


FIG. 16 (9).—*continued.*

$$\textcircled{523} = \frac{807}{\textcircled{12}}$$

Line ㊶ Tie from $\frac{2775}{\textcircled{34}}$ to $\frac{807}{\textcircled{12}}$

NOTE.—In order to save space, and also for convenience and rapidity of booking, the starting and finishing points and junction of one line with another are expressed as fractions; *e.g.* "Line ⑯ from $\frac{770}{\textcircled{3}}$ to $\frac{419}{\textcircled{18}}$" means that line ⑯ starts from a station left at 770 links along line ③, and ends at a station 419 links in line ⑱; the number of the line being enclosed in a circle and appearing as the denominator of the fraction.

To indicate a station, its length as read off from the chain line is enclosed in a circle; *e.g.* the length (609) in line ⑮ is a station, line ⑭ crossing at this point (1350) links from the starting-point (*i.e.* of line ⑭).

the station at 1010. In measuring from this to the intersection of line 34, two hedges are crossed, and the survey-line No. 14, so that there is little possibility of a mistake in identifying the station from which the measurement was taken. In the same way the position of the station 1550 on the base-line, where No. 26 ends, is found by remeasuring a portion of the base-line.

The same care has to be taken in crossing other lines, as, for instance, the tie-line No. 34 crosses lines 14, 1, 13, 29, 33, 32, and 27; and the position of the intersection of all these lines must be noted with the same care as was taken in noting the intersection on the base-line. By this careful noting of intersections, the detection of any error in the survey is a certainty; and not only that, but the place of the error is quickly discovered, and the length which has been inaccurately measured or incorrectly entered in the note-book can be measured over again, otherwise the surveyor might have to waste a great deal of time in remeasuring lines that had been accurately measured the first time.

Complete Note-book.—Figs. 15 and 16 give the whole of the survey-book from which the plan Fig. 14 has been plotted, and the student is recommended to plot this survey without referring to Figs. 14 and 17 until he has finished. Fig. 17 shows the order in which the triangles are plotted.

Railway Surveying.—The mining engineer has frequently to set out railways for mineral traffic, and every surveyor ought to understand how to survey the country where it is proposed to make a railway. Fig. 18 shows a piece of country through which it is proposed to make the line which is shown by the thick black curve, and it is necessary to make a plan of the fields, etc., through which it passes. The main lines of the survey are marked 1, 2, 3, 4, 5, 6, 7, 8. It will be seen that the proposed line of railway starts in a direction north-west, then turns to north-east, and again to south-east; and the piece of ground surveyed is a strip about 12 chains wide, following the curve of the railway. By the careful measurement of the triangles, line 4 is accurately placed in relation to

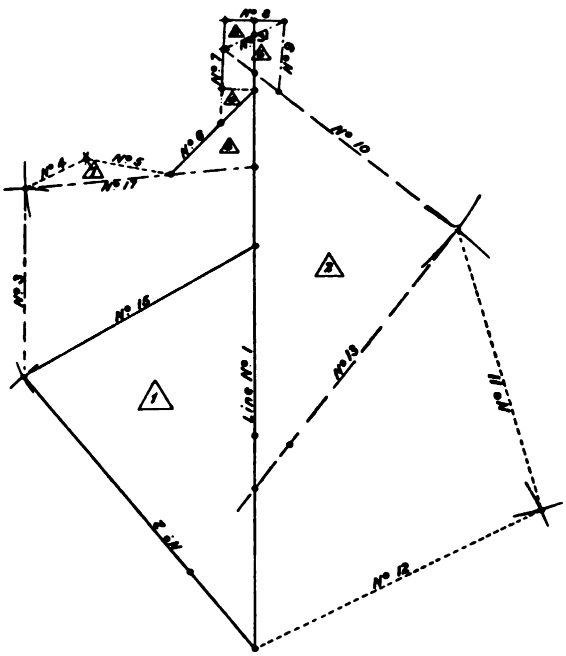


FIG. 17.—Showing the order in which the survey-lines given in Figs. 15 and 16 are plotted.

line 1, and line 6 is accurately placed in relation to line 4, and the survey may thus be continued for a good many miles with great accuracy. It is essential that all parts of the survey should be connected by two or more lines, so that all the measurements are checked. The lines Nos. 1 to 8 are the main lines; numerous other lines run alongside the fences and complete a network of triangulation that eliminates all chance of undetected errors.

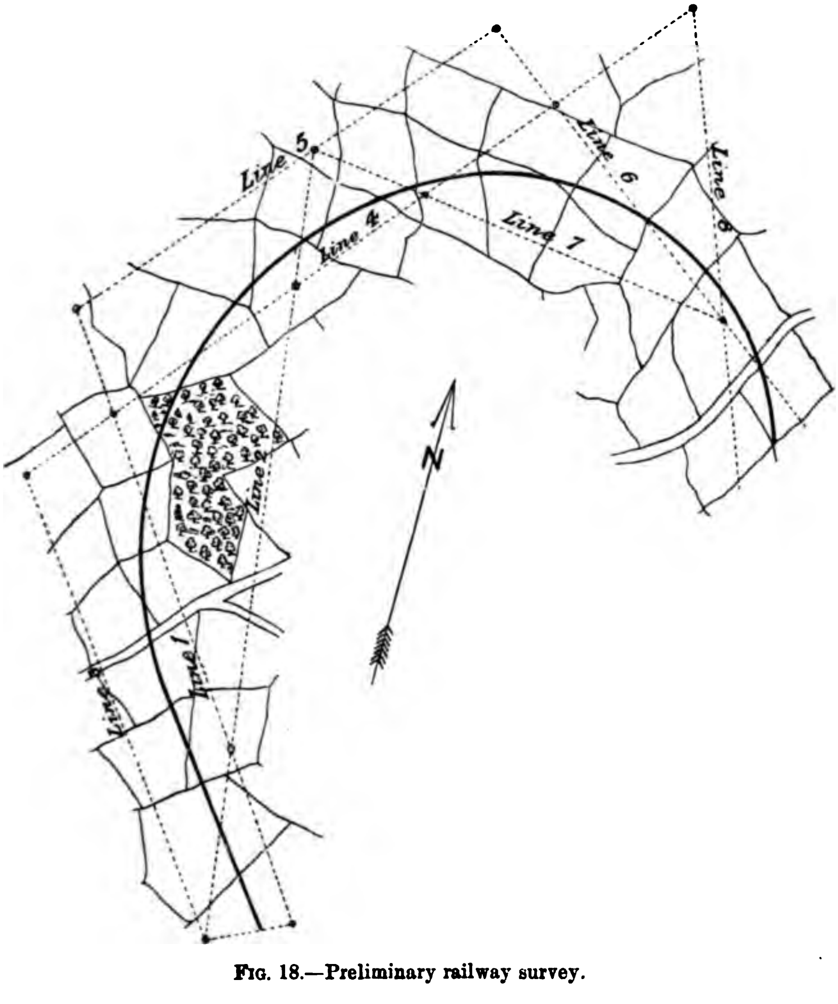


FIG. 18.—Preliminary railway survey.

When the student has once mastered the principles on which the plans Figs. 14 and 18 are made, he understands the whole theory of an ordinary surface survey of an estate; and practice, combined with the requisite physical and mental faculties, only is required to make him a competent land surveyor.

CHAPTER IV.

INSTRUMENTS FOR MEASURING ANGLES.

It is characteristic of the man of science to use every means at his command for testing the accuracy of his observations. Referring to Fig. 18, plan of a railway survey, it is obvious that if the bearings of some of the main lines were taken, that is to say of lines 1, 4, and 6, they would be a check upon the accuracy of the triangulation, especially if it were continued for a long distance, say 10, 20, or 100 miles. For this reason surveyors commonly use a magnetic compass to take the bearings of their main lines. With this information they can quickly correct any very serious blunder that might have been made either in the measuring or in the plotting of the survey.

Magnetic Compass.—The construction of the magnetic compass is based on the well-known fact that a very light steel bar, like a knitting-needle, which has been magnetized, will, if balanced at its centre on a fine point, turn so that one end points to the north and the other end to the south; whichever way the needle is placed originally, it is always the same end which seeks the north, that end is therefore called the north end (meaning the north-seeking end) of the needle, and the other end the south end of the needle. The direction in which the needle points is not, however, towards the north pole (*i.e.* towards the pole star), but it is towards the magnetic pole. To an observer in England this magnetic pole is west of the true or geographical north pole. A person standing at Greenwich and looking due north would have the magnetic pole a little to the left of his line of sight. The difference between magnetic and true north, or the angle between the magnetic meridian and the geographical meridian, is called magnetic declination.

Declination of the Needle.—On the 1st of January, 1901, the magnetic needle at Greenwich pointed in a line about 16° 26′ west of the true or geographical north. The magnetic pole is constantly moving its position. Three hundred years ago the magnetic north in England was east of true north; it moved gradually westward until the year 1818, when the needle near

London pointed about 24° 38′ west of the north pole. Since then it has been gradually returning eastwards. The movement in England is now approximately at the rate of 6′ to 8′ a year, or roughly 1° in 8½ years. Apparently the present rate of movement is rather slower than the average of the last 36 years, which has been fairly regular, averaging during that period about 8 minutes a year in the neighbourhood of London.[1]

Variation of the Declination.—The declination of the needle from the true north is not the same for all places; thus whilst the declination may be 16° 26′ west at Greenwich, and about the same at Worthing in Sussex, and Newmarket in Cambridge, it would be about 17° 34′ at Torquay, or Kidderminster, or Leeds, or Middlesborough, and 18° 35′ at Pembroke, or Conway, or Barrow, and 19° 30′ at Glasgow. The lines of equal declination (or isogonic lines) for the British Isles are shown on a map published annually as a supplement to the *Colliery Guardian.*[2] A somewhat similar map on a reduced scale is shown in Fig. 19. This map has been prepared by reference to the elaborate paper by Professors Rücker and Thorpe, published in the *Philosophical Transactions*, 1890. The isogonic lines drawn on this map represent average declinations; there are a great many local variations due to various causes, such as the magnetic character of the rocks, of which no account is taken in the diagram. The direction of these lines is north-easterly, and a person travelling along one of these lines, say from Torquay in Devonshire to Leeds in Yorkshire, and using the magnetic compass, would find the same declination from the true north along the whole line, but in journeying from London to Liverpool there would be a change in the declination every mile. By way of illustrating the use of this map, a surveyor in the Warwickshire Coalfield will find that the isogonal marked 19° in 1886 passes through that district, and that the declination in January, 1901, was 17° 15′. A surveyor in the Liverpool district is on another isogonal, marked 20° in 1886, and 18° 15′ in 1901. Half-

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[1] Every day there is a slight movement, known as the diurnal variation. According to Professor H. Stroud, M.A., D.Sc., the needle reaches its westerly maximum deviation at 1 p.m., and its maximum easterly at 10 p.m. (in the southern hemisphere east and west must be interchanged). This variation is about 10 minutes, or a sixth of a degree. For ordinary bearings this slight variation may be neglected, but when fixing the north point on a plan, or in other cases where extreme accuracy is desired, account must be taken of this variation. The reader is referred to the paper by Professor Stroud, in the *Proceedings Inst Mining Engineers*, vol. vii. p. 268.

[2] May be obtained from the *Colliery Guardian* office, 49, Essex Street, Strand, W.C.

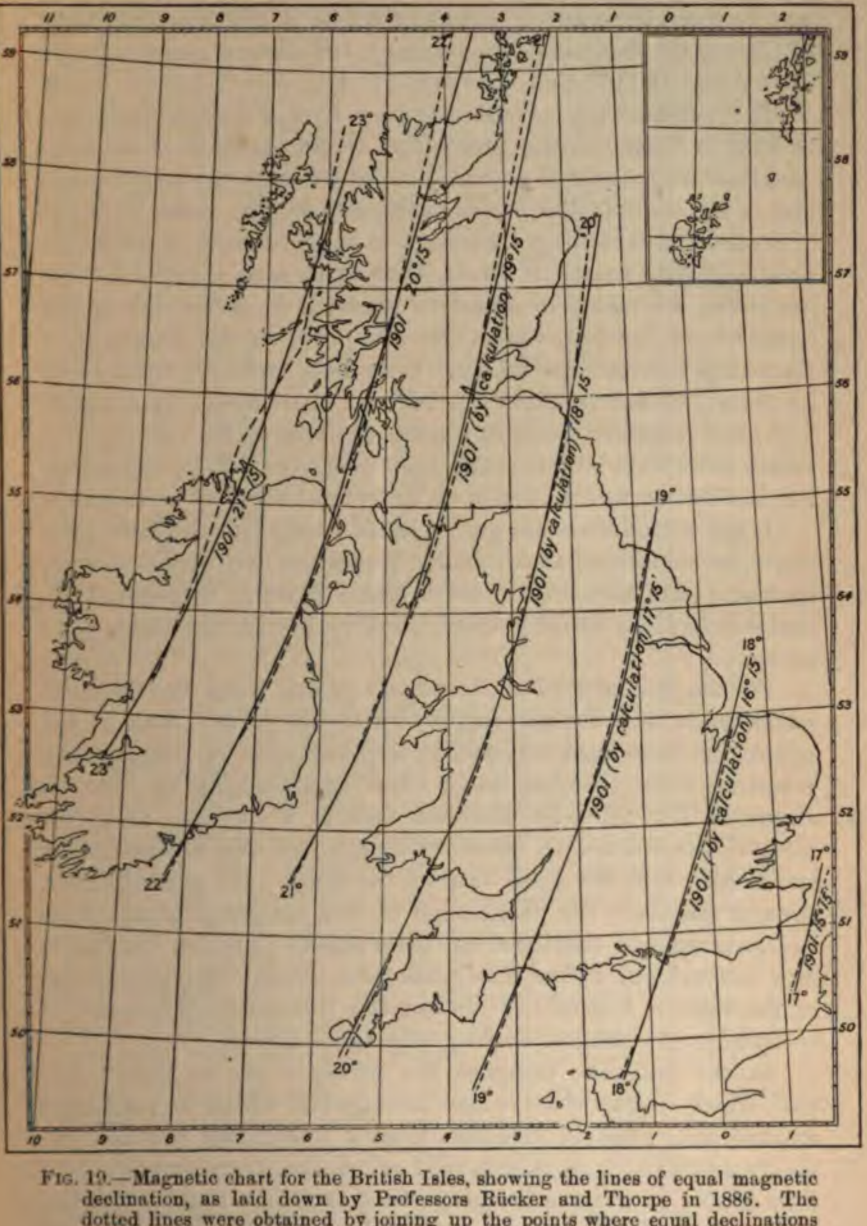

FIG. 19.—Magnetic chart for the British Isles, showing the lines of equal magnetic declination, as laid down by Professors Rücker and Thorpe in 1886. The dotted lines were obtained by joining up the points where equal declinations were found; the full lines show the average or mean lines. The figures printed at the ends of the curves are the declinations as appended by Professors Rücker and Thorpe in 1886. The present declination can be obtained approximately at any time by deducting 7 minutes per year for every year since that date.

way between these two isogonals, that is, in the neighbourhood of Crewe, the declination in January, 1901, was a mean between 17° 15′ and 18° 15′, that is to say, 17° 45′.

If a person were travelling along a line of latitude from east to west in England, the declination would increase as he went westward at the rate of about 1° in 100 miles on the south coast, and in the latitude of Berwick at the rate of 1° in about 70 miles. If instead of travelling from east to west, he were to travel (in England) from north to south, that is to say, along a line of longitude, the variation would be about 1° in 300 miles in the longitude of London, and 1° in 200 miles in the longitude of Falmouth and Milford Haven. If he were to travel north-west, as from London through Oxford and Cheltenham to Aberystwith, the variation would be on the average at the rate of 1° in rather more than 80 miles, and travelling from Whitby to Carlisle the declination would change at the rate of about 1° in 70 miles.

If the traveller should get on board a ship and sail round the world, he would find that in some places the declination is west, in others east, and that in some places there is no declination, that is to say, the needle points in the direction of the geographical north.

For the guidance of mariners and others, maps are prepared which show the declination of the needle in all parts of the world that have been explored by civilized man. A reduced map prepared from the Admiralty chart corrected up to 1900, is shown in Fig. 20.[1] In whatever locality a surveyor may find himself, the Admiralty chart will show him the declination of the needle, and the local rate of variation. He can, however, always ascertain the declination of the magnetic needle from the geographical meridian by observation, provided he knows how to mark out a line due north and south. For particulars of the various methods of finding the true north, the reader is referred to the chapter dealing with that subject.

In the magnetic compass the surveyor has an instrument with which he can observe how far any line which he may have marked out varies in direction from a line drawn towards the magnetic north, or, as it is usually called, the magnetic meridian; if he is able to observe the angle that each line makes with the magnetic meridian, he can easily calculate the angle that each line makes with the other lines, thus he can calculate the angles

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[1] The Admiralty charts may be had from the agent, 31, Poultry, London, E.C.

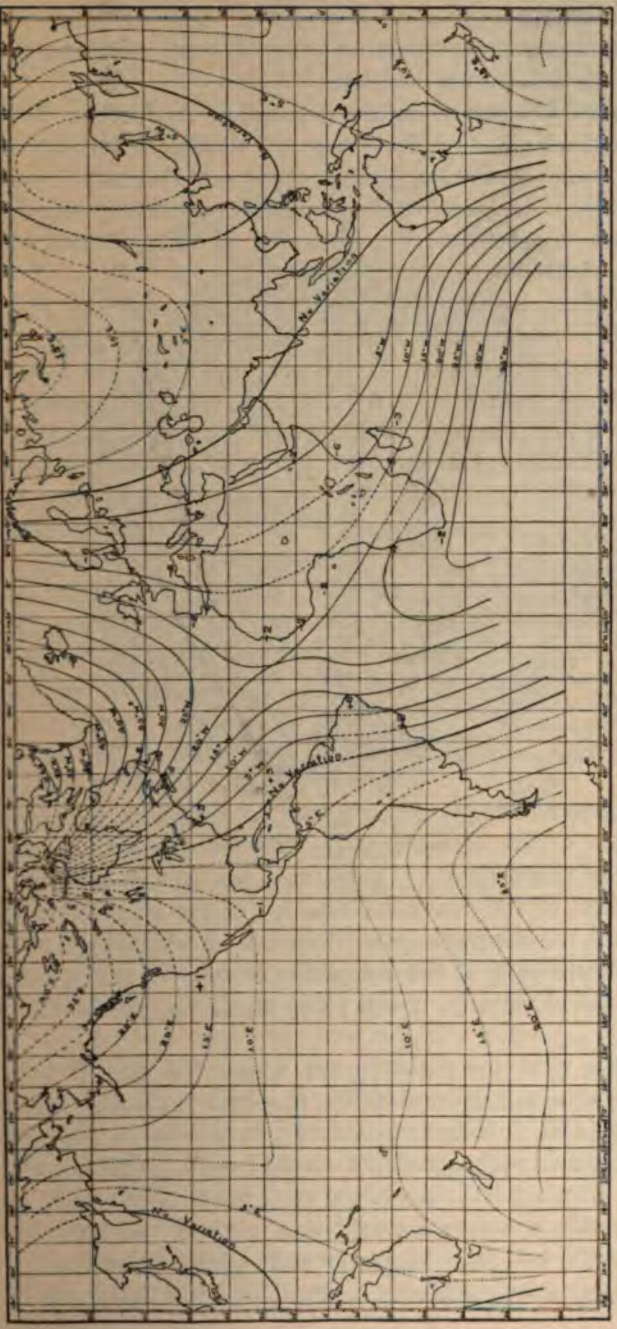


FIG. 20.—Map of the world, showing curves of equal magnetic declination. The values printed against the curves are for the year 1895. The annual change is given for various places, + sign meaning an annual increase, and − sign an annual decrease.

at the intersection of lines 1 and 4 in Fig. 18, and of lines 4 and 6, and can thus check the accuracy with which these lines have been laid down on the plan.

Mariner's Compass.—Before proceeding to further detail as to the method of using the magnetic needle, it will be well to describe some of the numerous forms of magnetic compass. The mariner's compass is the form most generally known, and of greatest use, because by means of it all the fleets of the world are steered across the ocean.

The novice, looking at a mariner's compass (see Fig. 21), might fail to learn that it had anything to do with the magnetic needle, because no needle is visible. The magnetized steel bar (or needle) is covered with a card, and, being supported at its centre on a sharp-pointed pivot, is free to revolve, and the card, being attached, moves with it; the instrument is enclosed in a brass case with a glass window, so that it is sheltered from the wind; the compass-holder is suspended in brass hoops (gimbals), so that the horizontal position of the card may not be disturbed by the motion of the ship. Inside the box or case are two marks above and in a line with the centre of the card, and on opposite sides of the card: these two marks are placed in a line parallel with a line drawn through the centre of the vessel. If, then, the ship is pointing towards the magnetic pole, these two marks will coincide with the direction of the magnetic needle; if the ship is turned to the right of the magnetic pole, these two marks will be pointing in a line north-east of the magnetic meridian; and if the ship is turned the other way, it will point north-west of the magnetic meridian.

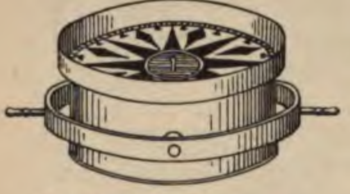


FIG. 21.—Mariner's compass.

In order that the direction in which the ship is pointing may be ascertained without delay, the card is divided by marks called "points;" there are thirty-two points in the circumference, eight in each quadrant, so that each point is an arc of 11° 15′. Thus: north, north by west, north north-west, north-west by north, north-west, north-west by west, west north-west, west by north, and west begins or ends another quadrant. The other quadrants are similarly divided, and the outer rim of the card is divided into 360°. The direction in which the ship travels is seen by reading on the card the position of the fixed marks in the box; one mark represents the bow of the vessel, and the other the stern; the bow mark is red. The card of the mariner's compass is shown in Fig. 22.

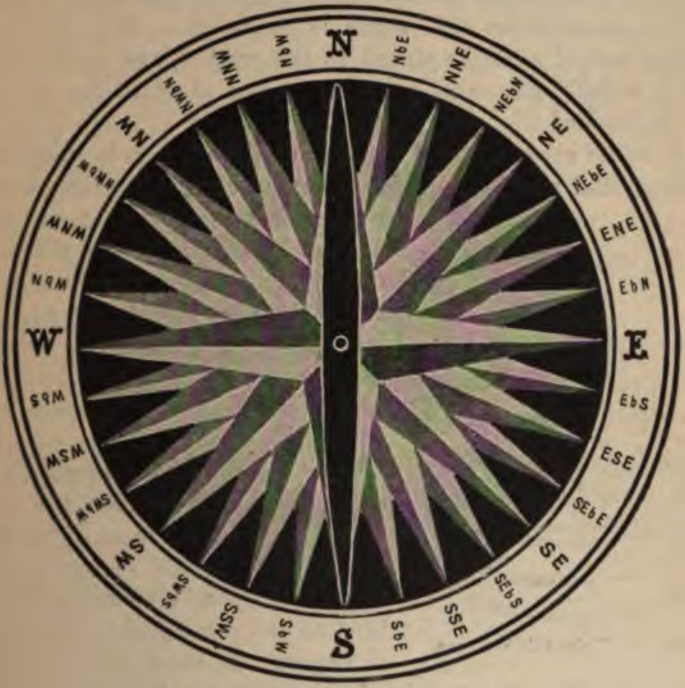


FIG. 22.—Card of mariner's compass.

Prismatic Compass.—A somewhat similar compass, called the prismatic compass (see Fig. 23), is used by land surveyors, but a light divided circle (generally made of aluminium) is often substituted for the card over the needle; it is also fitted with sights: one sight, **A**, has a slit; the opposite sight, **B**, has an opening, down the middle of which is stretched a hair or other fine thread. This slit and hair are placed in the direction of the line of survey, and the bearing is read by a pointer in the box, which is in the same line as the sights. The instrument is made so that the bearing may be read whilst it is held in the hand. In such a case it is necessary to read the bearing at the same instant that the sights come into the line. That this may be done, a reflecting prism is placed just below the top of the slit **A**. By means of this prism the marks on the graduated circle are reflected into the eye, and the mark which coincides with the line of sight is the bearing. This method, of course, only suffices for rough approximations to the bearing. Where accuracy is required, the compass must be placed on a stand, and in some cases this stand is made of a single stick, the pointed end of which is placed in the ground, and on the upper end is a ball-and-socket joint, by means of which the compass can be levelled.

In some cases a tripod stand is used, and this is suitable for underground work. In order that the correct bearing may be read, it is necessary that the circle should be marked as if the north end of the needle were the south end. Suppose the observer is looking towards a staff, light, or other mark north of him, the north end of the needle will, of course, be at the opposite side of the compass-box to the observer; therefore the observer can only read the south end. If this end is marked "south," the observer would be apt to book that reading, and afterwards imagine that he had proceeded in a southerly direction. To avoid such an error, the reading he observes should give him the direction in which he is moving, and therefore the letter N should be placed at the centre of the southern semicircle, and the letter S at the centre of the northern semicircle, and the east and west marks should be put in their correct positions relatively to the north and south marks, that is to say, the letter E will be at the side which is really the west, and the letter W at the side which is really the east.

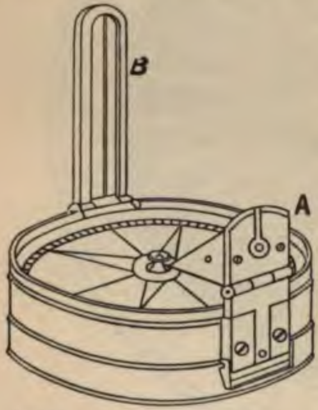


FIG 23.—Prismatic compass.

Graduation of Circle.—The division of the circle into points as used by the mariner is not required by the surveyor. The circumference is divided into degrees only, each degree being the three hundred and sixtieth part of the circumference. Counting from the N. end of the card, which is 0°, and proceeding, say, towards the E. mark, the first quadrant, up to 90°, is called north-east; the second quadrant, from 90° to 180°, is south-east; the third quadrant, from 180° to 270°, is south-west; the fourth quadrant, from 270° to 360°, north-west. In order, however, to facilitate the plotting, it is a common plan to count both ways, from both the south and the north ends; thus from north to west the degrees may be figured (on an inner ring of figures) from 0° to 90°, and from north to east also from 0° to 90°; from south to west in the same way the figures go from 0° up to 90°, and the same from south to east; so that the bearings are always read so many degrees from the meridian line, say 40° north-west or 40° north-east, as the case may be; or, if the observer is proceeding in a southerly direction, he might be going 30° south-west or 30° south-east, meaning that the bearing is the direction of a line proceeding from the centre pivot of the compass through a mark on the circumference 30° from the meridian line. The compass is made in various sizes from 1½ inch diameter up to 6 inches; the common size is about 2½ inches. The weight of the card or metallic circle on the needle is, however, some objection to the use of this form of compass.

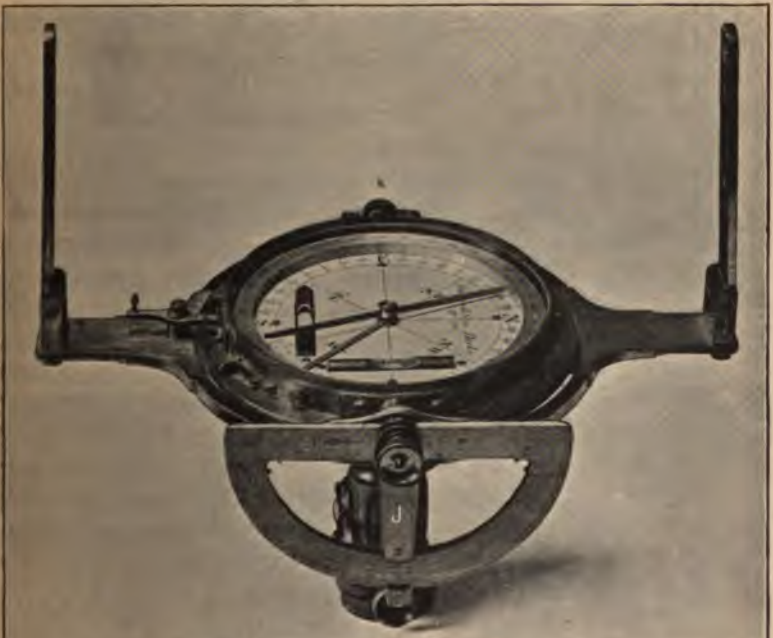


FIG. 24.—Hedley dial with outside vernier.

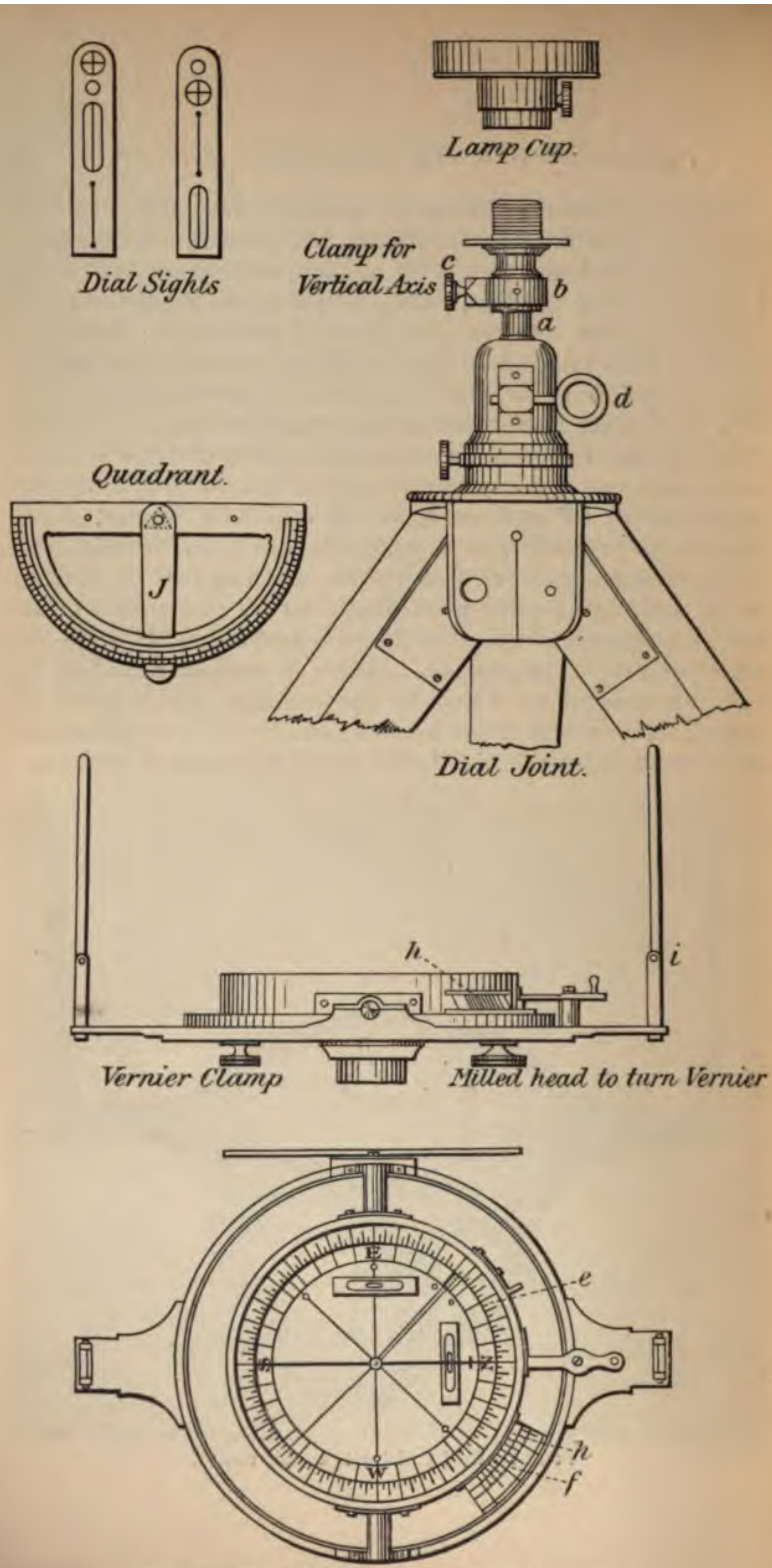


FIG. 25.—Details of Hedley dial with outside vernier.

Miner's Dial.—The dial is the instrument generally used by mining surveyors for taking bearings and angles. It differs from the two preceding forms of compass in this important respect—that the card or graduated circle is stationary, and the needle swings clear of it. One of general utility is shown in Figs. 24 and 25. Dials are made in various sizes, from a small pocket one, up to one carrying a needle 18 inches long (these large ones being for special work only). For general work the most usual size has a needle about 4½ inches long; occasionally a 6-inch needle is used.

In France the needle is usually a thin flat bar, wide at the centre, and the sides gradually converging to a point at the extremities (*a*, Fig. 26). In England it is common to use a needle rectangular in cross-section, and nearly the same thickness throughout. Just at the middle it is a little wider, and near the ends it is drawn down to a fine edge (*b*, Fig. 26). Sometimes, instead of drawing the end down to an edge, a line is marked on the top to represent the middle of the needle (*c*, Fig. 26). A piece of agate (stone) is securely fixed in a brass cap screwed into a hole drilled through the middle of the needle from top to bottom, and in this agate a conical hole is drilled from the under side nearly through; this agate rests on the sharp point of hard steel of the pivot that carries the needle. The agate is hard enough to resist the cutting effect of the steel point. The needle is free to revolve round the point in a horizontal plane. It is essential that the friction on the point should be reduced to a minimum, as the magnetic force is very small, and is insufficient to overcome any but the smallest frictional resistance.

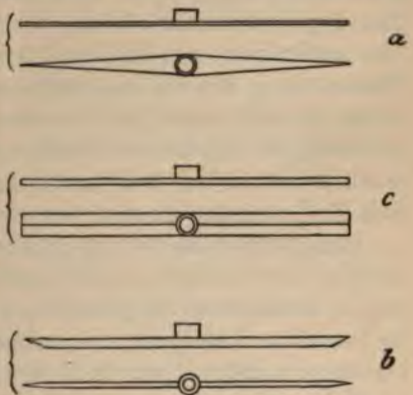


FIG. 26.—Varieties of compass needle.

The needle has to be so weighted that, *when magnetized*, it is evenly balanced on the steel point or pivot; a small piece of brass clipping the needle firmly, but capable of sliding along it, enables the balancing to be done accurately.

In course of years a needle is apt to lose its magnetism, and requires to be remagnetized. This may be done by taking out the needle and unscrewing the cap. The north pole of a strong permanent bar magnet is then stroked down the needle from the centre to the *south end.* The needle is then turned round, and the south pole of the magnet is stroked from the centre to the north end of the needle. The needle is then turned over, and the process repeated on its under side.

It is important that the agate cap of the needle, and the steel pivot on which it works, should be kept free from dust. The pivot should also be kept sharp, so as not to interfere with the free movement of the needle.

The top of the needle is level with the upper surface of a graduated circle which is fastened on to the dial-plate, and this upper surface is about $\frac{1}{4}$ inch above the bottom of the dial. The graduations are carried down the vertical side of the circle. This circle is divided into degrees, and if the end of the needle is not opposite one of the divisions, the surveyor has to estimate as nearly as he can the fraction of the degree beyond the last mark, thus: $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, and $\frac{7}{8}$; the bearing being, say, south-east $21\frac{1}{8}^\circ$ or $\frac{7}{8}$, as the case may be.

Many surveyors do not profess to read to eighths on a dial of this size ($4\frac{1}{2}$-inch needle), and would only book quarters, as $21\frac{1}{4}^\circ$. It is, however, possible, with a well-marked dial and a well-made and properly magnetized needle, to read to even one-eighth of a degree, which means that, supposing the bearing is booked by the surveyor as $21\frac{1}{4}^\circ$, it is possible that he may be deceived, and that the real bearing is $21\frac{1}{8}^\circ$ or $21\frac{3}{8}^\circ$; but the error need not be more than $\frac{1}{8}$ either way.

E. and W. reversed.—In the ordinary dial (Figs. 24 and 25) the letter E is put on the west side, and the letter W on the east side of the dial-plate or graduated circle. The graduations are read by the help of two systems of figuring. The outer set of figures are marked 10, 20, etc., up to 360. These figures go from north to east, and continue round the way the sun travels; thus W. is at 90, S. at 180, and E. at 270. The other system of figuring is on the inside ring, and refers to the quadrants, as 10, 20, 30, up to 90. Thus, counting from the north to the right hand are 10, 20, 30, etc., N.W.; starting from the north to the left hand are 10, 20, 30, etc., N.E.; starting from the south to the right hand are 10, 20, 30, etc., S.E.; and starting from the south towards the left-hand are 10, 20, 30, etc., S.W.

Mode of using the Dial.—There are two sights on the dial in a line with the north and south marks on the dial-plate. These are shown in Fig. 25, and consist of folding arms hinged at the point *i*, so as to fold down when not in use. Each sight has in it a broad opening and a slit, and down the centre of the broad opening is stretched a hair. The observer takes a sight by placing his eye at the slit, and moving the sights until the hair in the opening opposite is exactly in the centre of the object to which he is sighting. When taking inclinations, the circular holes shown are sighted in a similar manner. In using the dial the north (or N.) sight is always turned in the direction the surveyor is going. If he happens to be sighting a station *behind* him, then the south (or S.) end of the dial is turned towards this station; if he happens to be going in a direction magnetic north, the north end of the needle will point exactly to 360° or 0° of the graduated circle over the letter N; if he happens to be going north-east, the line of sight will be to the right hand of the north end of the needle.

To read the bearing, the surveyor looks at the north end of the needle, and reads the bearing against which it points, say 21° N.E.; but, whichever way the surveyor goes, he must bear in mind to turn that end of the dial which is marked N. (for north) in the direction in which he is going, and to read the bearing from the north end of the needle. The north end of the needle is indicated by a mark upon it; it sometimes consists of a notch, and sometimes of a brass cross-bar.

Hedley Dial.—The kind of dial most commonly used, and perhaps the most convenient form that is made, is known as the Hedley dial, and it is this form of dial which is illustrated in Figs. 24 to 28. The distinctive feature of this dial is that the sights are not fixed on the dial-plate, but to a separate ring outside, carried on bearings on each side of the centre dial-plate; the circle carrying the sights can thus be moved up and down through an arc of about 60° either way, so that a sight can be taken up or down a very steep place. Attached to the instrument is a semicircle for measuring vertical angles, the arm **J** (Fig. 24) is fixed to a projecting end of the axis which carries the movable ring to which the sights are attached. The semicircle is fastened by two studs to this ring, and is therefore inclined to the same degree as the line of sight, when it is taken through the small round eye-hole and the cross-hair of the opposite sight. The arm **J** always remains in a vertical position as long as the surface of the dial is kept level, and a pointer at the end of the arm enables the angle of elevation or depression to be read.

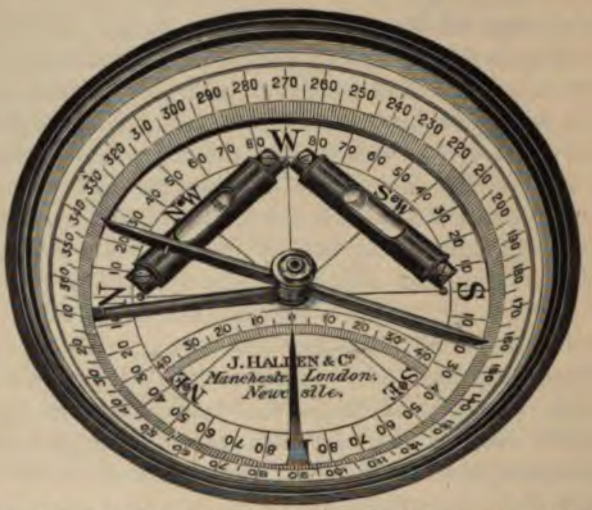


FIG. 27.—Face of dial, showing Halden's method of measuring inclinations.

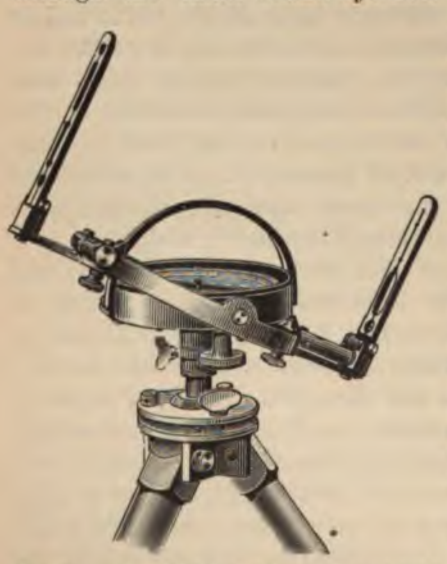


FIG. 27A.—Casartelli's dial, showing semicircle for measuring inclinations.

The semicircle is graduated in quadrants, zero being at the centre, and the graduations extending to 90° each way. There is a clamping-screw at the lower end of the arm **J**, by which the sights can be clamped at any desired angle of inclination.

Messrs. Halden make a dial with an improved arc for measuring vertical angles. Instead of an external attachment, which may get broken, the graduated arc is on the base of the compass-box, the traversing finger working on a centre near the E. point of the dial (see Fig. 27). Another form of inclinometer is shown in Fig. 27A. This is made by Messrs. Casartelli, and consists of a graduated brass semicircle, in the same line as the sights, which folds down on one side of the dial-box when not in use.

Dial with Inside Vernier.—The dial is generally so made that it could be used for measuring angles if the needle was taken away, or if, owing to the presence of iron or other magnetic metal or rock, it cannot be used. One form of this is shown in Fig. 28. On the inside of the dial-box is fastened an index or vernier, the 0 on the vernier being in the same line as the dial-sights. The dial-box is so made that it can be moved round independently of the dial-plate. If the dial-plate is firmly clamped and the sights moved to the east or west, the mark on the inside rim of the box moves with them, and the angle of movement can be read on the graduated circle.

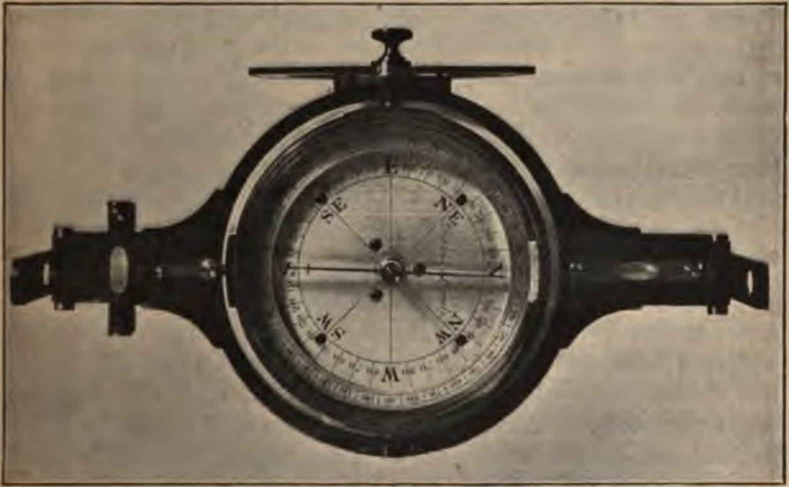


FIG. 28.—Hedley dial with inside vernier.

The use of the vernier is that fractions of degrees may be accurately read. The ordinary dial vernier reads to 3′, or $\frac{1}{20}$ part of a degree. The dial-plate is graduated and figured as already described. When using this dial for taking bearings with the needle, the mark in the centre of the vernier must be over the north or zero point of the graduated dial-plate (as shown in Fig. 28), and it can be kept in this position by means of a brass pin, which is put up through the bottom of the dial-box and the dial-plate. When it is desired to use the vernier for taking angles, this brass pin is pulled out and the clamping-screw slackened. The sights are moved by means of a milled head on a pinion, the teeth of which fit into a rack on the inner side of the dial-box. By means of this pinion the sights can be easily moved to the required extent.

Outside Vernier.—In another, and in some respects superior, form of this dial (Figs. 24 and 25) there are two graduated circles: one inside the dial-box, *e* (Fig. 25), to be used for taking bearings with the needle; and the other outside the dial-box, *f* (Fig. 25), to be used when taking angles without the needle. This outside graduated circle is immovably fixed to the vertical axis, while all the other parts of the dial above it can revolve (by the action of the rack and pinion). The outer graduated circle is covered by a brass rim outside the compass-box, which conceals it from view, except at one place where this rim is partly cut away so as to expose the graduations for a length of say 30°.

On the movable dial-box is fixed the vernier *h* (Fig. 25), on which is a centre-mark. For the sake of convenience in reading, this vernier is not exactly under either of the sights, but is a little on one side, and at the beginning of a survey the centre-mark of the vernier is placed opposite the zero on the graduated external circle. If the dial-sights are then looking north and south, any movement to the east or west will be measured in degrees and minutes by the movement of the mark on the vernier from the zero point.

The advantages of this form of dial are—first, that the outer graduated circle and vernier can be easily read; second, that the sights are always in a line with the north-and-south line on the dial-plate, and therefore the needle can always be swung and a true bearing observed (in case there is no attraction), whereas with the dial with inside vernier a loose-needle bearing could not be read until the ring carrying the sights had been put back into its original position, with the centre-mark of the vernier opposite the north-and-south line of the dial.

It is essential that the dial should be placed level, and for that reason two spirit-levels, at right angles to each other, are generally placed on the body of the dial (as shown in Figs. 24 and 25).

The spirit-levels may also be placed on the limb to which the sights are attached (as shown in Fig. 28), and, although more liable to get broken in this position, they do not interfere with the swinging of the needle.

Dials are generally made of brass, but aluminium dials are now being made, and are preferred by some on account of their great lightness.

Dial-joint.—The dial is generally carried on a tripod stand, to which it is attached by a coupling, having a ball-and-socket joint (see Fig. 25). Above the ball is a strong brass pillar, *a*, which fits into a socket, *b*, which may be screwed on and off from the under side of the dial. The dial and socket are free to revolve round this pin or vertical axis, but can be fixed in one position by means of a clamping-screw, *c*. Below the ball is a clamp, *d*, by means of which the vertical axis can be tightened in the required position, and by which it can be slackened to admit of adjustment. This ball-and-socket joint, and the upper swivel movement, generally give satisfaction if they are kept in good order, but it is necessary that they should be cleaned from time to time and used with care. Some surveyors of great experience condemn this joint because of the insecure attachment of the dial by a screw, which may lead to an inaccurate survey, and also because of the absence of any convenient mode of levelling the head of the tripod holding the lamp-cup.

There are, however, other modes of attachment. The ordinary parallel plates, such as are used with the theodolite (Fig. 37), may be substituted for the ball-and-socket joint. There is also the Hoffman joint, made by Davis of Derby (see Fig. 29). By turning the milled-head screws *a*, *a* from left to right, the two concentric balls **B** and **D** are liberated, and the dial can then be approximately levelled up; on turning the screws in the opposite direction, the joint is clamped, and the final adjustment may be made by turning opposite screws in reverse directions just as required.

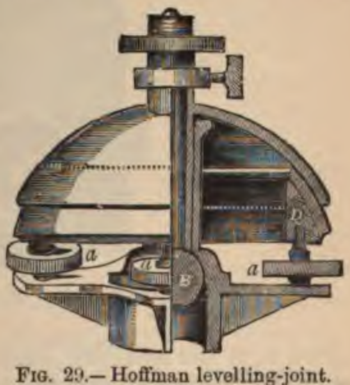


FIG. 29.—Hoffman levelling-joint.

Another variety of levelling-joint (shown in Fig. 27A) has a ball held between two plates which can be tightened or slackened by turning a thumb-screw. On the top of the ball is a strong brass pillar, fitting into a socket fixed on the under side of the dial; in the socket is a clamping-screw. Each tripod has fixed to it the levelling-joint. Two lamp-cups, fitted with cross-levels, are used to hold the object-lamps, and by means of these levels the brass pillar is set, so that when the dial (as in fast-needle work) is moved and placed upon it, its face will be level.

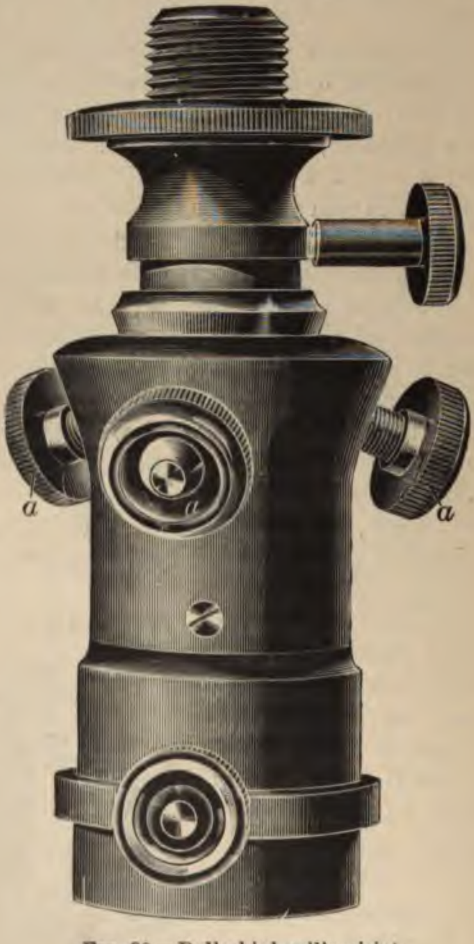


Fig. 30.—Bullock's levelling-joint.

Some surveyors who have used most kinds of dials strongly recommend the joint above described.

Bullock's ball-and-socket joint is shown in Figs. 30 and 30A.

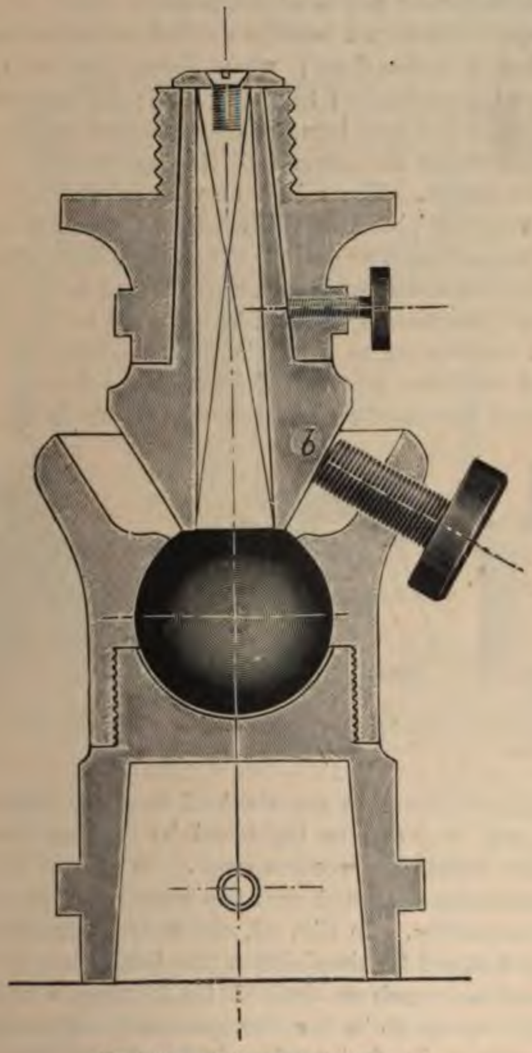


FIG. 30A.—Bullock's levelling-joint.

On reference to the figure, it will be seen that three adjusting screws, *a*, converge on to a cone, *b*, to which is attached the ball; then, by tightening or slackening these screws, the top may be thrown to any reasonable angle, and so enable the operator to obtain an accurate adjustment. One advantage of this joint is that when all the screws are touching the cone, the top cannot be thrown out of adjustment.

Dial-legs.—The tripod head is carried on three legs, usually about 4 feet 6 inches long; when these legs are spread out, the dial is at a convenient height to read; for low roads shorter legs are used. The long legs are often jointed in the middle, so that by unscrewing the lower half the legs remain about 2 feet 3 inches in length. These legs may be jointed again, for thin seams or very low places in the roads, for which places legs 12 or 15 inches in length are used. Telescopic legs are sometimes made, and are convenient in low, narrow, and rough places. It is important that the legs should be attached to the tripod head in such a manner as to preclude the possibility of slackness, whilst they must not be too stiff for convenient use. The three kinds of head commonly employed are shown in Figs. 31, 32, 33. In Fig. 31 the legs get slack if they are kept long in a dry place, but they can be tightened by soaking the joints in water, which causes the wood to swell. In Fig. 32 the split end of the legs can be tightened over the brass projection by means of the thumb-screw. In Fig. 33, which is the method adopted in the tripod stand for theodolites, the joints can be tightened or slackened as much as desired by turning a nut upon a screw. This seems to be the strongest and best method.

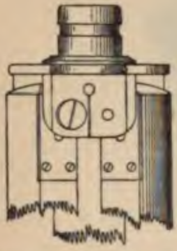


FIG. 31.—Ordinary dial tripod.

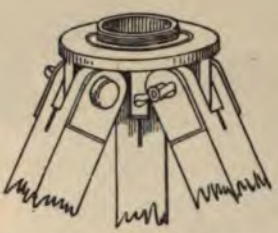


FIG. 32.—Improved form of dial tripod.

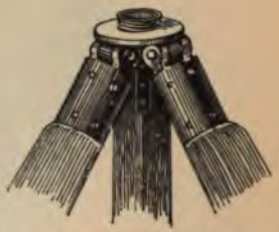


FIG. 33.—Theodolite tripod.

Lamp-cups. — In fast-needle dialling two lamp-cups are usually employed. These are shown in Fig. 25, and consist of shallow cups of suitable diameter to receive the lamp. One of the cups is provided with levels, and this is always used in fixing the front legs ready to receive the dial.

Various Dials.—Many modifications of the dial are made. In one of these a telescope is substituted for the simple slit and hair-sight, the sights being made detachable, so that the telescope may be taken off and the ordinary slit and hair-sights substituted, as shown at **A** and **B** (see Fig. 34). The telescope is advantageous for work where extreme accuracy is required, because the lamp, candle, or other mark can be clearly seen, and the meridian line of the dial turned precisely on the centre of the light, whereas with the slit and hair an error amounting to the thickness of the hair or the width of the slit may be easily made. The possible error, however, from this source, if the hair is properly fixed, is not more than 1 in 1200, or less than $\frac{1}{20}$ of a degree, so it is only for special cases, either where very long sights are taken or where special accuracy is required, that the telescope is useful.

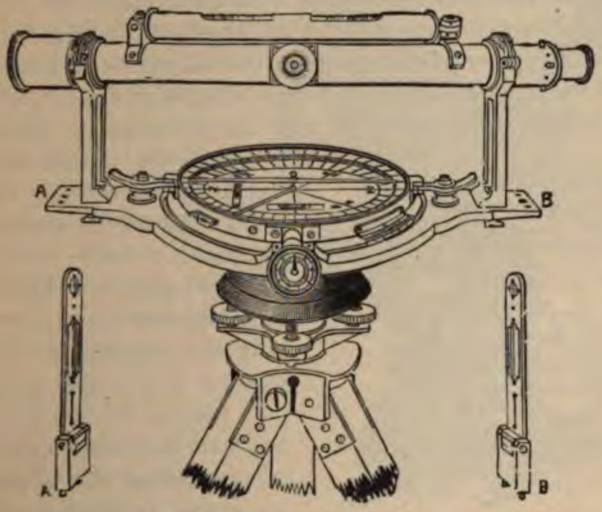


FIG. 34.—Hedley dial with telescope.

It is obvious that a single telescope is, in some respects, not so convenient as the ordinary sights, which are made double for looking either backward or forward, and where the telescope is supported in the manner shown in Fig. 34, it is necessary to take it out of the holders and reverse it for the back sight.

Dial with Eccentric Telescope.—In surveying without the needle—or "fast needle," as it is called—the angle can be read with great accuracy by means of the outside vernier; but if the needle is used there may be some difficulty in reading it, in case the line of sight should correspond with the magnetic north, as the needle will then lie immediately below the telescope. This difficulty is got over by placing the telescope on one side of the dial instead of over the centre, as shown in Fig. 35. There is, however, a drawback attending this form, because the line of sight through the telescope is not directly parallel with the direction of a line from the centre of the dial to the lamp. This, however, may be got over by placing the lamp to be looked at at an equal distance away from the mark, and on the same side of the mark.

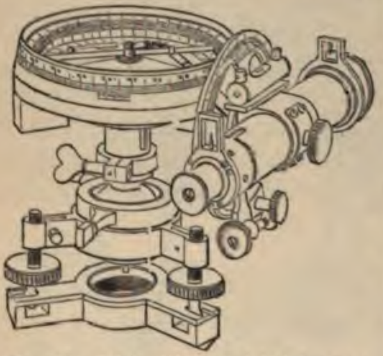


FIG. 35.—Dial with eccentric telescope. (*Kindly lent by Messrs. W. F. Stanley & Co., Ltd.*)

The plan of having the telescope on one side has not only the advantage of leaving the top of the dial quite clear and taking up less headroom, but has the further advantage of permitting the telescope to be moved through a complete circle, so as to look either backwards or forwards, up or down, as required, or at any intermediate inclination. When the telescope is fixed on one side of the centre, it is called eccentric. This eccentricity of the telescope need not be taken into account when reading the degrees on the vertical circle; it is only when measuring a horizontal angle or transferring a horizontal line of sight from a plane on another level either above or below, that the eccentricity has to be considered.

In dialling "loose needle" (that is, using the needle to read the bearings) for ordinary purposes, the eccentricity need not be considered, because the error in reading, whatever it may be in the back sight, is corrected in the fore sight. In fast-needle work, however, the eccentricity has to be considered. The surveyor must so arrange the lamp or other object viewed through the telescope that it is exactly as far from the centre of its tripod stand as is the telescope from the centre of the dial, and the object viewed must be on the same side of the centre of the stand as the telescope. Any failure to attend to this may lead to very serious errors. This liability to error has discouraged the use of this form of instrument.

By the use of an eccentric lamp-holder, the line of sight from the telescope to the lamp is exactly parallel to the line from the centre of the dial to the centre of the tripod stand which is in the line of survey, and therefore the eccentricity of the telescope leads to no error.

Combined Mining Dial, Level, and Theodolite.—This instrument, which has only recently been brought out, is shown in Fig. 35A.

The chief feature is the method of supporting the telescope in cranked gimbals, thus enabling a sight to be taken vertically upwards or downwards.

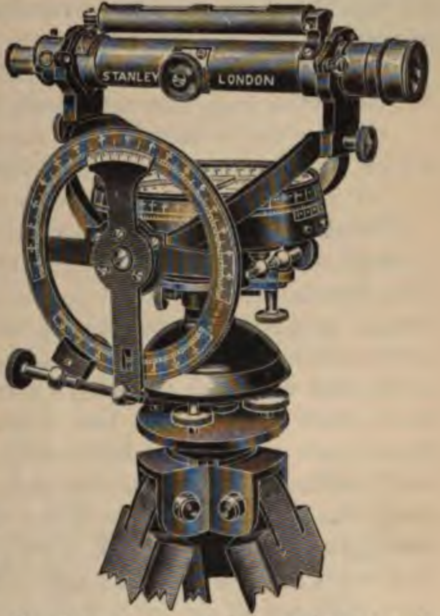


FIG. 35A.—Combined mining dial, level, and theodolite.
(*Kindly lent by Messrs. W. F. Stanley & Co., Ltd.*)

For fast-needle work, two outside verniers are used, reading to single minutes. The vertical circle has a clamp and tangent, and is also divided to read to minutes.

Hanging Compass.—An old-fashioned kind of compass, which is still used in some places, is shown in Fig. 36. In this case the compass-box, instead of resting on a tripod stand, is suspended by a cord in such a manner that the box is always level, and the needle free to revolve. The cord is stretched from end to end of the line of which the angle has to be taken, and the reading of the compass-needle shows the bearing of this cord.

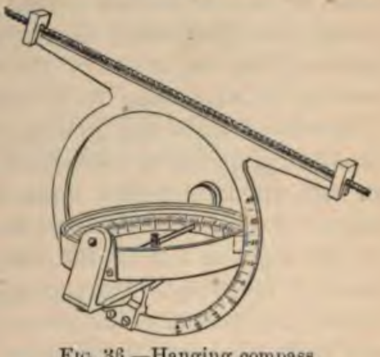


FIG. 36.—Hanging compass.
(*Kindly lent by Messrs. W. F. Stanley & Co., Ltd.*)

The following recommendations, in addition to those already made, may be of use to purchasers of dials: —

(1) There should be two verniers where very accurate work is required.

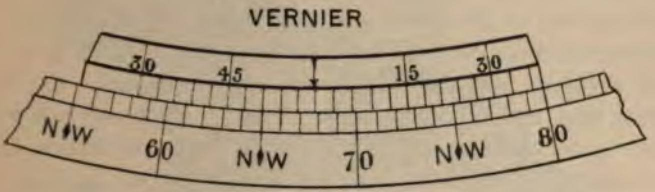
(2) The plate which carries the vernier should be clamped with a proper grip, and not merely by the point of a screw.

(3) The dial should not be attached to the stand by a screw which may unscrew unknown to the surveyor.

(4) Spirit-levels should have a white backing.

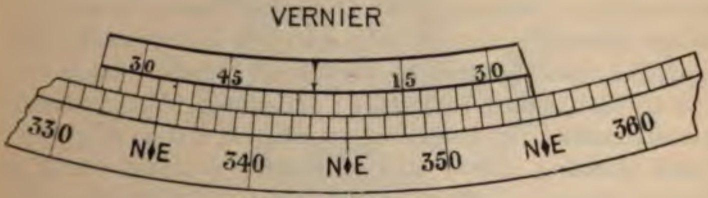
Vernier.—Called after the inventor Pierre Vernier. This is a small movable scale running parallel to the fixed scale on the dial, theodolite, protractor, barometer, etc. The use of the vernier is to facilitate the reading of the exact position of some mark which slides upon the scale or close to it. For instance, in the case of a dial the centre mark is indicated by an arrow-head which moves round the circumferentor when the sights of the dial are moved, as in fast-needle dialling. There is a similar mark indicated by an arrow-head on the plate that revolves above the graduated circle of the theodolite. In the case of a barometer, the moving mark is the top of the column of mercury. This mark may be placed exactly opposite one of the marks of the graduated circle in a dial or theodolite, or on a straight barometric scale, in which case no vernier is required; but if the mark comes to some position between the graduations, then the vernier is useful in reading the exact position between the two divisions of the scale. In the case of a dial, the sliding scale or vernier is fixed to that part of the dial which revolves round the graduated circle, and the arrow-headed mark is generally in the centre of the vernier, say 20 divisions on the vernier corresponding to 19 divisions on the graduated circle. If the divisions of the graduated circle are equal to one degree, then the divisions of the vernier are each equal to $\frac{19}{20}$ of a degree, so that when the centre mark on the vernier is set opposite one degree of the circumferentor, the nearest division of the vernier to the right or left of the centre mark will be $\frac{1}{20}$ of a degree short of reaching to the corresponding mark on the graduated circle. If, therefore, the arrow-mark is moved $\frac{1}{20}$ of a degree to the right, the next division on the vernier to the arrow-mark will coincide to the corresponding mark on the graduated circle. If the arrow-mark should be moved $\frac{2}{20}$ of a degree, the second division of the vernier will be in line with the corresponding division upon the graduated circle, and so on; therefore, in order to read the exact distances that the arrow-mark is from the degree from which it has moved, it is necessary to look for the line on the vernier that happens to coincide with one of the divisions of the graduated circle. If that division is 6 from the arrow-head, then the arrow-head is $\frac{6}{20}$ of a degree past the degree on the graduated circle from which it has been moved. Since the degree contains 60 minutes, the twentieth part of a degree

a.



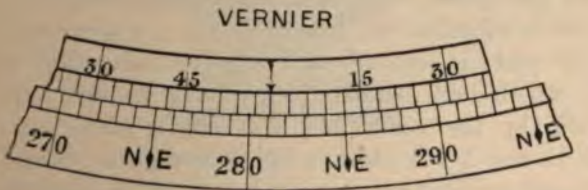
Reading 67° 51'.

b.



Reading 343° 15'.

c.



Reading 281° 12'

FIG. 36A.—Vernier readings.

is three minutes, and therefore if the sixth division on the vernier scale corresponds with a line on the graduated circle, the arrow-head is 18 minutes past the degree. It will be seen that the principle of the vernier is that the space between any two divisions on the vernier scale is a small fraction less than the space between any two divisions on the fixed scale, and therefore if one division line on the vernier scale is exactly opposite a line on the fixed scale, to bring the next line on the vernier scale opposite the next line on the fixed scale it must be moved through the small fraction above named. It should be noted that the divisions on the vernier scale may be spaced farther apart than those on the fixed scale. An illustration of three readings of the vernier is given in Fig. 36A.

Theodolites.—The principle of theodolite construction is similar to that of the improved Hedley dial, with outside graduated circle shown in Fig. 34; but the details of construction are very different, as may be gathered from Fig. 37. In the theodolite a telescope is always used, and mining theodolites are generally constructed as transit instruments; that is to say, the telescope can be turned all round in its bearings, so as to look either forward or backward. The telescope is generally carried on a vertical framework, *a* (Fig. 37), attached to and standing above the horizontal compass-box *b* at a sufficient height to allow the telescope to be reversed. The graduated circle *c*, for measuring vertical angles, is fixed on one of the telescope trunnions, while a pointer, *d*, carrying verniers is fixed to the framework. This circle can be clamped by means of the screw *x*. On the plate to which the telescope framework is attached are two verniers, *e*, *e*, at opposite sides; below this plate is another carrying the horizontal graduated circle *f*, which can be clamped to the vertical axis of the instrument by the screw *h*. The upper plate can also be clamped to the lower plate. Spirit-levels are placed on the telescope and on the upper horizontal plate. A 5-inch theodolite will read both vertical and horizontal angles to 1′, and an 8-inch theodolite to $\frac{1}{3}'$; a 12-inch theodolite will read to 1″. Mining theodolites are seldom bigger than 6 inches: the 5-inch is big enough for convenience (a 5-inch transit theodolite weighs from 12 lbs. to 14 lbs. without the legs). By a 5-inch theodolite is meant one in which the horizontal graduated circle is 5 inches in diameter. With this instrument, the compass-needle, being underneath the framework carrying the telescope, is not easily observed; it is therefore only occasionally used for taking the bearing of a base-line, or for noting the approximate direction of lines; the chief use of the instrument being for measuring the angles, both vertical and horizontal, made by one line with the next.

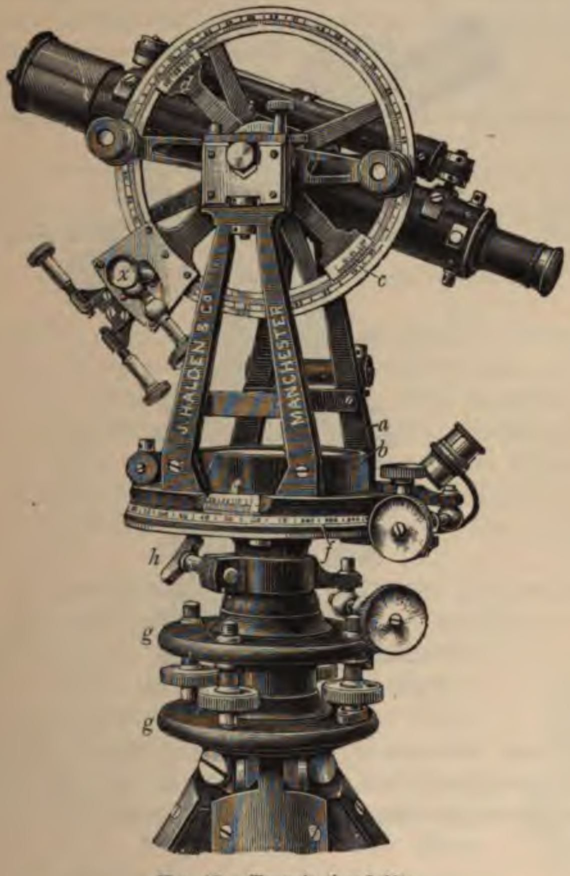


Fig. 37.—Transit theodolite.

Another variety of theodolite construction is shown in Fig. 37A. The standards carrying the telescope, which are usually made in separate parts screwed together, are here all in one solid casting. The axis and standards are also in one casting, so that displacement of the axis is impossible.

Instead of the ordinary compass-needle, a trough compass is sometimes substituted, shown in Fig. 38 (and shown in position in Fig. 37A). In this narrow box or trough the compass-needle is only free to revolve a few degrees on either side of the meridian, and it is merely used for fixing the theodolite in the magnetic meridian, this line serving as a base from which the bearings of the other lines can be calculated. Considerable accuracy may be obtained in fixing the instrument in the magnetic meridian, because it is possible to see a very slight divergence of the needle from the N. and S. marks on the compass-box.

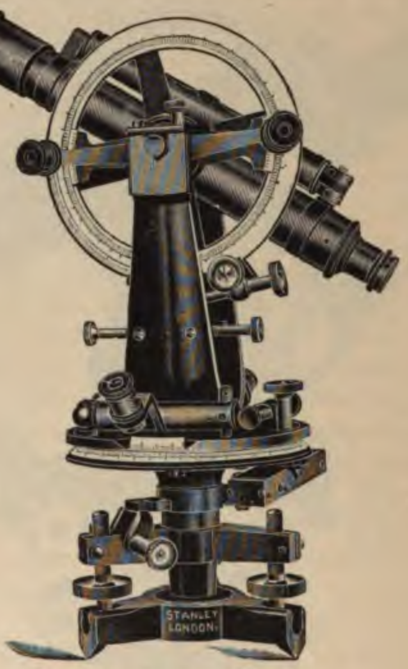


FIG. 37A.—Stanley's theodolite.

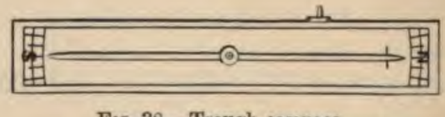


FIG. 38.—Trough compass.

Another kind of compass (Fig. 39) was made for the author, useful only for the purpose of setting the telescope in the meridian; it is fixed below the bottom plate of the theodolite. In this case the needle is very short—only 2½ inches—and is not suspended at the centre, but near to one end, the short end being thick and balancing the longer end, the thin end of which comes opposite a nick in the tube when the instrument is turned in the magnetic meridian, and the position of the needle is accurately observed by means of a microscopic eyepiece.

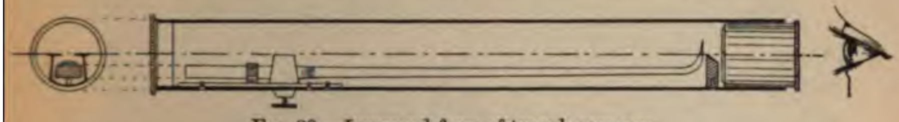


Fig. 39.—Improved form of trough compass.

Theodolites are generally made with parallel plates (see *g*, *g*, Fig. 37), by which the instrument can be levelled. A Hoffman head or other form of ball-and-socket joint, however, is sometimes used, which also has four adjusting screws. The ball-and-socket joint enables the instrument to be levelled whilst the tripod stand is on very irregular ground. With the parallel plates alone there might be some difficulty in adjusting the instrument.

Use of Theodolite Underground.—For the purpose of illuminating the cross-hairs of the telescope, which, owing to the darkness of the mine, would be otherwise invisible, one of the trunnions is made hollow, a lens being screwed into the outer end. Opposite this glass is fixed the bull's-eye of a small oil-lamp, the light from which passes down the hollow trunnion till it meets a reflector, consisting of a polished steel face about $\frac{1}{16}$ inch in diameter, placed within the telescope, by which the light is reflected on to the cross-hairs.

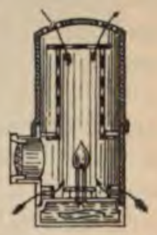


Fig. 40.—Lamp for illuminating the cross-hairs of theodolite.

For use in mines containing fire-damp, the small lamp for illuminating the cross-hairs must be enclosed in gauze, similar to that used for safety-lamps, and also shielded against the effects of strong currents, so as to comply with the conditions of the Mines Regulation Act (see Fig. 40).

In the absence of the hollow trunnions, the cross-hairs may be seen by the light of a lamp held near the object-glass.

Sextant.—This is an instrument for taking angles either in a vertical or a horizontal plane. It is used in surveying new countries, and for nautical and military surveying (Fig. 41). To measure the angle at the intersection of two lines, the telescope is directed upon an object in line No. 1. By means of a movable reflector fitted on the instrument and connected to the vernier, another object, in line No. 2, is at the same time brought into the same line of vision; the angle through which the reflector is moved is measured by the vernier, and the angle between the two objects is read on the graduated arc. Small sextants, called box sextants (Fig. 42), are often made about 3 inches in diameter, so arranged that they can be conveniently packed in a pocket-case. The instrument is carried in the hand, but, owing to the fact that two objects are brought simultaneously into the line of vision, the angle formed by the two lines of sight may be read with some approach to accuracy.[1]

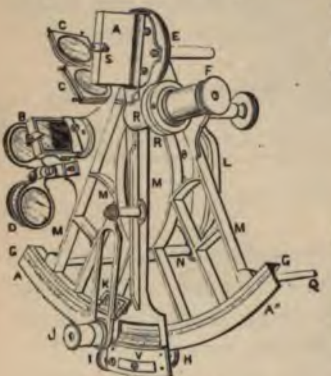


FIG. 41.—Sextant.

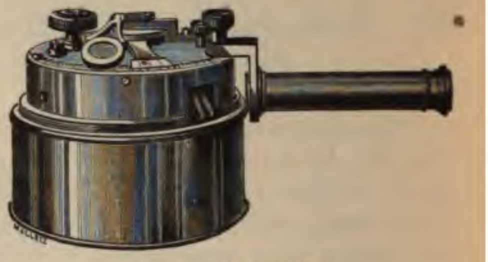


FIG. 42.—Box sextant.

(*Kindly lent by Messrs. W. F. Stanley & Co., Ltd.*)

Henderson's Rapid Traverser.—Mr. James Henderson has recently patented a very simple instrument (see Fig. 43) for measuring and recording the angles of a survey. It consists of a circular metal table, on the top of which is fixed, by means of several small brass screw-nuts and bolts, a thin disc of celluloid or other suitable material, about 10 inches in diameter.

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[1] For description of the sextant and mode of using, the reader is referred to *Hints to Travellers*, published by the Royal Geographical Society, also ***Surveying Instruments***, by W. F. Stanley.

Fixed on to the upper surface of the table and above the celluloid disc, by means of a centre-pin passing through, is a cross-bar, called an alidade, one side of which is bevelled. At each end of this cross-bar is a sight similar to the ordinary dial sight. By means of the usual clamping-screws, the table carrying the celluloid disc can be clamped to the stand, and the alidade, with the sights attached, can also be clamped to the table, when required. The disc is divided into five concentric rings, slightly scratched or grooved on the celluloid; and the bevelled edge of the alidade is notched out so as to afford to

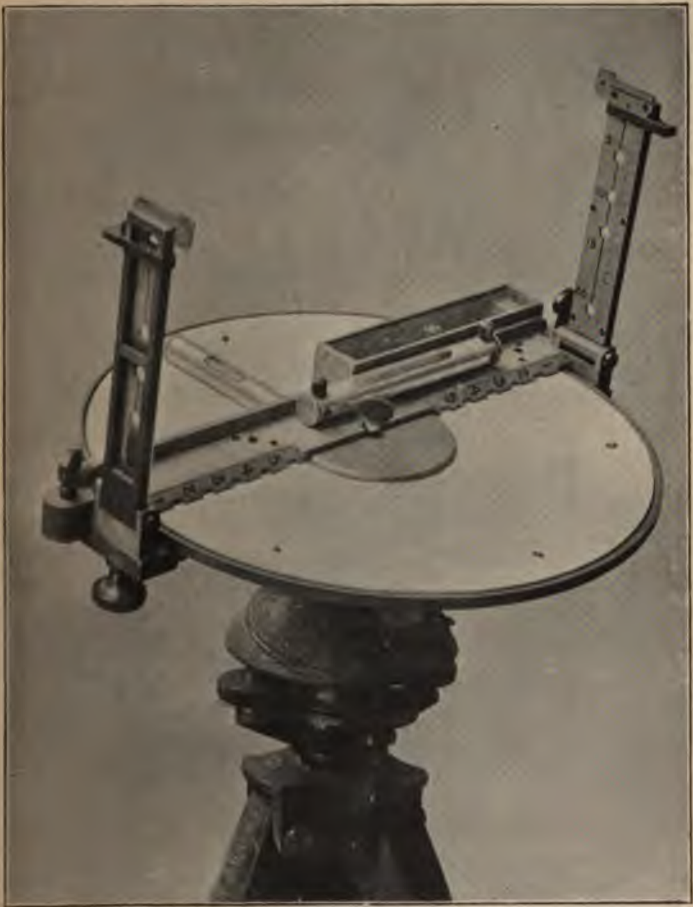

FIG. 43.—Henderson's rapid traverser.

each ring on the disc a certain length of bevelled edge, each length being distinguished by a number.

The object of these concentric rings is not only to permit separate surveys to be accomplished on one disc, but to avoid overcrowding of direction-lines in any particular spot on the disc. The semicircle for reading angles in a vertical plane with ordinary sights or telescope can be attached when required (see Fig. 44). The instrument is based on what is known as the plane-table system of surveying; unlike the plane table,

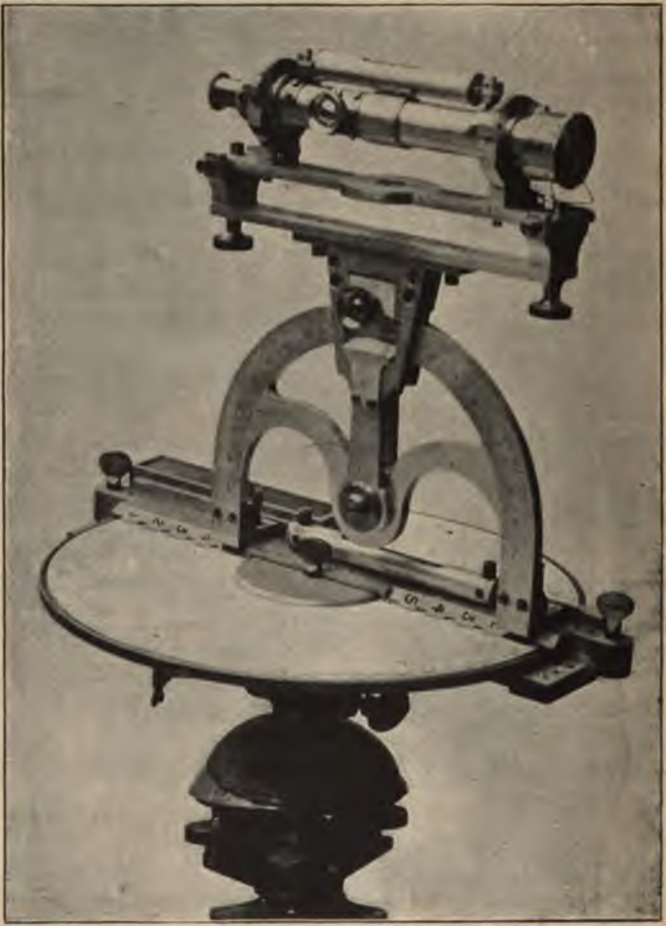

FIG. 44.—Henderson's rapid traverser, showing quadrant.

however, it is not intended that the rapid traverser should be used for plotting the survey in the field, but this is done afterwards, in the office, with the aid of a parallel ruler and scale.

The table is levelled by means of two spirit-levels, one of which is fixed on the alidade, and the other a small portable one which is carried in the pocket.

The magnetic meridian is taken, at any convenient point in the course of the survey, by means of a trough-compass placed temporarily against the back edge of the alidade. The actual direction of the lines of sight is indicated by making a pencil-mark on the disc, and at the conclusion of the survey the disc is taken off and the directions of the lines ruled off it on to the plan.

For future reference the disc itself may be kept, or else the magnetic bearings of the lines can be read off by means of a protractor and entered in the field-book, when the celluloid disc can be cleaned with soap and water or indiarubber, and so made ready for a future survey. The discs are now being made of enamelled zinc instead of celluloid.

Tacheometer (see Fig. 45).—This is an instrument used for measuring distances without a chain or tape. The ordinary tacheometer is similar to a theodolite, the only radical difference being in the telescope, in the diaphragm of which are fixed marks which can be directed to a graduated staff, such as a levelling-staff. The further the staff is from the instrument, the greater number of feet or inches will be seen between the two marks in the telescope. These marks may be made either of cobweb, like the ordinary hairs in the diaphragm of the theodolite, or of fine metallic points (in the later forms of instrument, lines engraved on a glass diaphragm are substituted for these hairs or wires); and they are placed at such a distance apart that the vertical height of an object between those two lines or points is some fraction, say 1 per cent., of the horizontal distance from the observer to the object. Thus if the vertical height on the graduated staff between the two points is 1 foot, the staff is 100 feet distant; if the vertical height is 10 feet, the staff is 1000 feet distant. According to the kind of work which it is intended to do, these points can be placed nearer together or further apart. The accuracy with which measurements can be made in this way depends upon the power

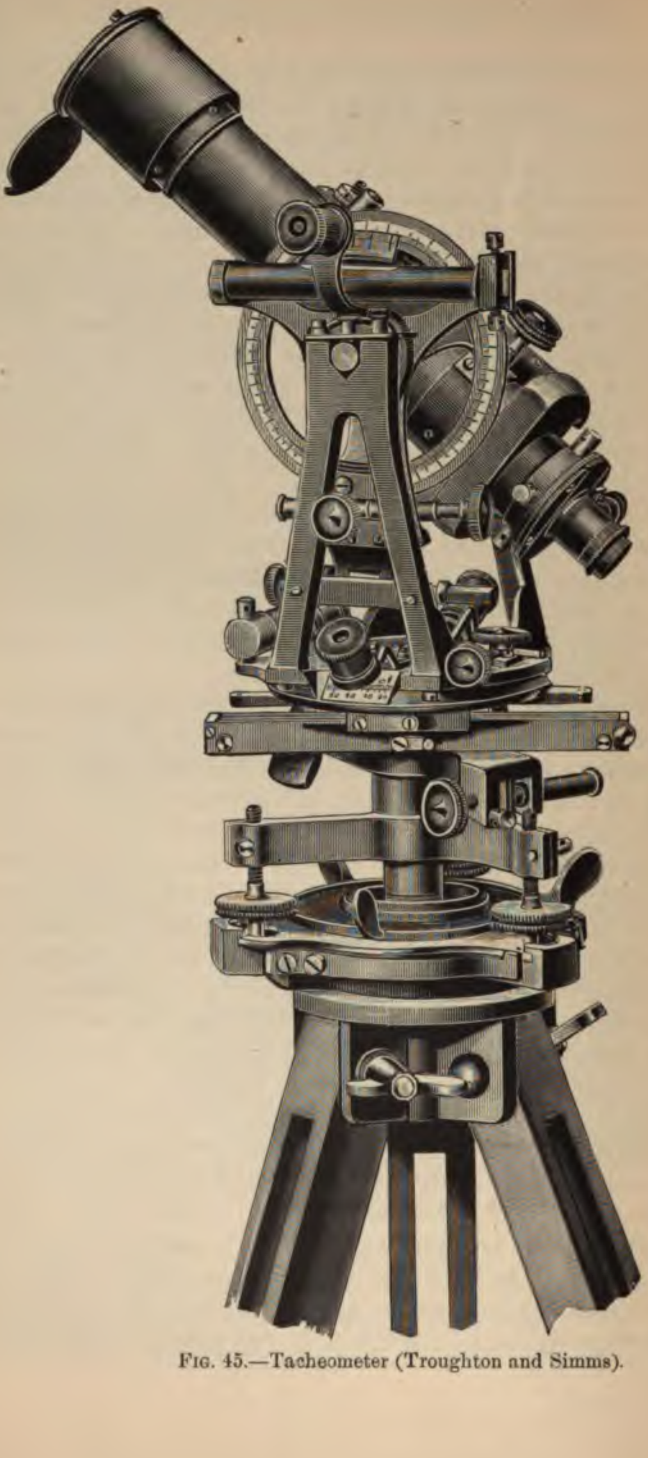

FIG. 45.—Tacheometer (Troughton and Simms).

of the telescope and of the microscopic eye-piece, and also upon the fineness of the points or cobweb used. Where it is possible to chain, the surveyor will, of course, employ this method in preference to the tacheometer, if great accuracy is required; but where, owing to the roughness or impassability of the ground, the measurement cannot be taken in this way, the tacheometer is of great use, and also for approximate measurements it is convenient.

With a telescope of moderate power (magnifying, say, fifteen diameters), and for distances not exceeding 500 feet, tacheometer-measurements, on a bright day, should be correct to 1 per cent.; for shorter distances, say under 300 feet, the error should not exceed ½ per cent.; with a more powerful telescope the error may be much less. Some engineers have claimed that the error has never exceeded 1 in 2000; but for such a degree of accuracy a very fine instrument and great care in using are necessary.

It is stated by surveyors of experience that a telescope magnifying fortyfold will read a staff to $\frac{1}{200}$ foot at a distance of 660 feet; and, supposing the arrangement of hairs in the diaphragm is such that 1 foot on the staff represents 100 feet horizontal distance, this means a possible error of ½ foot in 660, or an error of 1 in 1320. There is no doubt that with a good telescope great accuracy may be obtained with the tacheometer.

Measurement of Distances with Ordinary Theodolite.—It is possible to measure distances with the theodolite without the aid of two cross-hairs or other marks, by simply measuring the vertical arc subtended by a staff of given length. To measure lengths in this manner, direct the horizontal hair to the bottom of the staff or to some fixed mark above the bottom, and then, by means of the tangential screw, direct the horizontal hair to the top of the staff or some fixed mark, say 10 feet above the lower mark. Having read the angle, the distance can be calculated. Assuming that the staff is held vertically, and that the ground is level, the 10 feet will represent the chord of the arc. If the angle measured, for instance, was 1°, the natural chord is 0·017453; then the distance may be found by the following sum: 0·017453 : 1 : : 10 : 572·96. This method is not so handy as that with two hairs, because the calculation is longer, and it involves two readings with the telescope, and there is, perhaps, an additional chance of error; still, it is one that may be easily used in the absence of a tacheometrical attachment to the theodolite.

It follows, then, that when using a 10-foot staff, an error of one minute in the reading at a distance of 573 feet would mean an error of $\frac{1}{60}$ of that distance, or nearly 10 feet. The ordinary 5-inch theodolite is only graduated to read to minutes; but there is no reason why an error of one minute should be made in the reading. The error in the reading should not exceed half that; and it is not necessary that there should be any material error. The longer the staff, the less will be the error for a given length; but it is evident that for the accurate measurement of long lengths it is necessary to have a theodolite graduated to read to 10″. With such an instrument and a 10-foot staff, the error, instead of being 1 per cent., will be reduced to $\frac{1}{6}$ per cent., or 1 in 600.

In comparing the accuracy of tacheometer-measurements with that of ordinary chaining, it should be borne in mind that over rough ground, whether on the surface or in the mine, an error of half a link to the chain is very easily made, unless the surveyor gives the most careful personal attention to the laying out of the chain.

Some tacheometers are constructed on a slightly different principle. Instead of fixed points or cross-hairs at the diaphragm, between which is seen a length of a graduated staff, varying in exact proportion with the distance the staff is from the object-glass, a staff of fixed length is used, and at the diaphragm is a slide carrying a cross-hair, which can be raised or lowered by means of a screw until the whole length of the staff, or of two very clear marks on the staff, are included between two cross-hairs. The movement of this slide depends on the distance the staff is away; the further the staff is from the object-glass, the less the movement of the slide. This movement is measured by the turns of a screw, on the head of which is a scale; the distance corresponding with any given movement of the screw is marked upon the scale, so that no calculation has to be made.

In taking the observation, the two cross-hairs are so placed that one entirely obscures the other, and are directed towards one of the marks on the staff; the telescope is then clamped, and the requisite movement of the micrometer screw is made.

Many tacheometers are so made that the distance as read on the scale requires no correction; in others a correction is necessary, owing to the fact that the distance measured by a tacheometer of the simplest kind is from the principal focus of the object-glass, whilst the distance required is from the centre of the instrument at which the angles are measured; therefore the distance, as read off the staff, has to be corrected by the addition of a constant quantity equal to the sum of the focal distance of the object-glass, and the length from the object-glass to the centre of the theodolite. Thus, in using the theodolite with the fixed points, it is observed that the length of the graduated staff between them is, say, 2 feet; if the points have been adjusted so that the factor for length is 100, then the distance is $2 \times 100 = 200 +$ the length between the object-glass and the centre of the telescope (say 6 inches) + the focal length (say 12 inches), or the required length is 201·5 feet. If the lengths are required in links, the staff should be graduated in links and decimals.

Tacheometer Measurements in Hilly Country.—When the tacheometer is used for measuring lengths on a level country, the staff will, of course, be held in a vertical line. If, however, the ground is steeply inclined, then some consideration is necessary. In the first place, the telescope may be fixed quite level, and the staff held vertical, in which case the distance measured will be the horizontal length between the telescope and the staff (Fig. 46); of course, in this case, the length measured is limited by the height of the staff for the back sight, and the height of the telescope above the ground for the fore sight. In the second place (see Fig. 47), the telescope may be directed in a line parallel with the inclination of the ground, and the staff held at right angles to the inclination of

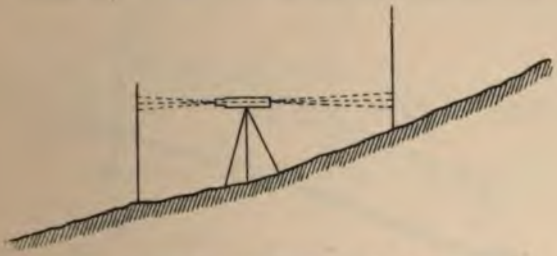

FIG. 46.—Tacheometrical measurements in hilly country.

the ground; then the distance measured will be the length of the slope and not the horizontal distance, which would have to be calculated by means of an observation of the angle made by the telescope with a horizontal line. In the third place (Fig. 48), the staff may be held vertical, and the telescope inclined at the same angle as the average slope of the ground, in which case the length measured will be greater than the length of the slope, and a correction will have to be made, owing to the greater length of the staff visible between the cross-hairs. Perhaps the best practice on steep gradients is to hold the staff at right angles to the incline; for moderate inclines the errors due to not holding the staff exactly in the correct position are very slight when this method is employed.

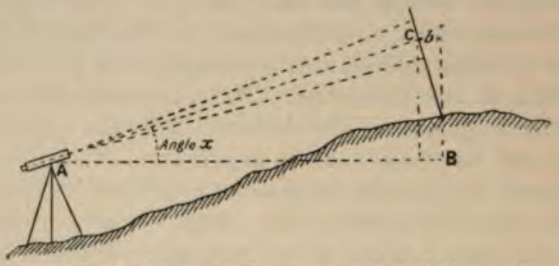

FIG. 47.—Tacheometrical measurements in hilly country.

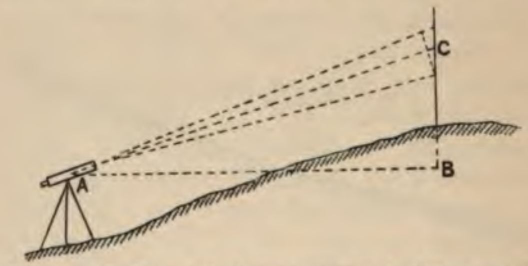

FIG. 48.—Tacheometrical measurements in hilly country.

For further information on the subject of tacheometry, the reader is referred to Mr. T. G. Gribble's excellent book on *Preliminary Survey* (Longmans, Green and Co.).

Prismatic Stadia-telescope.[1]—An ingenious modification of the ordinary stadia-telescope (tacheometer) is to use a glass prism or wedge. A ray of light passing through a prism is deflected, the amount of deflection depending on the angle enclosed by the two sides of the prism at their apex if prolonged. If, therefore, half the object-glass of a telescope is covered with a prism, and a graduated staff is observed, the figures on one side will be seen in their correct position; on the other side they will be seen out of place, owing to the deflection caused by the prism. Thus, if with the uncovered half of the object-glass the cross-hairs of the telescope appear to cut the figure 3, with the covered half the cross-hair may appear to cut the figure 5, showing that the deflection of the rays of light caused by the prism is measured by 2 feet on the staff if the staff is distant 100 feet. This deflection is equal to an angle of about 1° 9′. If, therefore, the staff were moved to a distance of 200 feet from the telescope, the deflection, being at the same angle, would cover 4 feet of the staff; and if the staff were moved to a distance of 300 feet, the deflection would cover 6 feet of the staff, and so on.

---

[1] Robert H. Richards, Boston, America, Inst. M.E. Montreal Meeting, February 1893; also Glen Summit Meeting, October, 1891.

This angle of deflection being ascertained, it follows that the distance at which the staff is held from the telescope can be calculated from the amount of deflection as read on the staff. Thus—

If the figure read with one half of the	telescope is	3,	and with the	other half	4,	the distance is	50
,,	,,	3	,,	,,	5	,,	100
,,	,,	3	,,	,,	6	,,	150
,,	,,	3	,,	,,	7	,,	200
,,	,,	3	,,	,,	8	,,	250
,,	,,	3	,,	,,	9	,,	300

and so on, every foot of deflection on the staff representing 50 feet of distance from the telescope, every $\frac{1}{10}$ foot representing 5 feet; $\frac{1}{100}$ foot, 0·5 foot; and $\frac{1}{1000}$ foot, 0·05 foot.

Mr. Robert H. Richards has tried various telescopes in which the deflection of the prism varies from 1 foot of staff in 100 feet in length, to 3 feet of staff in 100 feet in length. The greater the deflection, the greater the accuracy with which the amount of it can be read; on the other hand, the greater the deflection, the longer the staff required for any given distance.

Mr. Richards considers a telescope magnifying thirty diameters suitable for reading the staff at a distance of 1000 feet, and for distances up to 2500 feet, with a specially constructed sliding target staff. For a sight of 1000 feet and a prism deflecting 1 per cent., a staff about 12 feet long is required.

Mr. Richards also recommends the use of what he calls the optical vernier. This may be understood by reference to Figs. 49 and 50. This is a staff about 6 inches wide, and a height necessary for the distance it is intended to read; it is painted half white and half black. On the upper left-hand side is a vernier painted in white; the rest of the left-hand side of the staff is black, and the main scale is painted in black opposite this. It is divided into lengths representing 50 feet of horizontal distance, which are numbered 1, 2, 3, 4, 5, 6, etc.; this means six fifties, or 300 feet. Each fifty is divided by five equidistant diamond points, representing 10 feet. The vernier is also divided so that five points shall cover a space equal to four points on the main scale. The main scale is seen through the uncovered half of the telescope, the vernier through the prism. The prism deflects the vernier, and it is thrown down opposite some figure on the main scale.

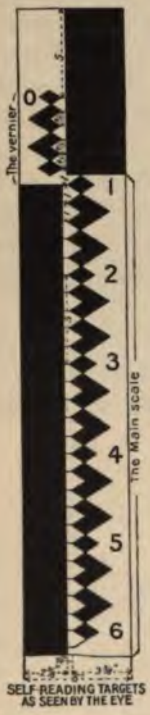

FIG. 49.—Staff used in connection with a prismatic stadia telescope.

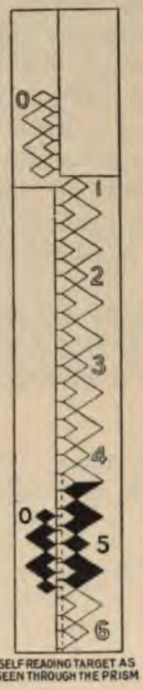

FIG. 50.—Method of reading ditto.

In Fig. 50 the zero of the vernier is apparently past the third point below 4; 4 means 4 times 50, or 200; the three points on the main scale are each 10 feet, therefore the distance is 230 + a fraction of 10. The second point of the vernier from zero is exactly opposite one of the points on the main scale; each point of the vernier counts 2, therefore the fraction is $\frac{4}{10} \times 10$, or 4 feet. So that the total distance is 234. The heights on the main scale, representing 50 feet of distance, have been found by experiments with the prism.

Tape Target.—At distances greater than 1000 feet, the figures on the staff cannot be read, and Mr. Richards recommends a tape target, the distance being read by the assistant carrying the tape. This target is shown in Fig. 51. The telescope is directed towards the centre of three diamond points on one target; the other target is moved along the tape, in accordance with signals given by the surveyor, until its deflected image becomes opposite to the image seen through the uncovered portion of the object-glass; the two centre diamonds of each set of three correspond when the targets are set at the correct distance apart; the two outer diamonds do not correspond, and the distance of their points apart should be equal for each pair, as shown in Fig. 52. The assistant reads the distance on the tape, and books the figure, and perhaps signals the reading to the surveyor.

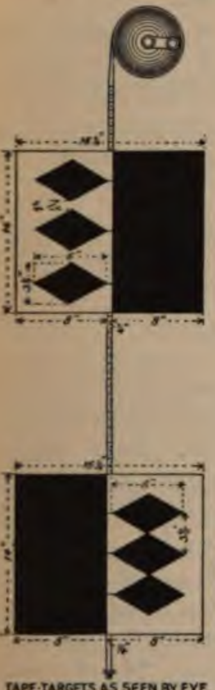

FIG. 51.—Tape with movable targets.

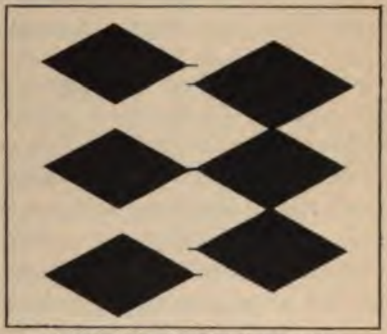

FIG. 52.—Method of reading movable targets.

All the systems of tacheometry above described necessitate the use of a staff on which the graduations can be read through a telescope, or on which are movable marks which can be read by an assistant who adjusts the marks in accordance with signals received by flags or otherwise from the surveyor.

It is, however, very convenient to use a range-finder, with which the surveyor is independent of any markings upon a staff. The ordinary method of triangulation with the theodolite from a measured base is a kind of range-finding, and for exact work it is the best method known.

For approximate calculations, such as are used sometimes by military engineers, a tape or cord of given length may be carried by two observers, each carrying a box sextant, and reading simultaneously the angle formed by the base-line and the object whose distance they wish to ascertain.

Range-finder.—An ingenious adaptation of the prism has been devised by Professors Barr and Stroud. In this instrument the measured base is a short tube, 3 feet long, held by the surveyor in his hand, or fixed on a tripod. The tube is held at right angles to the line of sight (see Fig. 53). It contains the equivalent of two telescopes, one at each end of the tube or base, with the requisite optical appliances for seeing the two fields of view in juxtaposition one over the other. Rays of light from the object viewed enter through openings, V_1, V_2, at each end of the tube, and are reflected at right angles along the axis of the telescope by means of the reflectors H_1, H_2. The observer places his right eye at the eye-piece K_2, and, by means of the arrangement of prisms at J, sees two images of the object, one above the other, but not in line with each other. By the movement of an achromatic glass prism, M, of small angle along the axis of one of the telescopes, the two images of the object whose range is required are brought into exact alignment, when the position of the prism furnishes a measure of the range, which is read off by the left eye on a scale, B, attached to the prism, and moving with it. The surveyor has, therefore, no calculations to make, but simply sets his instrument upon the object, such as a staff, church, house, tree, fence corner, candle, lamp, etc., and then, after adjusting the two images of the object in exact alignment, reads the distance as written on the instrument. A similar instrument has been adopted in H.M Navy, and is now installed on most of the battle-ships and cruisers.

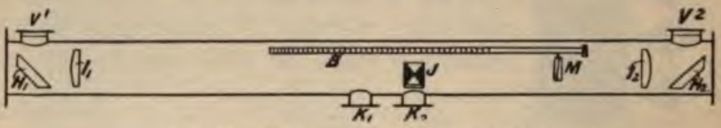

Fig. 53.—Barr and Stroud's range-finder.

CHAPTER V.

INSTRUMENTS FOR PLOTTING LENGTHS AND ANGLES.

In considering the use of instruments for plotting angles, it will be well to refer to the plan of an estate shown in Fig. 14. On this plan the bearing of No. 1 line is marked "North 10° East," which means that the direction of the line from the starting-point is going towards the north-east, and the exact bearing is 10° east of north; the bearing of No. 2 line is also given as N. 30° W., and line No. 12 is S. 74° W. If these lines are laid down according to the bearings so marked, and for the lengths measured, they will take up their correct position as regards each other, and it will not be necessary to use the compasses for the purpose of plotting them. If, however, the lines have been already plotted from the measurements only, the bearings can be used as a check on the accuracy of the survey and of the plottings, because the relative positions of the lines, as shown by the bearings, will be the same as that shown by the triangular measurements.

One use, therefore, of an instrument for taking these bearings is to check the accuracy of the survey; the second use is, perhaps, more important, and that is to ascertain the direction of the survey-lines with regard to the magnetic meridian, and for most mineral plans it is necessary to have the magnetic meridian, or "north point," as it is commonly called, marked with extreme care.

In the production of a plan, two distinct classes of instruments are necessary. These are, first, the instruments previously described for measuring lengths and angles on the ground, and second, the instruments for drawing or plotting upon paper the above-mentioned lengths and angles.

Scales.—The instrument for drawing the lengths is called the scale: it consists of a straight piece of hard material, either ivory, wood, metal, or cardboard; it is generally a little more than 12 inches long, and is divided into equal parts to suit the purpose required. For an ordinary English mining plan it is usual to have a scale of chains, the measurements being taken with the Gunter's chain. Thus it may be desired that 1 chain in length shall be represented by a length of 1 inch on the plan; then the scale will be divided into inches. If, however, this would produce too big a plan, ½ inch may be used to represent 1 chain, and the scale will therefore be divided into half-inches, or it may be divided into thirds, fourths, fifths, sixths, eighths, or tenths of an inch, each division intended to represent 1 chain. The most common scale for mining plans is that in which ½ inch represents 1 chain, commonly called a 2-chain scale, which means that 1 inch on the plan is equal to 2 chains measured in the field or mine.

In the Coal-Mines Regulation Act of 1887 it is mentioned that the scale of a colliery plan must not be less than 25·344 inches to the mile (which is equivalent to 3·157 chains to 1 inch).[1] This seems to give sufficient latitude as to the size of scale to be adopted; in many mines a scale of 3 chains to 1 inch is used; in others, a scale of 1 chain, and sometimes half a chain to the inch.

Having divided the scale into chain-lengths, each chain-length is then subdivided into tenths, each tenth representing 10 links. The surveyor, in plotting a length more than 10 and less than 20 links, must divide the space by his eye, as smaller graduations are not generally used. The edge of the scale is bevelled, so that the dividing marks on the edge of the scale touch the paper. It is found convenient to have on the opposite edge of the scale to that on which the chain-scale is divided, a feet-scale. This is a scale in which sixty-six divisions on the feet-edge measure the same distance as 100 divisions on the opposite or chain-edge. This enables the scale to be used for taking off measurements in feet from a plan which has been plotted in links. The use of feet and links on the same scale, however, often leads to confusion and error.

Another scale is also used, called an offset scale. It is generally 2 inches in length, graduated in the same manner as the long scale, but the divisions begin and end exactly at the ends of the scale. It is used in the manner shown in Fig. 54, to mark off lengths at right angles to the lines drawn on the plan. The scale is laid down on the paper along the line representing the survey-line; the offset scale is then placed so as to measure lines at right angles, and is moved along the scale to the division representing the required distance on the survey-line; the length of the offset can then be marked off by means of the shorter scale.

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[1] The exact wording of the Act of 1887 is as follows: "Every such plan must be on a scale of not less than that of the Ordnance Survey of twenty-five inches to the mile, or on the same scale as the plan for the time being in use at the mine."

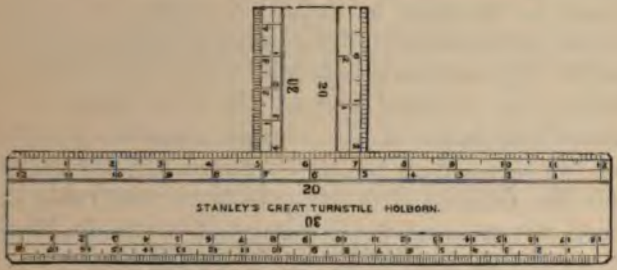

FIG. 54.—Scale and offset.

(Kindly lent by Messrs. W. F. Stanley and Co., Ltd.)

In constructing a plan, the scale is usually drawn upon it, and thus, if any serious shrinkage of the paper takes place, measurements may be made by means of this scale.

Ivory is much liked for scales, because of the clearness of the lines, but boxwood is cheaper and less easily broken; metal is not much used, partly, perhaps, because of its greater tendency to expand or contract with variations of the temperature. The expansion of brass between freezing point and boiling point is $\frac{1}{500}$ of its original length, which is equal to $\frac{1}{90000}$ part for each degree of temperature, or to the expansion of $\frac{1}{2250}$ part for a rise of 40° in temperature, that is to say, the scale would expand 1 inch in a length of 2250 inches, or $\frac{1}{22}$ part of an inch in a length of 100 inches. This amount of expansion is not very serious, especially as the temperature of a drawing office in England does not usually vary as much as 40°; it is seldom that drawing is done in an office of a less temperature than 50° or a higher temperature than 65°, hence the expansion would be only that due to 15°, or $\frac{1}{60}$ inch in a total length of 100 inches; therefore the expansion of brass does not seem to be a sufficient reason why it should not be used. A more practical objection is that metal scales soil the drawings.

Ordnance Maps.—The survey of the United Kingdom was commenced by order of the Government about the year 1784.

The survey has been published in maps of various scales, viz. 1 inch to the mile, or $\frac{1}{63360}$; 6 inches to the mile, or $\frac{1}{10560}$; and 25·344 inches to the mile, or $\frac{1}{2500}$. Town plans, on scales of 10½ feet to a mile and 5 feet to a mile, are also published of the principal towns. On these maps are shown the various boundaries of the counties, unions, parishes, etc. The first two series show the contour-lines, and are particularly useful for the purpose of deciding as to the best route to adopt for lines of railway, and the positions of shafts, buildings, etc. They also show the lines of latitude and longitude.

The $\frac{1}{2500}$ scale maps can be obtained either plain or with the buildings and rivers coloured. The fields are all numbered, and the area of each field in acres is either printed on the map or can be obtained for each parish, published in book form.

A plan made by mounting the various sheets of the $\frac{1}{2500}$ map covering the royalty is sometimes used on which to mark underground workings. By application to the Director-General of the Ordnance Survey Office, Southampton, however, tracings from the original plotted plans can be obtained, and these are much more accurate for this purpose, as the printed maps often shrink a good deal. Owing to this latter fact, measurements from the Ordnance plans should be made with the printed scale given on each sheet.

Geological maps are also published on the 1-inch and 6-inch scales, and give a great deal of valuable information as to the faults, dip of the measures, and other geological features of the country.

Compasses.—Compasses are generally used to set off the distances from the base-line as previously explained; these are shown in Fig. 55. They are made in various sizes, ranging from 2½ inches to 9 inches long. There are points at the end of each limb, needle-points are the best; one limb is jointed, so that the needle-point can be taken out, and a pencil, *a*, or pen, *b*, substituted; one or more lengthening pieces, *c*, can be added to this limb, so as to increase the length that can be set out. When this length is insufficient, beam compasses are used. These are formed with a beam, or piece of wood, and are shown in Fig. 56. At one end of this beam is fastened a screw-clip, *a*, carrying a point at right angles to the beam, and about 2 inches long. A similar clip, *b*, carrying a pencil is slipped on to the beam, and is moved along till the required distance from the point fixed at the other end is obtained. It is then clamped, and an exact adjustment for length is made with an adjusting screw on the point-holder; then, with the fixed point as the centre, a circle may be described with the pencil-point. Several beams of say 2, 4, and 6 feet in length are kept for use with these compasses.

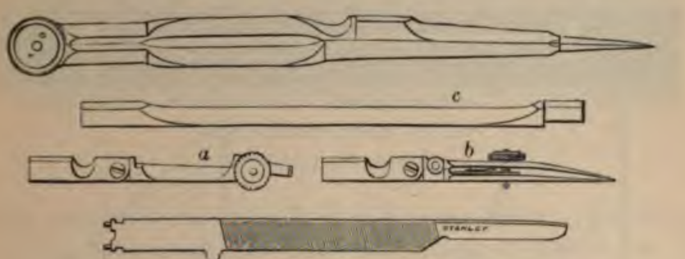

FIG. 55.—Compasses.
(*Kindly lent by Messrs. W. F. Stanley and Co., Ltd.*)

Straight-edge.—For the purpose of ruling a straight line from one point to another, a straight-edge is used; a metal straight-edge is the best, not being liable to warp. Steel straight-edges require to be kept bright, and are sometimes nickel-plated. Though not absolutely necessary, it is a good thing to have bevelled edges to the ruler.

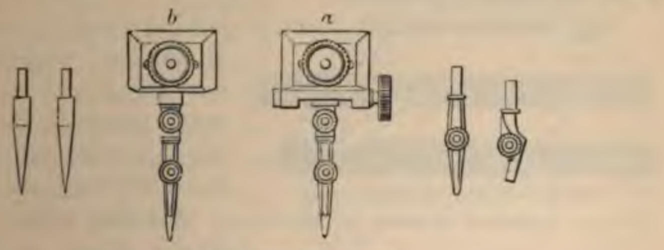

FIG. 56.—Beam compasses.
(*Kindly lent by Messrs. W. F. Stanley and Co., Ltd.*)

Parallel Ruler.—A parallel ruler (Fig. 57) is much used by mining surveyors; it is generally made of metal, as a considerable weight is advantageous; it consists of a bar from 2 inches to 3 inches wide, and from $\frac{3}{32}$ inch to $\frac{3}{16}$ inch in thickness, with bevelled edges, and varying from 6 inches up to 2 feet in length. In this bar are cut two holes within a short distance of each end; on the upper side of the bar are fixed two rollers, fixed on a long spindle, the ends of which are carried in brackets; the lower sides of the rollers project a little way through the bar, so that the bar may roll along. Each of these rollers is the same diameter, and is roughened with longitudinal cuts to prevent it from slipping. These rollers being the same diameter, if there is no slipping, the two ends of the bar will move at the same rate and the same distance when rolled along over the paper. Thus, if the ruler is held in a given position, and a line drawn, and it is then carefully rolled across the paper, and another line drawn, the two lines will be parallel one to the other.

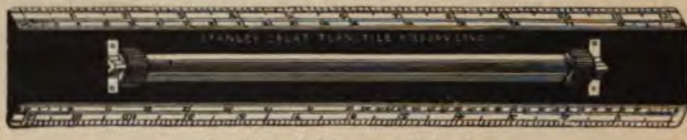

FIG. 57.—Rolling parallel ruler.

(Kindly lent by Messrs. W. F. Stanley and Co., Ltd.)

Fig. 58 shows another form of parallel ruler, known as the sliding-bar parallel ruler, but for the purposes of a mine surveyor the rolling parallel ruler will be found to be the most efficient.

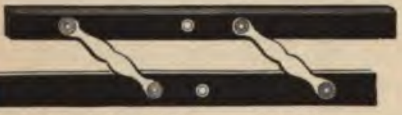

FIG. 58.—Sliding parallel ruler.

(Kindly lent by Messrs. W. F. Stanley and Co., Ltd.)

Drawing-pencil.—In plotting a survey the lines are drawn with a hard-lead pencil cut to a fine point. Pencils are made in varying degrees of hardness, the most useful being that marked H.H. The Koh-i-noor pencil is highly recommended.

Pricker.—Distances and stations are generally marked off the scale with a needle-pointed pricker, the point of the needle making a much finer and more permanent mark than the point of the pencil. In this way a length may be marked on the 2-chain scale with an error not exceeding 1 link; thus if the actual distance measured was 8 chains 55 links, the prick-mark made with the needle might possibly be 8 chains 54 links or 8 chains 56 links, but, in either case, it would be within a link of the correct distance.

Set-squares.—A large set-square is useful for marking out lines at right angles to one another; such lines are required for plotting lengths ascertained by trigonometrical computation; the larger this set-square, the greater the degree of accuracy with which the cross-lines can be drawn. The draughtsman is recommended to use one not less than 12 inches long on each of the square sides. The two most usual forms of set-square are shown in Fig. 59.

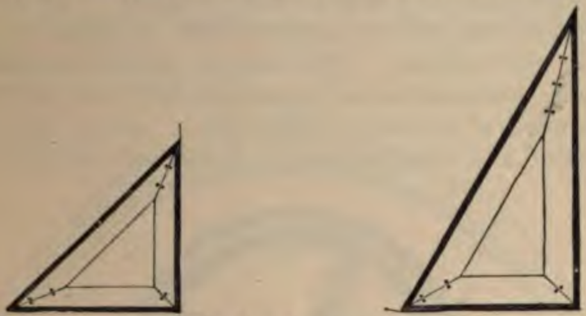

FIG. 59.—Set squares.
(*Kindly lent by Messrs. W. F. Stanley and Co., Ltd.*)

Protractor. — For plotting angles a graduated circle marked in a similar way to the dial, called a protractor, is used (see Fig. 60).[1] These may be made of brass, and vary from 8 to 12 inches in diameter; the 8-inch protractor is graduated to half-degrees, and the 12-inch protractor to quarter-degrees, smaller fractions of a degree having to be estimated; the protractor being so much larger than the dial, the fractions of a degree can be estimated with greater

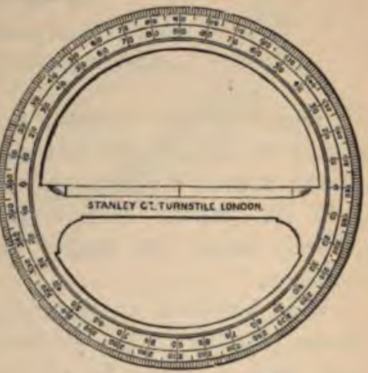
STANLEY GT. TURNSTILE LONDON.

FIG. 60.—Brass protractor.
(*Kindly lent by Messrs. W. F. Stanley and Co., Ltd.*)

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[1] For plotting circle readings of the needle, the numbers on the protractor should count the reverse way of those on the dial.

accuracy, and therefore there should be no serious errors in plotting from this cause.

It is, however, difficult with an 8-inch protractor to divide a degree without some error, which may very likely amount to $\frac{1}{8}$°; the thickness of a needle-prick is about $\frac{1}{8}$° on an 8-inch protractor, so that for very accurate work a simple 8-inch protractor is not sufficient. By using a 12-inch protractor the accuracy is increased in the proportion of 2 to 3; but for very accurate work a protractor fitted with a vernier with folding arms, clamp, and tangent-screw is sometimes used (Fig. 61).

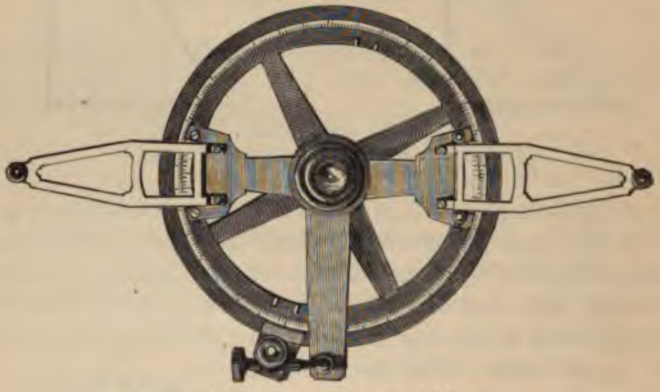

FIG. 61.—Brass protractor with folding arms.
(Kindly lent by Messrs. W. F. Stanley and Co., Ltd.)

By means of the vernier the arms may be adjusted to 1′, that is to say, to the sixtieth part of a degree. At the end of each arm is a sharp pricker, which can be pressed down to mark the paper. If this instrument is well constructed and properly used, the angles can be marked out with great accuracy.

It is, however, a common practice to use a cardboard protractor (Fig. 62). The graduated circle is printed on to a stout card, and is generally 12 or 15 inches in diameter. The divisions are made to read inward from the circumference, instead of outwards as with other protractors, the centre space of the card being entirely cut away.

In using cardboard protractors it is not necessary to prick off the angle, as the parallel ruler can be placed upon the protractor at the right angle, and then rolled to the required place, provided, of course, that the work is within the circumference of the protractor. For plotting underground surveys, where the lines are usually short and close together, these protractors are very convenient.

A modified form of cardboard protractor has been designed by Mr. R. F. Percy,[1] and is shown in Fig. 63. It is made of

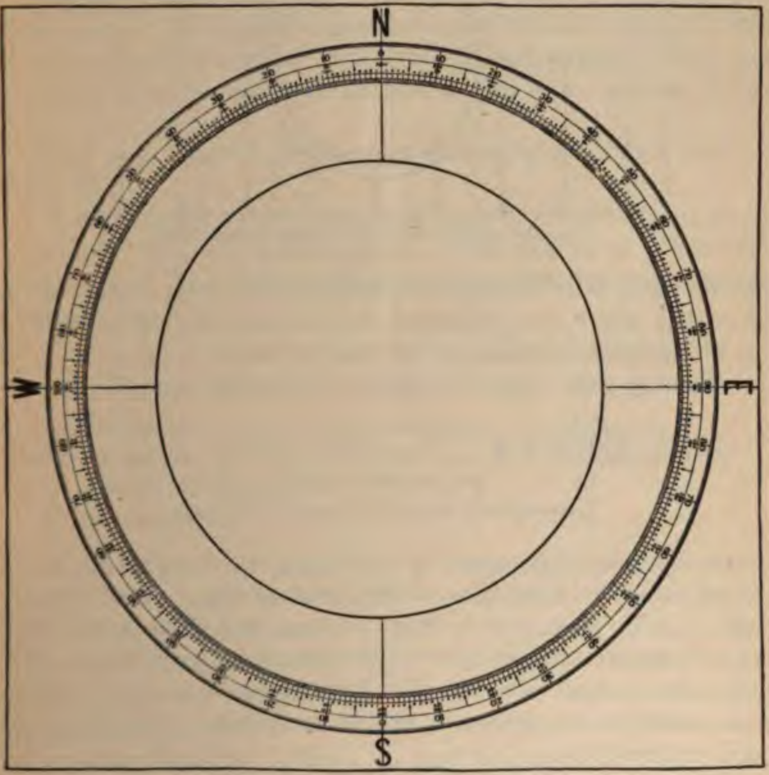

Fig. 62.—Cardboard protractor.

thin pasteboard. Parallel north-and-south lines are, with the greatest care and accuracy, drawn at intervals of 2 or 3 inches, and at the ends of all these parallels, on the left at the north edge, and on the right at the south edge, divergences of 1° and fractions of 1° are indicated (Fig. 63). The part within the divided circle is, as usual, cut away, and the plotting is executed within that space.

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[1] *Transactions Fed. Institute of Mining Engineers*, vol. xiii. p. 585.

The parallel meridians allow the protractor to be placed exactly where it is needed, very few meridian-lines being required on the plan. The divergences marked at the ends of the parallel lines will allow the protractor to be twisted for declination, so as to bring the meridian to the date of the survey.

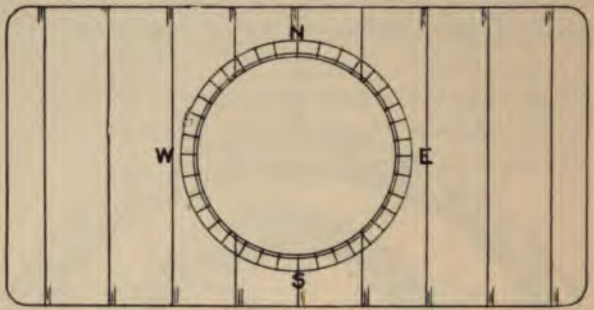

FIG. 63.—Percy's form of cardboard protractor.

Drawing-pens.—Fig. 64 shows a drawing-pen; it has two pointed blades, kept apart by a spring; the distance apart can be adjusted by turning a milled-head screw. It is supplied with ink by means of a brush or pen, and when used should be held nearly upright between the thumb and forefinger. After being used some time, the nibs become blunt, and will require sharpening on an oil-stone; this is an operation requiring some skill and practice.

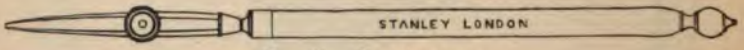

FIG. 64.—Drawing-pen.
(*Kindly lent by Messrs. W. F. Stanley and Co., Ltd.*)

Curves.—For drawing curved lines, such as railway curves, it is found useful to have ruling-edges made of pearwood or cardboard. These are cut to arcs of circles with radii varying from 1 to 250 inches, and are sold in sets.

Weights and Pins.—To hold the plan while working at it, drawing-pins may be used, but these injure the plan. Lead or iron weights are more commonly used by mine surveyors; they are of oblong form, and covered with cloth or leather so as not to soil the paper.

Colours and Brushes.—For inking-in the finished plan, Indian ink is used. This is generally sold in hexagonal or octagonal sticks, and is ground into liquid ink by rubbing with water upon some kind of palette. The rubbing is continued until a line drawn with the ink dries quite black. Lines drawn with the best ink, however, are liable to run when colour is washed over them, so the lines should be as fine as possible.

Liquid Indian ink may be obtained which overcomes this defect, but it is hardly so good to draw with as the stick ink. Water-colours are used for colouring drawings; they are supplied in cakes, and are ground in the same way as Indian ink.

The best kind of brushes for colouring are those made of sable hair.

Drawing-paper.—It is important that the best drawing-paper should be used for mining plans. That known as Whatman's is very good. The sizes in which sheets of drawing-paper can be obtained are—

Demy	20	inches	by	15¼	inches
Medium	22¾	,,	,,	17	,,
Royal	24	,,	,,	19¼	,,
Imperial	30	,,	,,	22	,,
Double elephant ...	40	,,	,,	27	,,
Antiquarian	53	,,	,,	31	,,

Mounted plan paper can also be obtained in continuous rolls in widths varying from 27 inches to 60 inches, or paper can be mounted to order to make a plan of any size. For a large permanent plan the best paper mounted on strong brown holland will cost as much as 5*d.* to 8*d.* a square foot. The thickness of the paper and holland together varies from about $\frac{1}{76}$ inch up to about $\frac{1}{32}$ inch; $\frac{1}{38}$ inch makes a very good plan.

Tracings.—Copies of drawings are usually made on tracing-paper or tracing-cloth, which are transparent. These may be obtained in continuous rolls the same as the drawing-paper.

CHAPTER VI.

GEOMETRY, TRIGONOMETRY, LOGARITHMS.

Before proceeding to consider the method of surveying on the surface by means of angles, or of underground surveying which is always done by means of instruments for measuring angles, it will be necessary to consider the relations of the sides and angles of a triangle to each other, which are ascertained by the science of Trigonometry.

A slight knowledge of Geometry is also necessary. The definitions given below are taken from Euclid's *Elements*.

Fig. 65 shows a circle; the point **A**, from which it has been described, is called the centre of the circle.

The diameter of a circle is a straight line drawn through the centre, terminated both ways by the circumference (**BC**, Fig. 65).

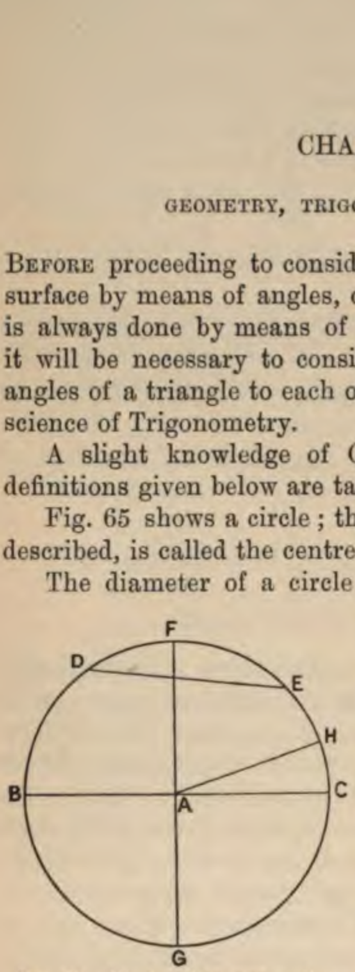

Fig. 65.—Circle: diameter, radius, chord, arc.

The radius of a circle is a straight line drawn from the centre to the circumference (**AB**, Fig. 65).

The circumference of a circle is the line described by the pencil of the compass when it is revolved round a point.

A chord is any straight line drawn across the circle from circumference to circumference, not passing through the centre (**DE**, Fig. 65).

An arc is that part of the circumference of a circle which lies between the two ends of a chord (**DFE**, Fig. 65).

An angle is formed when two straight lines, not in the same straight line, meet together. The unit adopted in measuring angles is the degree. The circle is divided into 360 degrees (written °); each degree is subdivided into sixty equal parts, called minutes (written ′); and each minute is subdivided into sixty equal parts, called seconds (written ″). The circle is also divided into four equal parts, called quadrants, each containing 90 degrees (**CAF, BAF, BAG, CAG,** Fig. 65).

The measure of any angle (**CAH,** Fig. 65) is the number of degrees covered by the arc **CH.**

A right angle encloses 90 degrees; a straight line at right angles to another straight line is said to be a perpendicular (Fig. 66, (1)).

An obtuse angle contains more than 90° (Fig. 66, (2)).

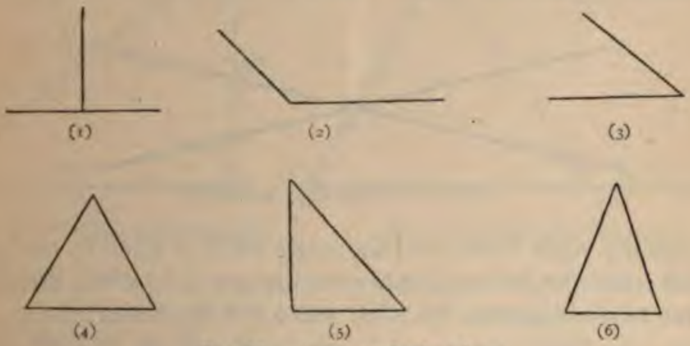

FIG. 66.—Angles and triangles.

An acute angle contains less than 90° (Fig. 66, (3)).

A triangle is a figure contained by three straight lines. An equilateral triangle has three equal sides, and three equal angles (Fig. 66, (4)); an isosceles triangle has two sides equal (Fig. 66, (6)); a right-angled triangle is that which has one of its angles a right angle (Fig. 66, (5)).

A square has four equal sides, and all its angles are right angles.

A rectangle has all its angles right angles, but not all its sides equal.

A trapezium is a plane figure contained by four straight lines, of which no two are parallel.

Parallel straight lines are those which, if produced both ways, would never meet.

The following theorems are also taken from Euclid, and should be thoroughly mastered:—

(1) When a straight line meets another straight line, the angles formed are together equal to two right angles. Referring to Fig. 67, the two angles **ABC** and **ABD** together equal two right angles, or 180°, so that if we know the number of degrees in one angle, we can find the magnitude of the other by subtraction.

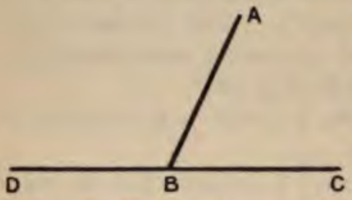

FIG. 67.—Elementary geometry.

(2) If two straight lines cut one another, the vertical or opposite angles are equal. Thus in Fig. 68 the angle **AEC** equals the angle **DEB**, and the angle **AED** is equal to the angle **CEB**; and the four angles are together equal to 360°; therefore, if one angle is known, the other three can be calculated.

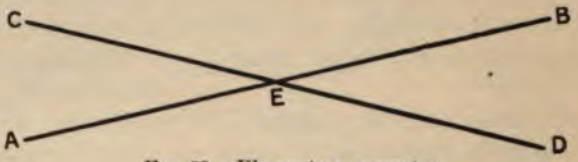

FIG. 68.—Elementary geometry.

(3) If a straight line, **EF**, fall on two parallel straight lines **AB** and **CD**, the angles **AGH** and **GHD** are equal, the angles **EGB** and **GHD** are equal, and the two angles **BGH** and **GHD** are together equal to two right angles (see Fig. 69).

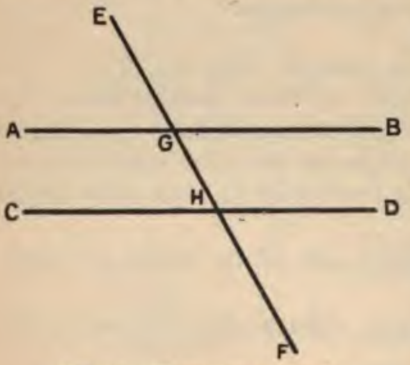

FIG. 69.—Elementary geometry.

(4) The angles at the base of an isosceles triangle (see Fig. 66, (6)) are equal to one another.

(5) The three angles of a triangle are together equal to two right angles, or 180°; therefore, knowing the two angles, we can get the third by subtraction.

(6) Any two sides of a triangle must be together greater than the third.

(7) In any right-angled triangle, the square which is described on the side opposite the right angle is equal to the sum of the squares described on the sides containing the right angle. Fig. 70 shows a right-angled triangle; then $\mathbf{AB}^2 = \mathbf{AC}^2 + \mathbf{BC}^2$. Suppose **AC** is 80, **BC** is 100; then to find **AB**—

$$\mathbf{AB}^2 = (80)^2 + (100)^2 \qquad \therefore \mathbf{AB} = 128{\cdot}1$$

In the same way, we can find **AC** or **BC**, if we have the other two sides of the triangle given.

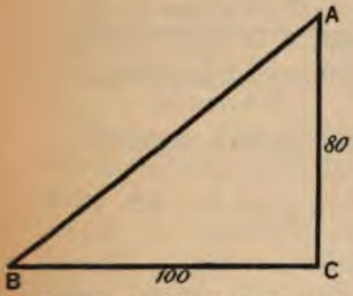

FIG. 70.—A right-angled triangle.

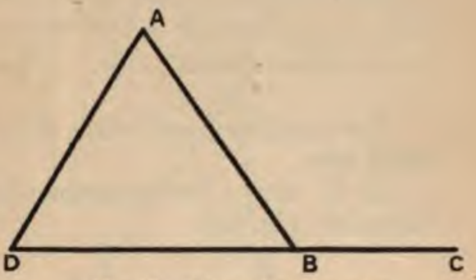

FIG. 71.—Elementary geometry.

(8) If one side of a triangle be produced, the external angle is equal to the sum of the two opposite internal angles. The angle **ABC** is equal to the sum of the angles **DAB** and **ADB** (Fig. 71).

(9) In every triangle equal sides subtend (or are opposite to) equal angles, the greatest side subtends the greatest angle, and the least side the least angle.

Practical Geometry.

(1) To bisect a line **AB** (Fig. 72); that is, to divide it into two equal parts. From **A** and **B**, with any radius greater than the half of **AB**, describe arcs cutting each other in *c* and *d*. From *c* draw a straight line to *d*, and it will bisect the line **AB**.

(2) To draw a line perpendicular to a given line **AB** at a point **C** in the line (Fig. 73).

From **C**, with any radius, cut the line **AB** in *c*, *c*; from *c*, *c*, with any radius greater than half *cc*, describe arcs cutting in *d*; draw the line **C***d*, and it will be perpendicular to **AB**.

215374B

(3) To draw a line perpendicular to a given line **AB**, from a point **C** above or below the line (Fig. 74).

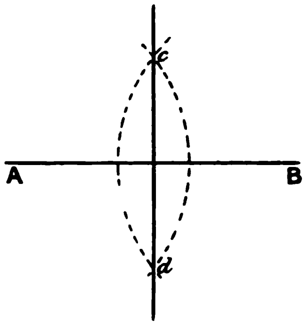

FIG. 72.—Method of bisecting a line.

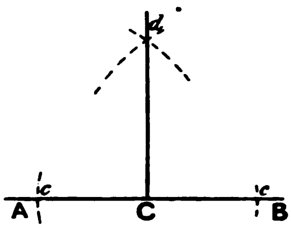

FIG. 73.—To draw a perpendicular line.

The description and letters of the last problem apply to this figure also.

(4) To draw a line perpendicular to a given line **AB**, at its extremity (Fig. 75).

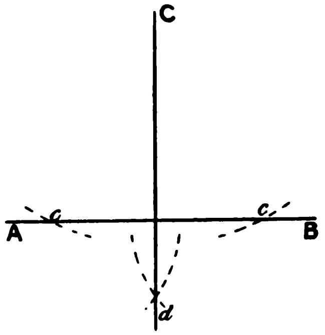

FIG. 74.—To draw a perpendicular line.

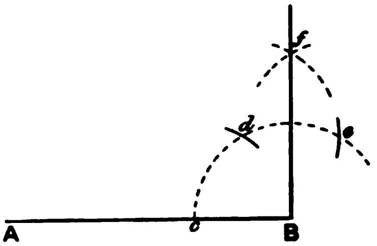

FIG. 75.—To draw a perpendicular line.

From **B**, with any radius, describe an arc having its extremity *c* in the line **AB**.

From *c*, with the same radius, cut the arc in *d*; and from *d*, with the same radius, cut the arc in *e*.

From *d* and *e*, with the same radius, describe arcs cutting in *f*.

Draw the line *f***B**, and it will be perpendicular to the line **AB** at its extremity.

(5) Through a given point **C** to draw a straight line parallel to a given straight line **AB** (Fig. 76).

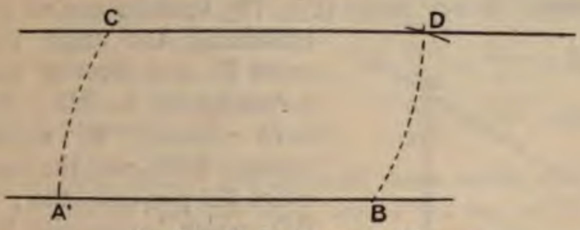

Fig. 76.—To draw a parallel line.

From any point **B** in the line **AB** describe an arc **CA**, and from the centre **C**, with the same radius, describe the arc **BD**, and make the arc **BD** equal to the arc **AC**.

Then the line joining **CD** is parallel to the line **AB**.

(6) To construct a triangle, its three sides being given (Fig. 77). Let the sides be 50, 75, and 60. Draw a line **AB**, and mark off the length **AC** equal to 50 on the scale; then, with centre **A** and radius 75, draw an arc, and from the centre **C**, with the radius 60, draw another arc, cutting the first arc in **D**. Then join **AD** and **CD**, and **ACD** is the required triangle.

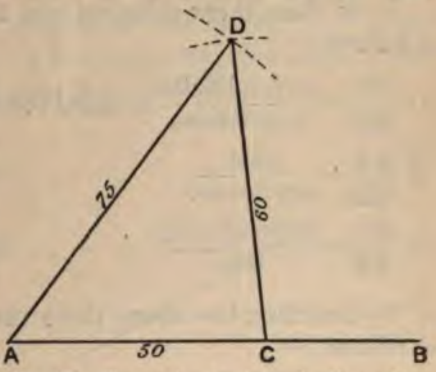

Fig. 77.—To construct a triangle, three sides being given.

(7) To construct a triangle when two of its sides and the angle between them are known (Fig. 78).

Let the two sides be 30 and 40, and the angle included 45°. Then draw a straight line **AB**, and mark off a length **AC** equal to 40, and, by means of a protractor, make the angle **CAD** equal to 45°, and make **AD** equal to 30. Then, by joining **DC**, the triangle is completed.

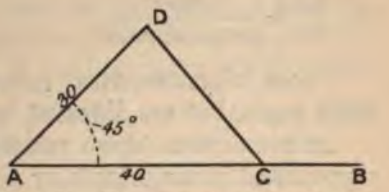

Fig. 78.—To construct a triangle, two sides and the included angle being given.

Trigonometry deals with the relative measures of the sides and angles of triangles.

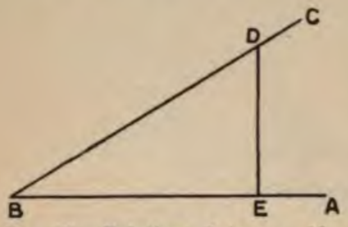

FIG. 79.—Relations between sides and angles of a triangle.

Let **ABC** be any angle (Fig. 79), then in one of the lines containing the angle take any point **D**, and from **D** draw **DE** perpendicular to **AB**. Then we have formed a right-angled triangle **BDE**, and the side **DE** is called the perpendicular; the side **BD**, which is opposite the right angle, is called the hypotenuse, and the side **BE** is called the base.

From these three sides we can form six ratios or fractions as follows:—

(1) $\frac{DE}{BD}=\frac{\text{perpendicular}}{\text{hypotenuse}}$ is called the **sine** of angle **EBD** (or **ABC**)

(2) $\frac{BE}{BD}=\frac{\text{base}}{\text{hypotenuse}}$,, ,, **cosine** ,, ,,

(3) $\frac{ED}{BE}=\frac{\text{perpendicular}}{\text{base}}$,, ,, **tangent** ,, ,,

By inverting the above three ratios, we obtain three more, as follows:—

(4) $\frac{BD}{DE}=\frac{\text{hypotenuse}}{\text{perpendicular}}$ is called the **cosecant** of the angle **ABC**

(5) $\frac{BD}{BE}=\frac{\text{hypotenuse}}{\text{base}}$,, **secant** ,, ,,

(6) $\frac{BE}{ED}=\frac{\text{base}}{\text{perpendicular}}$,, **cotangent** ,, ,,

These trigonometrical ratios are always the same for the same angle, but are different for different angles.

In some cases these ratios—*i.e.* sine, cosine, etc.—may be represented in magnitude by single lines.

For instance, referring to Fig. 80, suppose the circle to have been drawn with a radius of 1—

Then the sine of the angle **ABC** is $\frac{FD}{BD}=\frac{FD}{1}=FD$

and the cosine ,, ,, $\frac{FB}{BD}=\frac{FB}{1}=FB$

and the tangent of the angle **ABC** is $\frac{AC}{AB} = \frac{AC}{1} = AC$

,, cotangent[1] ,, ,, $\frac{HE}{HB} = \frac{HE}{1} = HE$

,, secant ,, ,, $\frac{BC}{BA} = \frac{BC}{1} = BC$

,, cosecant[1] ,, ,, $\frac{BE}{HB} = \frac{BE}{1} = BE$

It will be seen that by referring all these ratios to a radius of 1, we are able to measure their values for any angle. Thus in Fig. 80 the angle **ABC** is drawn 60°, and if the line **FD** be measured with the same scale that **AB** was drawn with, it will be found to be 0·866; therefore the sine of 60° (to radius 1) is 0·866. In the same way the other ratios can be arrived at.

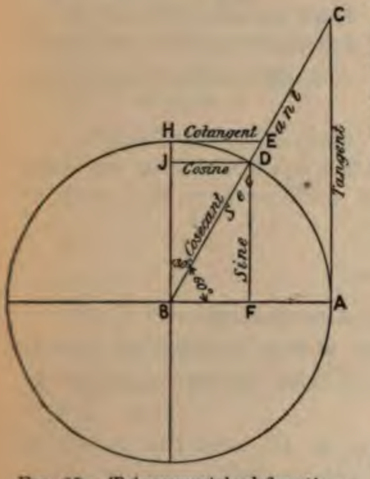

FIG. 80.—Trigonometrical functions.

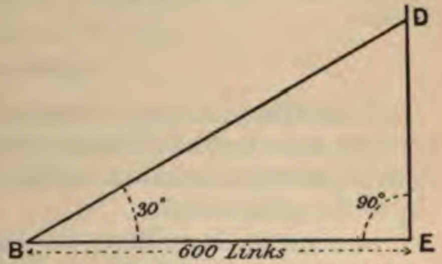

FIG. 81.—Use of trigonometrical ratios.

Tables of these ratios may be got in which the values of the natural sines, cosines, etc., have been worked out for all angles.

The word "natural sine" is used to distinguish it from the logarithmic sine. The natural sines are the actual values of the ratios, while the logarithmic sine is the logarithm of that ratio.

EXAMPLES.—(1) Let Fig. 81 represent a triangular field. The base **EB** is known to be 6 chains, also the angle **EBD** 30°; then to find the side **ED**.

We know that $\frac{ED}{EB}$ = tangent of **EBD**. On referring to our book of tables, we find the natural tangent of 30° is 0·5773503.

Then $\frac{ED}{EB} = 0{\cdot}5773503$; but **EB** = 6 chains = 600 links. Then **ED** = 600 × 0·5773503 = 346·41. *Ans.*

---

[1] Since the angle HEB = the angle ABC.

In a similar manner, by working out the equation $\frac{BE}{BD}$ = cosine 30°, we can find the other side **BD**.

(2) At a point 100 yards from the foot of a building, I measure the angle of elevation of the top, and find that it is 23° 15′: what is the height of the building?

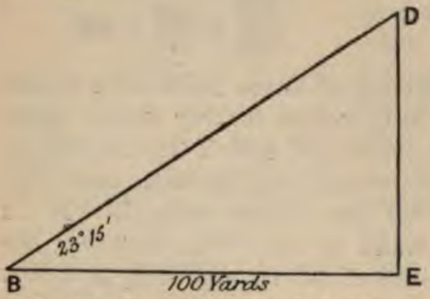

FIG. 82.—Use of trigonometrical ratios.

Let Fig. 82 represent the problem; **ED** is the unknown height. The length **BE** is known to be 100 yards, and the angle **EBD** to be 23° 15′.

Then $\frac{ED}{EB}$ = tan 23° 15′.

From the table of tangents we find that tan 23° 15′ = 0·4296339.

$\therefore$ **ED** = 100 × 0·4296339
= 43 yards (nearly)

which is the required height.

Of course, both these problems could have been solved by plotting; but unless the scale had been very large, the results would not have been nearly so accurate.

Logarithms.

Logarithms are used to facilitate calculations.

The logarithm of a number is the power to which an invariable (or constant) number, called the base, has to be raised to equal the given number.

In common logarithms the base is 10, and the power to which 10 has to be raised to produce any number is the logarithm of that number. Thus—

$$10 \times 1, \text{ or } 10^1 = 10 \quad \therefore 1 = \log. 10$$
$$10 \times 10, \text{ or } 10^2 = 100 \quad \therefore 2 = \log. 100$$
$$10 \times 10 \times 10, \text{ or } 10^3 = 1000 \quad \therefore 3 = \log. 1000$$
$$10 \times 10 \times 10 \times 10, \text{ or } 10^4 = 10000 \therefore 4 = \log. 10000$$

and so on.

It is proved by algebra that $10^0 = 1 \quad \therefore 0 = \log 1$

and $0{\cdot}1 \text{ or } \frac{1}{10} = 10^{-1} \therefore -1 = \log. 0{\cdot}1$

and $0{\cdot}01 \text{ or } \frac{1}{100} = 10^{-2} \therefore -2 = \log. 0{\cdot}01$

$0{\cdot}001 \text{ or } \frac{1}{1000} = 10^{-3} \therefore -3 = \log. 0{\cdot}001$

and so on.

Thus we see that the logarithm of a number greater than 1 and less than 10 is a positive decimal; and the log. of a number between 10 and 100 is greater than 1 and less than 2; that is to say, will be 1 + a decimal, and so on.

We see also that the logarithm of any number between 1 and 0·1 is negative, and would lie between 0 and -1, and can be written $-1 +$ a decimal; and the log. of a number between 0·1 and 0·01 can be written $-2 +$ a decimal; and so on.

A logarithm consists of two parts—the integral, or whole-number part, which is called its *characteristic;* and the decimal part, which is called the *mantissa.*

The *mantissa* of the logarithm may be found in a table of logarithms, but the *characteristic* is found as follows:—

(*a*) If the number whose logarithm is sought is greater than unity, the *characteristic* is always one less than the number of figures it contains; thus—

The logarithm of $43758 = \overset{(c)}{4}$[1]$\cdot\overset{(m)}{6410575}$
„ $4375{\cdot}8 = 3{\cdot}6410575$
„ $43{\cdot}758 = 1{\cdot}6410575$
„ $4{\cdot}3758 = 0{\cdot}6410575$
etc.

(*b*) If the number is less than unity, the characteristic is minus or negative, and is found by adding one to the number of cyphers between the decimal point and the first significant figure; thus—

Log. $0{\cdot}43758 = \overset{(c)}{\bar{1}}\cdot\overset{(m)}{6410575}$
„ $0{\cdot}043758 = 2{\cdot}6410575$
„ $0{\cdot}00043758 = \bar{4}{\cdot}6410575$

Many good tables of logarithms can be obtained; the author often uses Chambers's,[2] which, in addition to giving the logarithms of all the numbers from 1 to 108000, contain an excellent explanation of their use, from which some of these illustrations are taken.[3]

I. To perform multiplication by logarithms.

Add the logarithms of the factors, and the sum will be the logarithm of the product.

---

[1] c = characteristic; m = mantissa.

[2] Chambers's *Mathematical Tables*, published by W. & R. Chambers.

[3] Babbage and Callet's Tables give logarithmic sines, cosines, etc., worked out to 10 seconds.

EXAMPLES.—(1) Multiply 9999 by 999.
Log. 9999 = 3·9999566
" 999 = 2·9995655
Sum = 6·9995221, which is the log. of 9989001. *Ans.*

(2) Multiply 0·03902, 59·716, and 0·00314728.
Log. 0·03902 = $\bar{2}$·5912873
" 59·716 = 1·7760907
" 0·00314728 = $\bar{3}$·4979353
Sum = $\bar{3}$·8653133, which is the log. of 0·007333533. *Ans.*

II. To perform division by logarithms.

From the logarithm of the dividend subtract that of the divisor, and the remainder will be the logarithm of the quotient.

EXAMPLES.—(1) Divide 371·49 by 52·376.
Log. 371·49 = 2·5699471
" 52·376 = 1·7191323
Difference = 0·8508148, which is the log. of 7·092752. *Ans.*

(2) Divide 241·63 by 4·567.
Log. 241·63 = 2·3831509
" 4·567 = 0·6596310
Difference = 1·7235199, which is the log. of 52·90782. *Ans.*

III. To raise a number to any power by logarithms.

Multiply the logarithm of the given number by the index of the power to which it is to be raised, and the product will be the logarithm of the required power.

(1) Find the cube of 30·7146, written thus: $(30{\cdot}7146)^3$.
Log. 30·7146 = 1·4873449
3
4·4620347, which is the log. of 28975·75. *Ans.*

(2) What is the value of $9{\cdot}163^4$?
Log. 9·163 = 0·9620377
4
3·8481508, which is the log. of 7049·38. *Ans.*

IV. To extract any root by logarithms.

Divide the logarithm of the given number by the index of the root to be extracted, and the quotient will be the logarithm of the required root.

(1) Find the cube root of 12345, written thus: $\sqrt[3]{12345}$.
Log. 12345 = 4·0914911.
3)4·0914911
1·3638304, which is the log. of 23·11162. *Ans.*

(2) Find the fourth root of 0·0076542.

$$\text{Log. } 0\ 0076542 = \bar{3}\ 8838998$$
$$= \bar{1}\cdot4709749$$

To divide a negative characteristic, add such a quantity to the characteristic as will make it divisible without a remainder, and prefix an equal number to the decimal part of the logarithm. Thus, in the example, add 1, and you get—

$\bar{4} + 1\cdot8838998 \div 4 = \bar{1}\cdot4709749$, which is the log. of 0·295784. *Ans.*

In calculations in which sines, cosines, etc., occur, and logarithms are to be used, then the logarithmic sine, cosine, etc., must be used. They can be obtained from Chambers's Tables.

The logarithmic sine is obtained by finding the logarithm of the number representing the natural sine, and adding 10 to its characteristic.

For example, if the reader refers to his book of tables, he will find that the natural sine of 30° is 0·5000000. The logarithm of 0·5 is $\bar{1}\cdot6989700$, but to avoid the inconvenience of the negative characteristic, 10 is added, and so we arrive at log. sine 30°, which is equal to 9·6989700.

In using log. sines, cosines, etc., the 10 which has thus been added is always deducted again, as in the following example :—

To find **ED**, page 104, Example 2.

$$\mathbf{ED} = 100 \times \text{tangent } 23^\circ\ 15'$$
$$\therefore \text{log. } \mathbf{ED} = \text{log. } 100 + \text{log. tan } 23^\circ\ 15' - 10$$
$$= 2 + 9\cdot6330985 - 10$$
$$= 1\cdot6330985, \text{ which is the log. of } 42\cdot964$$
$$\therefore \mathbf{ED} = 42\cdot964. \quad \textit{Ans.}$$

The Solution of Triangles.—In every triangle there are six parts, viz. three sides and three angles. If any three of these parts are given, *one of which must be a side*, the remaining parts can be found, the process being known as the "solution" of the triangle.

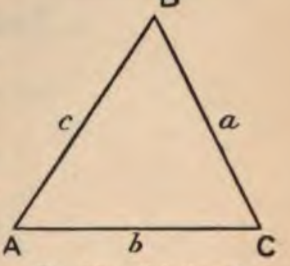

FIG. 82A.—Solution of triangles.

It will be at once seen that this information is of great service to the surveyor, who is able, by observing the angles of his triangles, to calculate the lengths of the sides, and thus check the measured distance. In cases also where it is not practicable or necessary to measure one of the sides, its length can be calculated from the other known parts of the triangle.

In order to shorten the formulæ, the three angles of the triangle will be referred to as A, B, and C, and the three sides opposite them a, b, and c, respectively (see Fig. 82A).

Case 1.—*Given the three sides a, b, and c, to find the angles.*

Let s = half the sum of the three sides.

$$\text{Then } \tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\tan\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$$

These formulæ will give us the angles A and B. The angle $C = 180^\circ - A - B$.

Example.—The three sides of a triangle are: $a = 750$ links; $b = 835$ links; and $c = 679$ links. Find the angles A, B, and C.

$$\text{Here } s = \frac{750 + 835 + 679}{2} = 1132$$

$$\text{then } \tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$= \sqrt{\frac{(1132-835)(1132-679)}{1132(1132-750)}}$$

$$= \sqrt{\frac{297 \times 453}{1132 \times 382}}$$

$$= \sqrt{0{\cdot}3111321}$$

$= 0{\cdot}5577921$, which is the natural tangent of the angle $29^\circ\ 9'\ 9''$

$$\frac{A}{2} = 29^\circ\ 9'\ 9''$$

$$\text{and the angle } A = 58^\circ\ 18'\ 18''$$

$$\tan\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$= \sqrt{\frac{(1132-750)(1132-679)}{1132(1132-835)}}$$

$$= \sqrt{\frac{382 \times 453}{1132 \times 297}}$$

$$= \sqrt{0{\cdot}5147053}$$

$= 0{\cdot}7174290$, which is the natural tangent of the angle $35^\circ\ 39'\ 24{\cdot}5''$

$$\frac{B}{2} = 35^\circ\ 39'\ 24{\cdot}5''$$

$$\text{and the angle } B = 71^\circ\ 18'\ 49''$$

$$\text{and } C = 180^\circ - 58^\circ\ 18'\ 18'' - 71^\circ\ 18'\ 49'' = 50^\circ\ 22'\ 53''$$

Case* 2.—*To solve a triangle, having given two angles and a side.

In any triangle the sides are proportional to the sines of the opposite angles.

$$\text{Thus } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Let A and C be the given angles and b the given side. Then the angle B = 180° − A − C.

To find the sides—

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\therefore a = \frac{b \sin A}{\sin B}$$

from which we get the side a.

$$\text{and } \frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\therefore c = \frac{b \sin c}{\sin B}$$

from which we get the side c.

Example.—In a triangle ABC, the angle A = 50°, the angle C = 66°, and the side a is 1000 yards. Find the remaining sides and angle.

The angle B = 180° − 50° − 66° = 64

To find the sides—

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\therefore \frac{1000}{\sin 50^\circ} = \frac{b}{\sin 64^\circ}$$

$$\therefore b = \frac{1000 \times \sin 64^\circ}{\sin 50^\circ}$$

$$b = \frac{1000 \times 0\cdot8987940}{0\cdot7660444}$$

$$= \frac{898\cdot7940}{0\cdot7660444} = 1173\cdot29$$

$$\text{and } \frac{c}{\sin C} = \frac{b}{\sin B}$$

$$c = \frac{b \sin c}{\sin B}$$

$$c = \frac{1173\cdot29 \times \sin 66^\circ}{\sin 64^\circ}$$

$$= \frac{1173\cdot29 \times 0\cdot9135455}{0\cdot8987940}$$

$$= 1192\cdot5$$

Case 3.—Given any two sides b and c, and the angle A *between them, to find the remaining side and angles.*

The angles $(B + C) = 180° - A$, from which we get $(B + C)$, and $\tan \frac{B - C}{2} = \frac{b - c}{b + c} \cot \frac{A}{2}$, from which we get $(B - C)$; and $(B + C) + (B - C) = 2B$, thus we find the angle B; and $C = 180° - A - B$, from which we get the angle C. We have now got all the angles, and can find the remaining side by Case 2.

Example.—The two sides of a triangle are 135 yards and 105 yards, and the angle between them is 60°: find the remaining side and angles.

Let A be the angle; then b and c are the sides.

$$\begin{aligned}
\text{The angles } (B + C) &= 180° - A \\
&= 180° - 60° = 120° \\
\tan\left(\frac{B - C}{2}\right) &= \frac{b - c}{b + c} \cot \frac{A}{2} \\
&= \frac{135 - 105}{135 + 105} \cot 30° \\
&= \tfrac{30}{240} \times 1{\cdot}7320508 \\
&= 0{\cdot}21650635, \text{ which is the tangent of } 12° \; 12' \; 59' \\
\therefore B - C &= 24° \; 25'' \; 58'' \\
\text{and } B + C &= 120° \\
\text{their sum} = 2B &= 144° \; 25' \; 58'' \\
\therefore B &= 72° \; 12' \; 59'' \\
\text{and } C &= 180° - 72° \; 12' \; 59'' - 60° \\
&= 47° \; 47' \; 1''
\end{aligned}$$

To obtain the side a—

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\text{then } \frac{a}{0{\cdot}8660254} = \frac{135}{0{\cdot}9522168}$$

$$\text{and } a = 122{\cdot}7$$

Case 4.—Given two sides and the angle opposite one of them, to find the remaining side and angle.

Let the two given sides be b and c, and the given angle B.

$$\text{then since } \frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\therefore \sin C = \frac{c \sin B}{b}$$

When C is found, $A = 180° - B - C$.

$$\text{and } a = \frac{b \sin A}{\sin B}$$

In must be noted, however, that when the angle B is acute, and the side b is less than the side c, there are two solutions to the angle C. This will be understood on reference to Fig. 82B, in which **B** is the given angle and c one of the given sides; the other given side b may be in either of two positions **AC** or **AC$_1$**, thus forming two triangles **ABC** and **ABC$_1$**. The angle **ACB** being the supplement of the angle **AC$_1$B**, both would have the same sine, and therefore either of these triangles would be permissible.

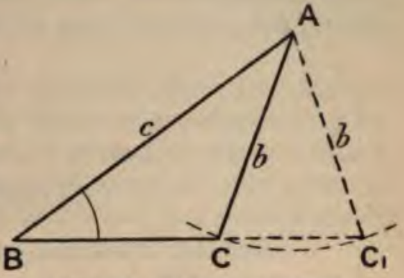

FIG. 82B.—Solution of triangles.

In practical work, however, there will be no doubt as to which value to take, as the surveyor generally knows the shape of the triangles he is working on.

It is impossible within the limits of this work to go at greater length into the subject of Trigonometry, but the reader is referred to one or other of the standard works on the subject.[1]

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[1] *Elementary Trigonometry*, by J. Hamblin Smith, published by Longmans; *Trigonometry for Beginners*, by J. B. Lock, published by Macmillan & Co.

CHAPTER VII.

SURFACE SURVEYING WITH THE THEODOLITE.

In order to make the plan of a large estate with accuracy, or of a small estate with extreme accuracy, or of portions of an estate across which lines cannot be measured at will—as, for instance, where the ground is occupied with buildings, as in a town—it is advisable to use an instrument for taking angles. A transit theodolite is generally used. There is nothing in the theory upon which the theodolite is designed to make it more accurate than a miner's dial; but theodolites are generally used by persons requiring extreme accuracy, and the instruments are therefore made with great care. A 5-inch theodolite is an instrument of very convenient size for an ordinary land surveyor's use, and is graduated to read to angles of 1′. The larger the horizontal circle of the theodolite, the greater the degree of accuracy with which the theodolite can be read, thus a 12-inch theodolite reads to 1″; a 6-inch theodolite may be constructed to read to angles of 20″; 8-inch theodolites are graduated to read to 10″. For very important work special theodolites have been made up to 36 inches' diameter, reading to $\frac{1}{10}$ of a second. The larger instruments are only required for very large surveys, such as the Ordnance Survey of the United Kingdom, or for very important railway surveys, where a long tunnel is projected to pierce a mountain, as, for instance, some of the tunnels through the Alps. The size of the instrument to be used depends on the nature of the work in hand. It may easily be calculated that the sine of an angle of 1″, to a radius of 100 miles, is equal to about 2 feet; and the sine of an angle of 1′ for a radius of 100 miles is equal to 153·59 feet. It is evident, therefore, that for the accurate fixing of places at a distance of 100 miles by means of the theodolite—supposing it to be possible that a sight of this length could be taken—it would be necessary to have one reading to seconds or fractions of a second; but for fixing a point at a distance of only 1 mile, an instrument reading to minutes or fractions of a minute would have a corresponding accuracy.

Fig. 83 shows a plan of an estate in which the principal distances are to be ascertained by means of the theodolite. The first step is to select some piece of ground on which a line of 100 yards or more in length can be measured over a smooth and level surface. It does not matter where this piece of ground is, but the nearer it is to the centre of the estate the better; it might, however, be on one side or even outside the estate. It should also be in a good position for obtaining sights to numerous parts of the estate to be surveyed. Having selected this ground, let the base-line **AB**, say 2000 links in length, be measured. It is essential that the measuring should be done with extreme accuracy; this may be effected by using a carefully tested chain, or a steel tape. In order that the steel tape may be used with accuracy, the direction of the line should be

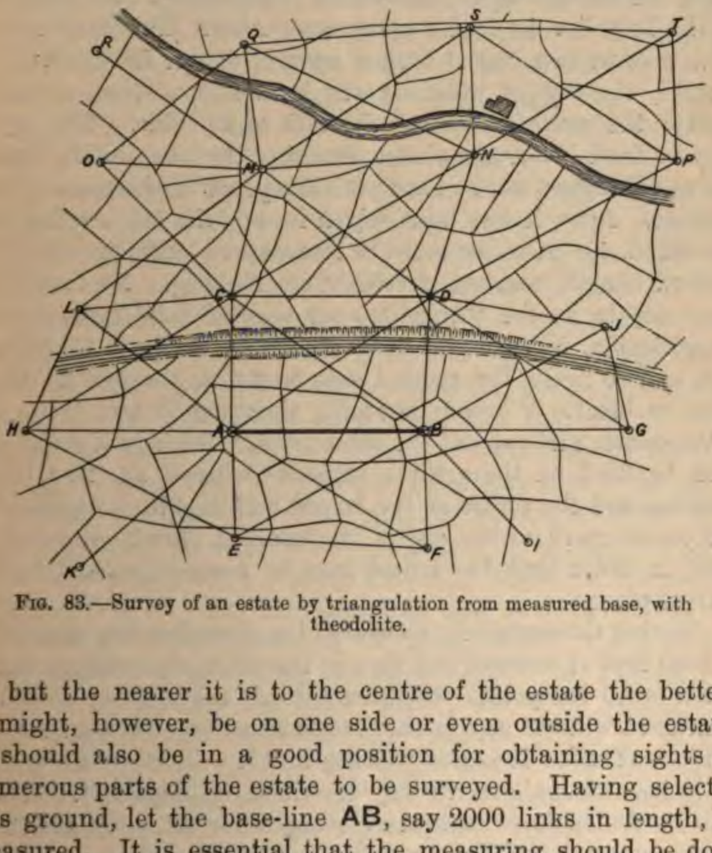

FIG. 83.—Survey of an estate by triangulation from measured base, with theodolite.

first carefully marked out by means of pegs or otherwise, and the distance approximately measured with a chain or common tape; then, at the end of every chain-length, a piece of smooth wood, or stone, or slate must be placed in the ground; so that, in measuring with the steel tape, the end of every chain-length can be carefully marked with a fine-pointed pencil. If the line is not perfectly level, the inclination must be measured, so that the length as measured may be reduced by calculation to that of a horizontal line. The temperature at the time of measurement should also be noted, so that corrections may be made for expansion or contraction if necessary. At each end of the base-line is fixed a permanent mark. This may consist of a wooden peg, say 4 inches square, driven firmly into the ground, the top of which is cut level and smooth, so as to receive the mark indicating the end of the line. The peg **A** may be fixed before the measurement of the base-line is begun; the peg **B** should not be fixed until the point has been accurately marked. After it has been driven in, and the top levelled and smoothed, the base-line must be remeasured, and the end of the desired length marked precisely on the peg. As it will be necessary to fix the theodolite over each of these pegs, it may be advisable, in case the weather is wet, or likely to become wet, and so make the ground soft, to fix in the ground three pegs or blocks of wood or stone, each equidistant from the centre-mark, and each at the same level. The tripod stand can then be fixed on these three supports without any fear of its yielding, and the centre of the tripod will thus be brought over the centre-mark at the end of the line. A plumb-bob must be used, in order that the tripod may be accurately fixed in the correct place.

Having thus carefully measured the base-line, the theodolite is fixed first at one end and then at the other, observations being taken to all the points or stations which are visible; these are then fixed without any further measurements, although, for the purpose of what is called "filling in," it will save time to use the poles and chain in the way described in Chapter II. It will also be desirable to measure certain other distances, checking the accuracy of the distances obtained by calculation.

It would save time, of course, to use two theodolites, one fixed at each end of the base-line, *i.e.* at **A** and **B**; but it is not everybody who possesses two of these instruments. If only one theodolite is obtainable, two tripod stands may be used with advantage, one fixed at **A**, and the other at **B**, and the instrument moved from one to the other, so saving the time otherwise lost in correctly centering the instrument over these points.

Stations fixed by Angles.—It is now necessary to fix a number of other stations, for instance, **C, D, E, F, G, H, I, J, K, L.** At each of these places a pole is fixed, and the angle made with the base-line is read by means of the theodolite. The stations are so chosen that no angle shall be less than 30° or more than 120°. Stations **C, D, E,** and **F** are fixed by reading the angles **CAB, CBA, DAB,** and **DBA,** also the angles **EAB, EBA, FAB,** and **FBA.** It will be seen that in the triangle **CAB** one side, **AB,** and the angle at each end are known, and therefore the remaining sides and angle can be calculated (see p. 107), and the position of the point **C** found; the positions **D, E,** and **F** can similarly be fixed with great accuracy. The places **G, H, I, J, K,** and **L** are not absolutely fixed, only one observation being made from the nearest end of the base-line. The theodolite may now be moved to the station **C,** and the calculated distance **CD** used as a base-line for further observations; the angle **ACL** is now taken, fixing the point **L,** and a further sight is taken to the point **M,** the angle **DCM** being read; the position **N** is then observed, the angle **DCN** being read, and the station **O** by reading the angle **DCO**; the position **H** will also be fixed by reading the angle **ACH.** The theodolite may now be moved to **D,** and the position **M** fixed by reading the angle **CDM**; **N** is fixed by reading the angle **CDN**; **J** is fixed by reading the angle **JDB**; and **G** is fixed by reading the angle **GDB.** A new station is projected at **P.** The theodolite may now be moved to **M,** and the position **O** fixed by reading the angle **CMO**; a new station is projected at **Q** by reading the angle **QMO**; the station **R** is projected by reading the angle **OMR,** and **S** by reading the angle **SMN.** The theodolite may now be moved to **N,** and station **Q** fixed by reading the angle **QNM,** and the station **S** fixed by reading the angle **SNM, P** fixed by the angle **PND,** and checked by the angle **SNP**; station **T** is projected by the angle **SNT.** The theodolite may now be moved to **S,** and the station **T** fixed by reading the angle **TSN.** The accuracy of the station **Q** is tested by reading the angle **QSN,** also the accuracy of the station **P** by reading the angle **NSP.**

Triangles.—In this way the whole estate may be covered with a series of triangles, and no single station should be placed at a greater distance than is convenient for accurate sighting of the staff, this distance depending on the power of the telescope. The stations at the end of the system of triangulation, as, for instance, the stations **S** and **T**, may be quite out of sight of the original base-line **AB**. At every station all the reasonable angles should be observed, and by this means every station will be observed several times and the accuracy of the work tested.

Where the direction of a line of fence corresponds with the direction of a line connecting any two stations, the length can be measured with a chain in the ordinary way, offsets being taken to the fence; the measured length should agree with the calculated length, and form a check upon the accuracy of the work.

Great care is necessary in fixing the stations, the angle of which is to be read. The common method of marking a station is to fix a surveying-pole in the ground at the required spot, and, after its position has been recorded, to mark the place by means of a peg driven into the ground. This peg may be round and of the same diameter as the pole, in which case it will fit into the same hole; the centre of the hole is marked by a cross on the top of the peg, and with care considerable accuracy may be attained by this method; but errors are liable to creep in, owing to the staff not being fixed quite vertically, and to the peg not being driven quite into the centre of the hole. Where possible, observations should always be made to the bottom of the pole, in which case the fact of the pole being placed a little out of truth will not lead to any error.

It would be possible to overcome these errors by fixing the positions of the stations by means of pegs in which a hole was bored to receive the pole.

Tripod stands could also be used for sighting to, which could be accurately fixed up over each station.

One objection, however, to the use of a tripod stand in place of a pole is that in a level country it might be rendered invisible by hedges and walls of moderate height, whereas the top of an ordinary surveying-pole can be seen over these obstructions. Another objection is the cost and weight of such a contrivance, which therefore preclude its use except for very special purposes.

In the course of a day's work, a surveyor might wish to observe twenty stations, and not to disturb the poles in any one. Where the distance is considerable, a slight error in fixing the pole may not be observed. Suppose, for instance, that the centre of the pole at **D** (Fig. 83), as observed from **A** and **B**, is 1 inch distant from the centre of the mark on the peg at **D**, over which the theodolite is subsequently placed; if the distance **BD** is 5 chains, this will be an error of 1 inch in 3960, equal to an error of about 1 minute, and such an error, of course, must not be deliberately risked. Therefore, after placing the station peg at **D** and marking the centre, the pole should be held vertically over the mark, in order to see if it corresponds with the station as observed, and if it does not correspond, the observations must be repeated. If, however, the length **BD**, instead of being only 5 chains, had been 12 chains, an error of 1 inch would only be about 22 seconds, and might be neglected in a survey of this size.

When a survey-line crosses a stone wall or wood fence, it is a good plan to make a notch at the junction; by so doing the line is much more easily found on subsequent occasions.

The position of such objects as church spires, mill chimneys, and corners of large buildings may be fixed by observations with the theodolite, thus checking the "filling-in" process done by means of the chain.

The method of triangulation above described is similar in principle to that adopted in the Ordnance Survey of the British Isles, the whole of the stations being fixed by triangulation from two principal base-lines, one on Salisbury Plain, about 7 miles long, and the other on the shore of Lough Foyle in Ireland, about 8 miles long;[1] but for this survey specially large instruments were used—theodolites 3 feet, 2 feet, and 18 inches in diameter. According to Mr. Bennett H. Brough, the exact length of the Salisbury Plain base was 6·97 miles, and that of the Lough Foyle base 7·89 miles; the length of the latter base was calculated by triangulation carried from the Salisbury Plain base, and the difference between the calculated and measured length of the Lough Foyle base was only 5 inches. The sides of some of the principal triangles measured here from 80 miles up to 111 miles in length; the principal triangles were divided into secondary triangles with sides 10 to 15 miles in length, and these again into tertiary triangles with sides averaging 1½ mile in length. It is within these last that the chain surveyors work. For the smaller triangles smaller theodolites were used.

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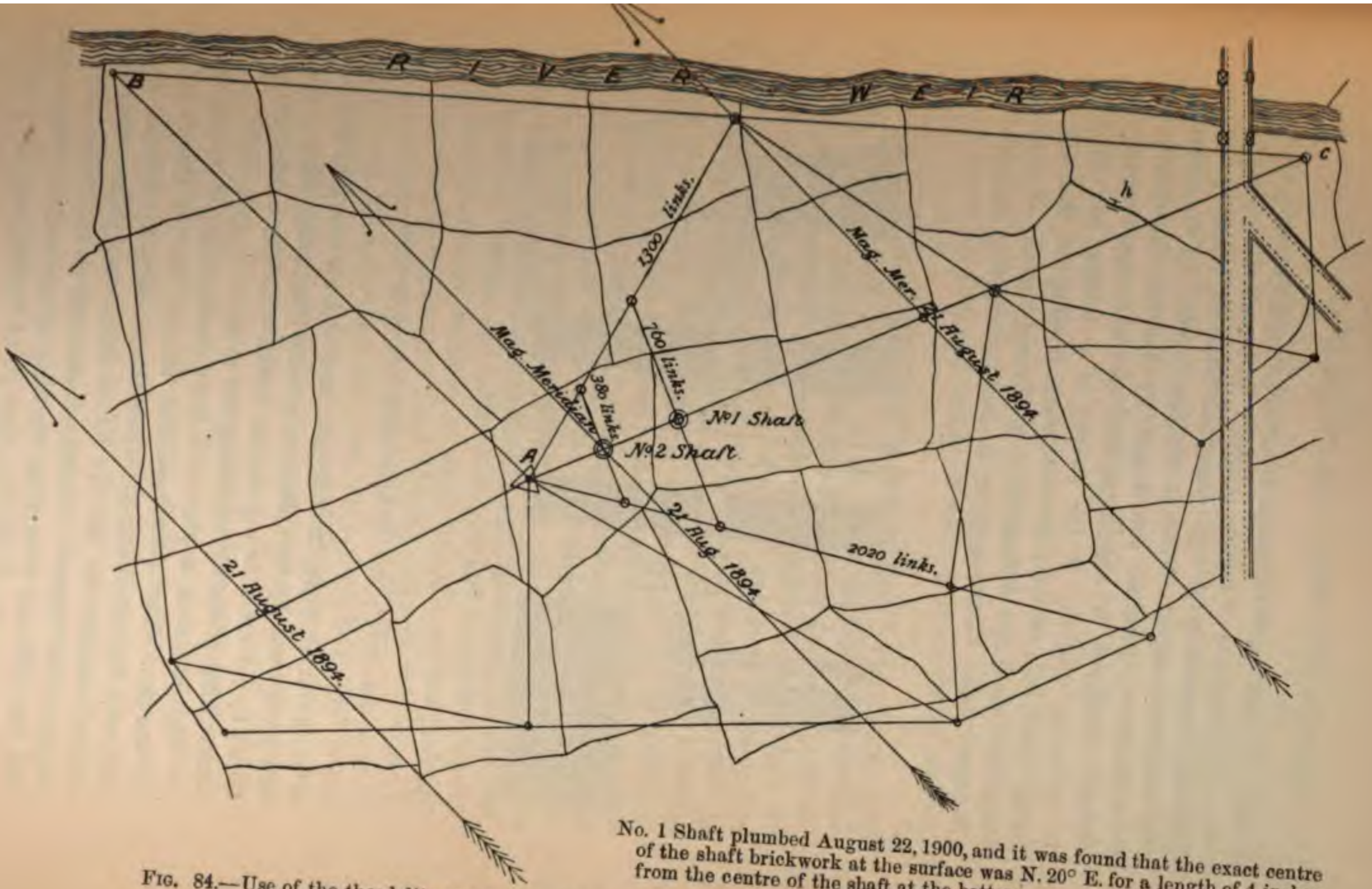
[1] *Ordnance Survey of the United Kingdom*, by Major Francis P. Washington, R.E.

The theodolite is often used, not as the chief means of fixing the station, but as a check upon the accuracy of the measurements made with the chain, and to facilitate the ranging of a long line of poles. With regard to the ranging of poles in the manner already described on p. 14, Chapter II., there is a possibility of the line becoming crooked, owing to the poles being fixed imperceptibly out of the true line, and so causing a gradual, but in the end considerable, divergence; this may be caused by the poles being blown by the wind or otherwise caused to lean on one side after being truly fixed. If, however, the line is ranged with the aid of a theodolite fixed on some level piece of ground, any slight divergence from the true line, when passing across a field in a hollow or hidden from view by high hedges, is at once corrected with the aid of the telescope as soon as the line reappears in the line of sight. The angles that the main lines make one with another are also observed, as shown in Fig. 84, where thirteen theodolite stations are shown at which the angles of the various lines are observed. In the large triangle ABC there shown, the three angles are measured as well as the three sides; if, however, one of the sides has been measured, the lengths of the other two sides could be calculated; but as all three sides are measured, the accuracy of the measurements can be checked by calculation, and therefore, if the work is honestly done, it is impossible for an error to escape detection, that is to say, it is impossible that the fact of an error existing somewhere shall escape detection, though it may take a little trouble to ascertain exactly where the error occurs. It is evident that one fine day's work with the theodolite will accomplish more in the way of checking the accuracy of a triangulation already made with chain and poles than five days' work of actual measurement with the chain across the fields.

Use of the Theodolite over Rough and Impracticable Ground.—It frequently happens that it is impossible to measure the sides of triangles in the way shown in Fig. 84, because of obstructions consisting of rivers, woods, precipices, and buildings; in such cases the use of a theodolite or similar angle-measuring instrument is necessary, as the distance between various stations, visible from some position of advantage outside



No. 1 Shaft plumbed August 22, 1900, and it was found that the exact centre of the shaft brickwork at the surface was N. 20° E. for a length of 4 inches from the centre of the shaft at the bottom.

FIG. 84.—Use of the theodolite as a means of checking the accuracy of a survey made with chain and poles.

the line to be measured, can be accurately fixed by triangulation from some base-line of known length. In towns it is, generally speaking, impossible to run diagonal lines, or to take sights from corner to corner of the rectangles formed by the streets; it is, however, possible sometimes to fix the theodolite on elevated positions or buildings, and so take observations to the principal stations in the survey of a town; but for the filling-in of the streets, the accurate reading of the angles formed by one street with another is the only means of making a correct plan. The surveyor, in fact, traverses a number of rectangular or polygonal figures, and the accuracy of his work can be proved by the plan so obtained agreeing with stations found by the main triangulation, which has been conducted from some rising ground overlooking the town.

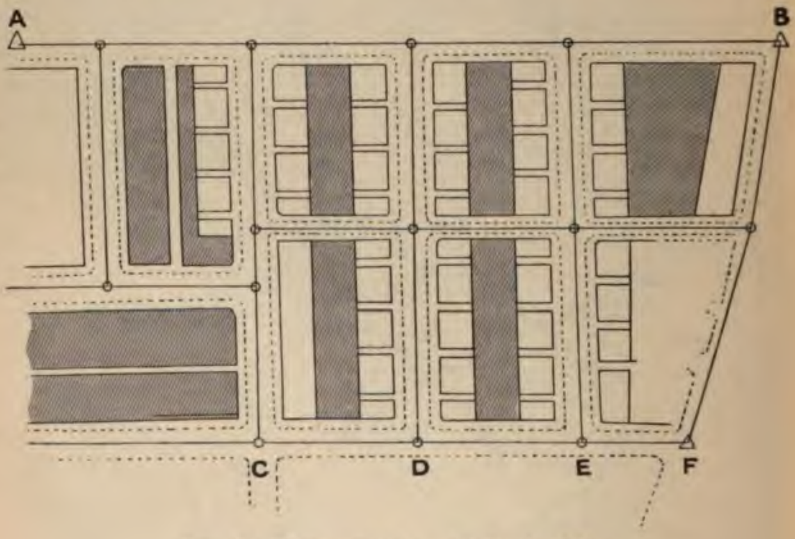


FIG. 85.—Town surveying with the theodolite.

Fig. 85 shows a plan of a few streets which it is necessary to survey. The points **A** and **B** are two of the principal stations, the positions of which have been fixed from rising ground outside the town. From these two points, and from intermediate stations between them, angles are taken between the line **AB** and other lines running up the centre of the streets to the points **C, D, E, F.** The point **F** being also a principal station, the accuracy of the intermediate survey is thereby checked.

Angles are likewise taken at the intersections of the cross-streets. The lines between the various stations are then measured, off-sets being taken to all buildings; and from the angles and measurements taken it will be possible to plot the survey thus made.

It does not often happen that the mining surveyor has to prepare a plan of the whole of a large town; but it not infrequently happens that he requires an accurate plan of part of a town or village through which he cannot easily range diagonal lines of survey.

Use of Miner's Dials, etc., on the Surface.—The dial is often conveniently used for many of the purposes for which the theodolite is better applied. The theodolite, being an expensive and cumbrous instrument, is not infrequently left at home when the dial will serve the purpose in view; for instance, the boundary of an estate may be traversed with the dial, and angles read as in Fig. 84, and the bearings of each line may be taken with the loose needle. In this way the accuracy of the chaining is checked, and it may be fairly argued that the instrument which is sufficiently accurate for underground work is sufficiently good for the surface, and this, with certain limitations, is true, if the dial is a good one and the distances are not long. For filling in a few fields, buildings, rows of houses, and for taking the bearings of main lines, the dial is a very convenient and useful instrument.

It must, however, be borne in mind that the ordinary miner's dial only reads angles to 3′, and that an error of 3′ is equal to 8·7 in 10,000; but it is not necessary to make such an error,—it need not be more than one half, or say 4·3 in 10,000, which in round figures is equal to an error of 3½ links in a mile. Where greater accuracy than this is necessary, the dial cannot be used for measuring angles.

In preferring, however, the use of the theodolite and of the chain and poles for surface work, the mining engineer is guided by the thought that by means of these instruments he can make a plan which is practically correct without an error of even half a link in a mile, and therefore he will not incur the risk of an error of 3½ links to the mile except for those portions of the underground survey where he is compelled to trust to the accuracy of his reading of the magnetic needle.

Magnetic Meridians.—With the exception of a few mines where some magnetic ore is worked, every mining plan has marked upon it the magnetic meridian, and in the vast majority of mines the underground survey is made with the magnetic needle; therefore the correct fixing of the meridian is of the very highest importance. For this purpose, when a new survey is being made, the bearings of all the main lines should be taken with the greatest possible care by means of the same dial that has been used for the underground survey. Of course, before using the dial, it should be carefully examined to see (1) that the needle swings freely and has sufficient magnetization to cause it to overcome any friction there may be on the pivot, and seek the north without undue delay; (2) that the needle is quite straight, and that both ends read the same, it being evident that if the needle is quite straight and the graduation of the circle quite accurate, the readings of the two ends of the needle will agree; (3) that the lines of magnetic force in the needle are parallel with its centre-line drawn lengthwise. The first two can be at once seen, and if the needle or graduated circle is wrong, another needle or circle must be obtained. Any error due to No. 3 can only be ascertained by comparing the bearing of a certain line with the bearing given by another needle; therefore, for an important survey, the magnetic needle on the dial or theodolite should be compared with several other instruments. If the same instrument is used for both surface and underground surveys, the error due to No. 3 may be neglected.

It has been already said that the ordinary 4½-inch compass needle can only be read to about ⅕ of a degree; it follows, therefore, that the meridian, as laid down from one reading of the needle, may possibly be misplaced to the extent of ⅕ of a degree. Perhaps the most accurate way of observing the bearing of a line is to turn the sights of the dial until the point of the needle corresponds with some mark on the graduated circle—either the zero mark or the degree nearest to the bearing of the line. It is easier to observe whether or no the point of the needle corresponds exactly with the division on the scale, than to estimate with precision, without the aid of a vernier, the proportional part of a degree to which the needle points. If, however, the dial is clamped with the needle pointing to some mark on the graduated circle, the sights can then be moved through the fractions of a degree necessary to bring them into the line of observation, and the fraction of a degree can then be read with the vernier, of which the readings are say to 3′, the error in the vernier being not more than one-half of that, that is to say, 1½′. The exact fixing of the needle upon one of the marks in the graduated circle may be done by means of a magnifying-glass or microscopic eye-piece, and when a surveyor has the advantage of broad daylight, there seems no reason why there should be any error; at any rate, the error need not exceed $\frac{1}{20}$ of a degree, thus the sum of the errors from one reading would be, say, 4½′. The observation of the bearing in this way should be made, say, three times, and the average taken, and if the bearings of six different lines are each read with an equal amount of care, and the error in any single case is not more than 4½′, it follows that if the meridians laid down from these six observations differ from each other, they can only differ to the extent of 4½′, and a mean may be almost perfectly accurate, or at any rate it is probable that the error may be greatly reduced, and amount to say only 2′, or an error of six links in 10,000.

To overcome any possible error from the diurnal variation of the magnetic needle, the meridian should be taken at the same time as the survey was made in the pit.

The writer has seen in Germany a station on the surface near a mine for observing the daily fluctuation of the needle. A magnetic needle was delicately suspended in a dark chamber, and a ray of light was reflected by a mirror on the needle on to a graduated arc of large diameter; the slightest movement of the needle, being at once apparent, could be easily recorded. In a similar way the variation of the needle is obtained at Greenwich, but in this case photographic records are obtained.

Having once laid down the magnetic meridian on the plan in the careful manner above described, it should not be altered unless an equal amount of care is used. Perhaps the simplest way of correcting the meridian is to compare the magnetic declination, as ascertained at the Royal Observatory at Greenwich from year to year; thus, in the year 1900 the magnetic declination was 16° 32′ at Greenwich, and in the year 1901 it was 16° 26′, so the meridian as laid down on the mining plan might be corrected to an equal amount. For the ordinary extensions, however, of a mining survey it is not generally considered necessary to correct the meridian every year; it should, however, be done at least once in two years.

The common way of correcting the meridian is to fix two or more pegs in some conveniently situated field on a line, the bearing of which has been accurately observed and recorded on the plan; upon a subsequent occasion the dial can be fixed in the direction of these marks, and any variation in the needle observed. It must, however, be noted that whilst this is useful for the purpose of checking meridians, the minute accuracy of the process depends, first, on the care with which the original bearing of the line was observed; second, the accuracy with which the pegs were fixed in that line; and third, upon the immovability of these marks, and the care exercised in fixing the instrument over them.

Some surveyors advocate the marking on the plan of the geographical north and south meridians or lines of longitude. The geographical meridian, of course, never changes, and the magnetic meridian can always be obtained for the purpose of plotting the survey by setting off the correct declination.

It must be remembered that great care must be exercised in altering the meridian by means of marks upon the plan, because, when a plan has been used some years, it is possible that, owing to shrinkage or bending or breaking of the paper, marks upon the plan may become a little misplaced. If, in the case of an existing plan, it is sought to ascertain the true magnetic meridian, it can only be done by ascertaining the bearing of lines between various points on the surface which are marked on the plan, and it must be borne in mind that, although the plan may be on the whole an exceedingly accurate and excellent one, it is quite possible that, owing to difficulties of draughtsmanship and slight extensions or contractions of the paper, or slight errors in the original survey, any particular part may be inaccurate to the extent of 5 or 10 links or more, and it is important to observe that this may cause a very serious error in fixing the meridian. Suppose the length of the line observed be only 5 chains in length, and the two stations as marked on the plan were each only 5 links out of their true position in opposite directions, this would make an error of direction of 10 links in a length of 500, or 1 in 50, which is equal to an error of 1° 9′. If, however, the distance, instead of being only 5 chains, was 50 chains, the error would be proportionately less; it is therefore important that the marks on the plan between which the line of bearing is observed should be a long distance apart, but, in order to reduce the probable error, the bearing of several other lines must be observed, both ends of which are quite distinct from the first line. Supposing the plan to be on the whole accurate, and that five or six lines are taken, each 20 chains in length, it is probable that the errors in the position of one line as marked on the plan will balance those of another, and that the meridian obtained as the average of the observations will be fairly accurate.

Position of the Shafts.—It is, of course, necessary to fix with extreme accuracy the position of the shafts, and their position should be indicated, not merely by an accurate delineation of them as circular or rectangular pits, as the case may be, but by the intersection of lines as shown in Fig. 84. All the principal survey-lines should be drawn on the plan in thin lines of some colour, say red or blue, and the length of each line written upon it; particularly should this be done in reference to the lines intersecting the centre of the shafts.

It is a common plan to rule only one magnetic meridian upon the plan, and that is commonly ruled through the centre of the downcast shaft; upon this line should be written the words, "magnetic meridian," and the date upon which it was observed; but the writer thinks that it would, perhaps, be better practice, upon the construction of a new and carefully made plan, to rule several parallel magnetic meridians. It is easy upon a new and unused plan to rule parallel lines, but some years later, when the underground workings have extended to portions of the estate perhaps 30 inches distant from the original meridian, it is not so easy to rule the new meridian strictly parallel to the one ruled through the shaft. Of course, the meridian first ruled has by that time become antiquated, but the new meridian can be drawn through each of the old meridians, the variation being the same in each case, either from observations made upon the variation from some fixed marks on the surface, or by adopting the variation as given by the Astronomer Royal.

In addition to making a plan showing correctly every object upon the surface, the surveyor should mark on it the position of lines of sewers, or drains, the property of any sanitary authority, also the position of lines of gas and water-pipes. The lines of fences shown are supposed to represent the centre of the hedge or wall, unless there is a ditch, in which case the line shown on the plan should be the centre of the ditch, but the position of the hedge should also be noted by a little mark upon the line, as shown at *h* (Fig. 84). If a wall is the boundary of a property, it is generally all upon one side; in that case the line shown upon the plan will be the side of the wall that represents the boundary. In the case of a river dividing two properties, the boundary-line is generally in the centre of the river; in the case of a public road dividing two properties, the boundary-line of the minerals is generally the centre of the road; but this is not always the case, and the correct boundary-line of the mineral property may have to be determined by reference to the title-deeds.

Reduction of Lengths for Inclination.—As before mentioned (p. 11, Chapter II.), it is necessary, in chaining, to measure the horizontal distance between various stations for the purposes of producing a plan in which all the objects are shown upon the same horizontal plane. Where the measurements are obtained with the chain or tape, this can be done in the manner referred to, by ranging a series of vertical poles in the line to be measured, and holding the chain or tape as nearly as possible level when measuring the distance from pole to pole. For the purposes of ordinary accuracy, it is not necessary that this chain or tape should be absolutely level, because at moderate inclinations the differences between the length of the line as measured on the slope, and as measured strictly level between the two poles, is very slight; thus at an inclination of 2½° the difference is rather less than 0·1 per cent. This, of course, would be a serious matter for long lengths, or for the very accurate fixing of some particular point, but for the ordinary filling-in of a survey it is sufficiently accurate. This method of measuring should only be resorted to either in the absence of instruments for taking the inclination or for the case of short slopes, banks, or terraces.

One of the chief uses of the theodolite is to facilitate the taking of the vertical angle formed by the slope of the line to be measured, and a line in a horizontal plane; in order that the true horizontal distance may be calculated. The method of reduction generally adopted may be explained with the aid of Fig. 86. Here the distance measured on the slope is, say, 1562, the angle of inclination is 2°. It is evident that the horizontal distance is equal to the cosine of the angle, if the slope is considered as the radius, and the vertical height of the upper end of the slope above the lower end is equal to the sine of the angle. On referring to a book of mathematical tables, it appears that the natural cosine of 2° is 0·9993908, and the natural sine is 0·0348995; therefore the length measured on the slope has to be multiplied by the decimal fraction representing the cosine; thus if the length had been 100, the cosine would be 99·939; if it had been 1000, the cosine would be 999·39; in this case the decimal fraction has to be multiplied by 1562, and the actual cosine, which is the horizontal distance, is 1561·0484296, and the length of the sine is obtained by multiplying the decimal fraction by 1562; therefore the actual sine or altitude is 54·5130190.

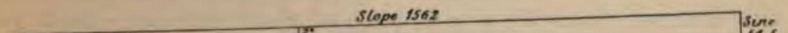


FIG. 86.—Reduction to horizontal distance of lengths measured on a slope.

In taking the inclination with the theodolite, the vertical circle is clamped with the vernier at zero, and the telescope is fixed horizontally by means of the levelling-screws; the telescope is then unclamped, and fixed upon the station of which the altitude has to be observed, the vernier reading giving the angle of inclination. It is important that the cross-hairs of the telescope shall be fixed upon a mark which is the same height above the ground as the centre of the telescope, and for that purpose a cross-bar or piece of paper should be fixed upon the pole at the proper altitude.

Average Inclination of Slope and Steep Undulations.—It must be borne in mind that with the theodolite the average inclination of a slope is measured; this may be a moderate inclination, say 4°, as shown in Fig. 87. Here, supposing the length of the straight line measured down the average slope along a line

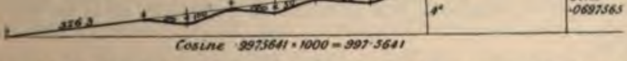


FIG. 87.—Reduction to horizontal distance of lengths measured over undulating ground.

stretched tight from top to bottom, to be 1000 links, then the reduced length is equal to 1000 links multiplied by the decimal fraction representing the natural cosine of 4°, which is 0·9975641, or the reduced length is 997·5641 links.

But in this particular case the average slope is compounded of a number of shorter slopes, some of which are very steep, as shown in the following table :—

	Length measured on the slope.	Inclination in degrees.	Cosine.	Reduced length.
No. 1	80	4	0·9975	79·80
No. 2	100	28	0·8829	88·29
No. 3	100	20	0·93969	93·969
No. 4	50	27	0·8910	44·55
No. 5	50	14	0·97029	48·51
No. 6	50	28	0·8829	44·14
No. 7	100	8	0·99026	99·026
No. 8	100	15	0·9659	96·59
No. 9	80	15	0·9659	77·27
No. 10	326·3	4	0·9975	325·48
	Total 1036·3			Total 997·62

Here it will be observed that the total length measured along the undulations is 1036·3, and therefore it would be very misleading to reduce the length so measured by the reduction due to the average inclination for 4°; it is necessary to measure the inclination of each slope, unless the method described in Chapter II. of measuring in horizontal steps is adopted.

The student will gather from this example that one short bit of steep incline, say 28° in 100 links, may cause a greater error in measurement than a gentle slope such as 2° would cause in 1 mile.

For the purposes of precise accuracy in a large survey, it is necessary to take the inclination of the gentlest slopes, but it is far more important to be careful in the chaining of short pieces of rough ground; and, where perfect accuracy is required, it is necessary to stretch the chain, or steel tape, or steel wire from station to station, the precise inclination of this wire being observed.

Measurements can be obtained with great accuracy by the system of triangulation shown in Fig. 83, as by this method all the errors due to the roughness of the ground are eliminated, except so far as they may affect the shorter lines between the main stations.

CHAPTER VIII.

UNDERGROUND SURVEYING.

In the collieries of Great Britain the shafts are generally sunk vertically, in which case the centre of the shaft at the bottom should be vertically below the centre of the shaft at the top; but it sometimes happens that the shaft has got a little twisted in sinking, and therefore, in starting the survey of a new colliery, it is necessary to hang a plumb-line down the shaft, in order to transfer the centre-mark from the surface to some beam at the bottom of the shaft, and when this has once been carefully done, it is desirable to make a written record of it upon the plan (see Fig. 84). If the shaft is not vertical, the bearing and inclination must be taken in the same manner as any other highly inclined passage.

Surveying with Miner's Dial or Compass.—The following is the method of making an ordinary colliery survey with the Hedley dial shown in Fig. 24. Assuming that there is no iron or other substance to attract the needle from the meridian, the dial is placed in the centre of the road of which the direction is required. A mark is fixed in the centre of the shaft, say a lamp; if this lamp cannot be conveniently fixed in the centre of the shaft, it may be moved nearer to or further away from the dial, but it must be placed on some part of a straight line which passes through the centre of the shaft and the dial. The distance at which the dial is placed from the mark, or the length of the sight, is, generally speaking, as far as the nature of the case permits. Supposing the road to be straight for a considerable distance from the shaft-bottom, the dial may be placed, say, 5 chains from the mark; but the distance must not be so great as to prevent the surveyor seeing the lamp clearly through the slit of the sight, or holding convenient communication with the other members of the party who are making the measurements and fixing the lights under his direction. The dial and lights should always, where practicable, be fixed in the centre of the roadway to be measured, and then the line of survey will correspond exactly with the direction of the roads. When this is not done, offsets must be taken to the side of the road.

The dial being now fixed and levelled, the ball and socket-joint, or other arrangement for levelling, is clamped; the needle is unclamped; the sights are turned upon the candle or lamp to be observed; the surveyor looking through the slit, and cutting the lamp-flame with the vertical hair. As soon as the needle is steady, the bearing can be read. The dial should be so placed that the side of the graduated circle which has the letter N engraved on it is turned in the direction in which the survey is proceeding; that is to say, in this case, in the direction from the shaft towards the dial. If the north end of the needle now points exactly to the zero mark under the letter N on the graduated circle, the direction of the line is due (magnetic) north; if, on the other hand, the north end of the needle points to the graduation at 180°, or to the zero mark under the letter S, the direction of the line is due south; if the needle points to the 90th degree, the direction of the line is due west; and if it points to the 270th degree, or to the zero mark under the letter E, the direction of the line is due east; if the needle points to the 45th degree between the letters N and W, the direction of the line is north 45° west; if it points to 20, it is north 20° west; if it points to some place between 20 and 21, say a quarter of the distance from 20, the direction is north $20\frac{1}{4}°$ west. The bearing so observed is booked as No. 1 bearing. A light is now fixed at a point further along the road, and the sights of the dial are turned upon this, care being taken that the sight which is on that side of the graduated circle on which is marked the letter N is turned towards this light, because that is the direction in which the survey is proceeding. The bearing is then read in the manner described for No. 1 bearing, and is booked as No. 2 bearing. The measurements are now taken, a Gunter's chain being generally used. If it is desired to note the exact width and every slight bend in the sides of the road, then the chaining may be done on a line kept straight from the dial to the light, by ranging lamps or candles in the line, in the same way as poles are ranged in a line on the surface, and offsets can be taken to right and left of this line. The length at which roads branch off is also noted. When the measurements have been made, the surveyor proceeds with the dial and legs past the forward light along the road which he is surveying, till he has got a convenient distance, or till he comes to some turn in the road which would hide the light from his view if he went further; he again fixes the dial in the centre of the road, and, sighting back to the light he has left, takes No. 3 bearing, and sends a light forward for No. 4 bearing; then the measurements of these two lines are taken. In this way the surveyor proceeds throughout the mine. He will, perhaps, survey back to the shaft by another road, and his last sight may be taken to the identical spot on which the first light was placed; in that case there is what is called "a tie." During the course of the survey, the surveyor will probably leave marks opposite the centre of some of the roads branching out to the right or left, from which he can start to survey the branch road. In "loose-needle" surveying only one set of legs is required, and this is used for the dial, the lamp or candle to which the sights are taken being put on the floor of the mine.

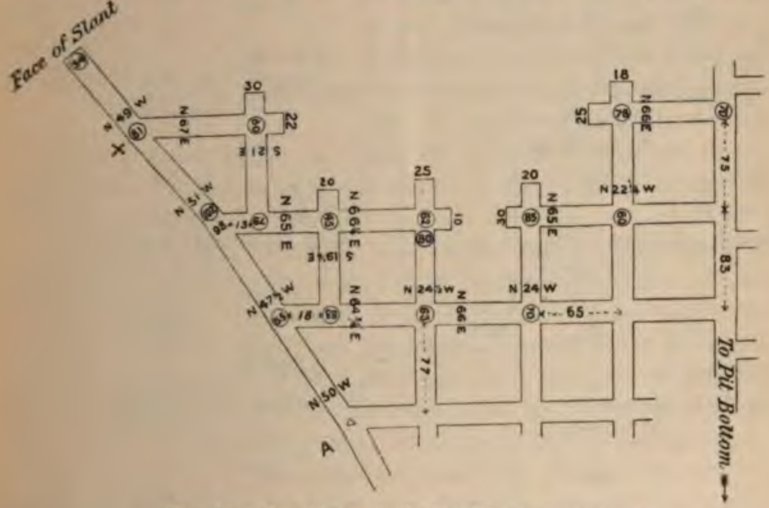


FIG. 88.—Graphic method of booking a survey.

NOTE.—A, Station left in previous quarterly survey. Roads driven 15 links wide. All measurements in links. ×, Station left for next quarterly survey.

Booking.—There are several ways of booking or recording the bearings and measurements. One is shown in Fig. 88. This may be called the graphic method: the note-book contains a sketch; very little attempt is made to make the sketch according to scale, but it shows the turns of the road, branch roads, etc.,

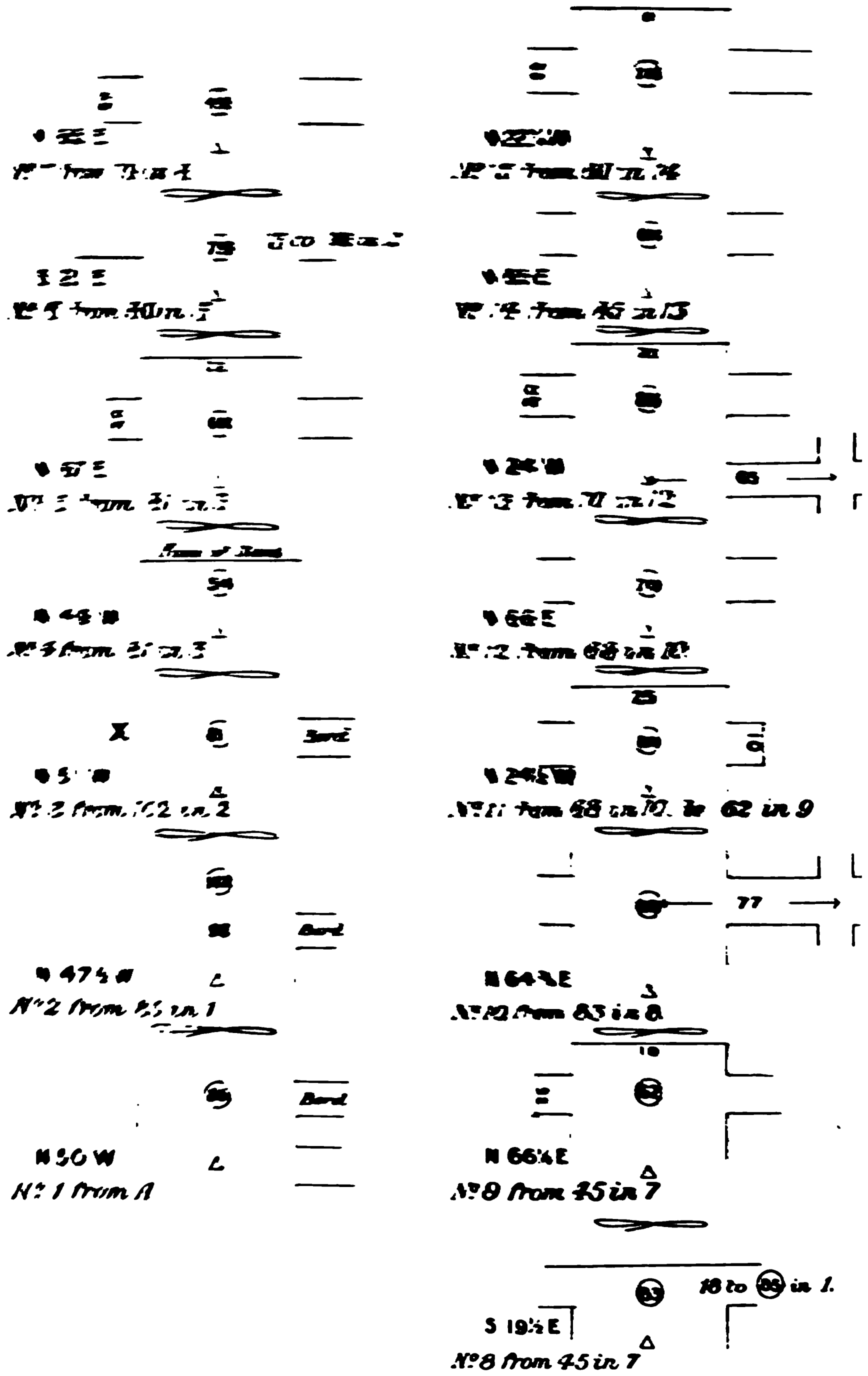


FIG. 89.—Written method of booking a survey.

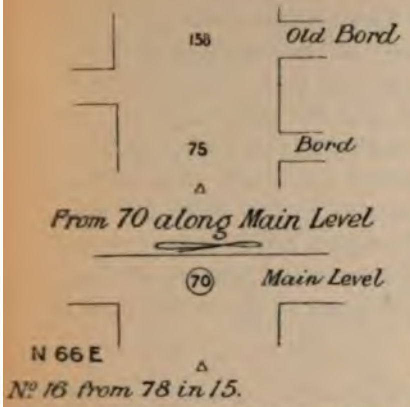


FIG. 89A.—Written method of booking a survey.

and to some extent facilitates plotting. The same survey may be booked in consecutive writing, as shown in Fig. 89; or, again, in the form of a table which may be printed so that the columns only have to be filled up. An instance from another mine is shown in Fig. 90. These three methods, the graphic, the written, and the tabular, have their different advocates; but the experienced surveyor may use all three methods at various times. Fig. 91 is the plan plotted from the survey notes given in Figs. 88 and 89. Fig. 92 is the plan plotted from the bookings given in Fig. 90. The method of plotting is given in Chapter IX.

Fast-needle Dialling with Dial with Outside Vernier.—It happens very frequently that, owing to the occurrence of iron or other source of attraction, the needle cannot be used near the bottom of the shaft, and perhaps the only place in which a correct bearing can be obtained is in some old road or in some working place from which the rails can be removed. When this is the case, the survey may be made in one of two methods.

No. 1 method: The surveyor proceeds at once to the old road or other place where he can obtain a loose-needle sight; this is, perhaps, a quarter of a mile from the pit-bottom. He there fixes his tripod stand firmly, levels the dial, and lets the needle swing; he then looks forward in the direction in which he intends to proceed, and observes the bearing. This forward light is fixed upon a tripod stand similar to the dial-stand, which is placed at a convenient point on the line to be surveyed, the lamp being placed in a cup which has been carefully levelled; the cup should be of such a diameter as just to contain the lamp without difficulty; in this way the centre of the lamp is made to coincide with the centre of the stand.

The dial is now moved forward and placed on the stand previously occupied by the lamp; the lamp-cup and lamp being removed and placed upon the stand from which the dial has been taken; a third tripod with cup and lamp is sent forward along the road to be surveyed and fixed at a convenient distance. The dial being now in a place where there is attraction, the needle is no use, and may be clamped; hence the term "fast-needle." The vernier circle is now unclamped, and the zero mark on the vernier fixed at a mark on the external graduated circle corresponding with the loose-needle bearing last read, thus if the bearing was N. 89° 30′ W., the vernier is put to N.W. 89° 30′, and is clamped in that position. The sights are now fixed upon the light where the dial was previously, and the bearing as read on the vernier circle is, of course, the same as previously, that is, N.W. 89° 30′. The vertical axis of the dial is securely clamped, the vernier circle is then unclamped, and the sights directed, by means of the milled head on the pinion, upon the forward light, bearing in mind that the sight upon that side of the dial where the letter N is marked is always turned in the direction in which the survey is proceeding; then, having fixed the sights upon the forward mark, the vernier is read, say, N.E. 314°. The vernier circle is then securely clamped, the vertical axis is unclamped, the dial is removed from the tripod,

Number of sight.	Distance.		Inclination.	Bearing.	Remarks.
	Measured	Plotted.			
(1)	355	350	10° rise	N. 50° E.	Commenced in A slant at bottom of No. 11 gate, and left mark to return to. At 160, No. 10 gate, to the left; at 330, No. 9 gate, to the left.
(2)	200	200	—	N. 50° E.	From 355 in (1), down A slant to coal-face.
(3)	140	140	—	S. 70° E.	From 200 in (2), down coal-face to old goaf at right of slant.
(4)	183	183	—	N. 80° W.	From 200 in (2), along face to left of slant. At 180, No. 9 gate.
(5)	256	256	—	N. 89½° W.	From 183 in (4), along coal-face. At 132, No. 10 gate.
(6)	200	200	—	S. 70° W.	From 256 in (5), along coal-face to edge of goaf.
(7)	398	385	15° fall	S. 1¾° E.	From 256 in (5), down No. 11 gate to mark left in A slant at commencement of survey.

FIG. 90.—Tabular method of booking a survey.

the lamp and lamp-cup from the tripod behind are brought forward and put in the place of the dial, the dial is taken forward and substituted for the lamp and cup on the forward tripod, and levelled. The back sight is now taken to the stand

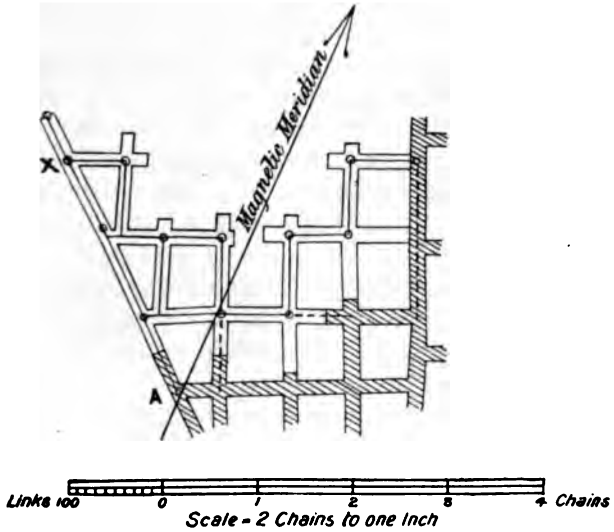


FIG. 91.—Plan plotted from the survey notes given in Figs. 88 and 89.

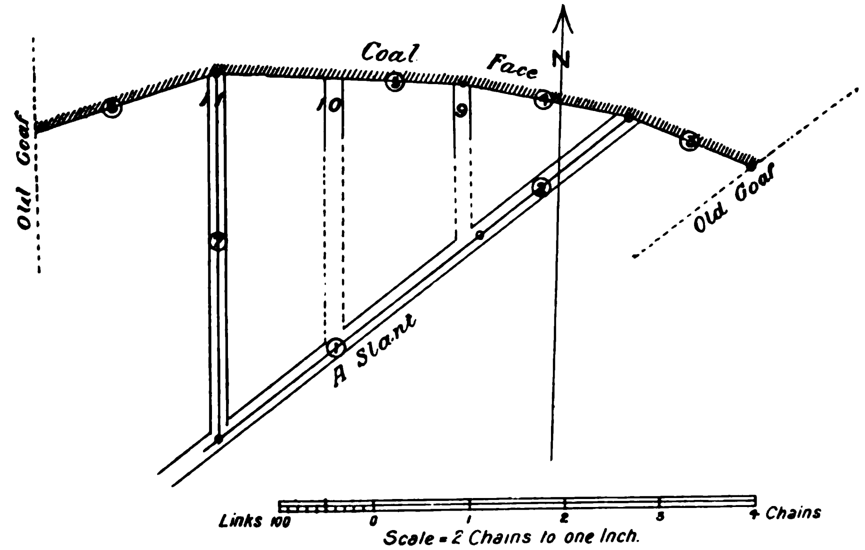


FIG. 92.—Plan plotted from the survey notes given in Fig. 90.

where the dial was last fixed, and the vertical axis is clamped with the sights on this line, the bearing on the vernier circle reading, of course, as before, N.E. 314°. The vernier circle is now unclamped, a tripod with lamp and cup are fixed forward, and the sights turned on to the forward light, and the bearing is read by the vernier, say N.E. 314° 30′. The survey is continued throughout in this manner.

If, during the course of a survey, the dial is fixed at some station where there is believed to be no attraction, the needle may be unclamped and the bearing of the needle read. This should agree with the bearing shown by the vernier; if it does not agree, it is a proof either that there is attraction, or that the survey has been inaccurately made; if it agrees, and if there is no attraction, it is a proof that the angles have been accurately taken. The more loose-needle sights that can be obtained in a fast-needle survey the better, because by this means the accuracy of the work is proved.

No. 2 method: Instead of the preceding mode of beginning the survey, another, which is perhaps in some respects more accurate, may be adopted. Fixing the dial similarly where there is no attraction, the needle is unclamped, and the dial turned until the north end of the needle corresponds with the zero mark under the letter N on the graduated circle. With the help of good lights the needle may be adjusted to this mark with great accuracy, with an error of say not exceeding 1′; but to attain this degree of accuracy, the surveyor must take great pains. The vertical axis of the dial is now clamped and the needle again observed, to make sure that it still points to

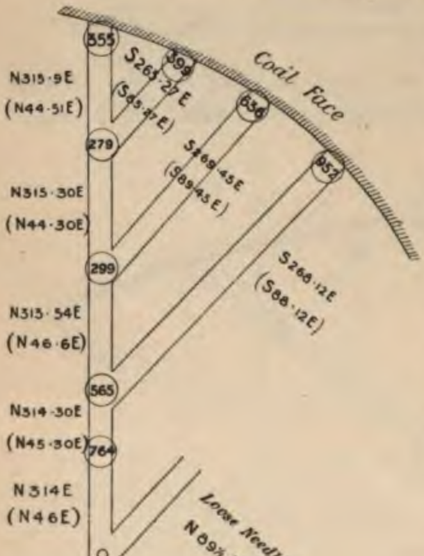


Fig. 93.—Method of booking a fast-needle survey.

N.; the vernier, of course, also corresponds with zero on the outside circle. The vernier circle is now unclamped, and the sights directed, by means of the racking pinion, on to the forward mark, and the bearing read as before, say N.W. 89° 30′. The survey and bookings then proceed as before. The booking of this survey (by the graphic method) is shown in Fig. 93.

There is no difference between the booking of a fast-needle survey and a loose-needle survey, except that it is a good practice to book the angle of the quadrant as well as the angle of the circle, as this forms a check upon the accuracy of the bookings. The outer graduated circle on which the vernier works is divided continuously from 0° to 360°, and is not subdivided into quadrants, so that the angle of the quadrant has to be obtained by a mental calculation, as follows:—

Angles as read on circle.	Quadrant.	Quadrant angle.
Between 0° and 90°	N.W.	Same as circle-reading
" 90° and 180°	S.W.	Subtract circle-reading from 180°
" 180° and 270°	S.E.	Subtract 180° from circle-reading
" 270° and 360°	N.E.	Subtract circle-reading from 360°

The quadrant angles are shown on the bottom of the dial-box and act as a check on the calculation.

The practical difference between the making of a fast-needle survey and a loose-needle survey is that the dial has to be fixed twice when using the fast needle for once that is required in using the loose needle, because the back sight in the fast-needle survey is required as a base-line from which to measure the angle of the forward sight, whereas in the loose-needle process the magnetic meridian always forms the base, and bearings can be read both of back-sight and fore-sight from the same station.

Fast-needle Survey with Dial with Inside Vernier.—Many dials are made without the outside graduated circle and vernier, the vernier being inside, moving round the circle with the sights as shown in Fig. 28. If with this kind of dial the vernier is moved from the zero, the sights are out of position, and the vernier must be restored to the zero mark before taking a loose-needle bearing. The process of fast-needle surveying with this dial is as follows: Placing the dial, as in the last instance, on a tripod stand where there is no attraction, the needle is unclamped, and, when it has settled, the dial is turned on the vertical axis (the vernier being at zero) until the zero on the graduated circle is opposite the north end of the needle. The sights are now in the magnetic meridian; the vertical axis is then clamped and the sights turned, by means of the racking pinion, upon the forward mark, reading N. 89° 30′ E. It will be remembered that in the survey last described the bearing was put down as N.W., but this time the bearing read by the vernier is N.E., that is because the vernier is fixed on the same circle as that used for the needle, and for convenience in reading the needle (in loose-needle surveying) the east and west have been transposed on the circle. Having read the bearing thus, N.E. 89° 30′, it is booked as N.W. 89° 30′. The surveyor now moves the dial to the forward stand, placing a lamp where his dial was fixed; looking back towards this lamp and clamping the vertical axis, the bearing still reads, according to the dial, N. 89° 30′ E. He now turns the sights to the forward light, which reads N. 46° W., and is booked N. 46° E. He now clamps the vernier circle, unclamps the vertical axis, and moves the dial on to the forward legs, and fixes the sights in the direction of the back sight before he unclamps the vernier screw to take the forward sight. He proceeds with the survey as in the previous instance, but with this difference, that he always books the bearings as read from the vernier with the E. or W. reversed. If he arrives at some place where there is no attraction, he can loosen the needle, the north end of which should then come to rest at 0° under the letter N on the graduated circle; if it does not, it is a sign that there has been some mistake in taking the angles.

Fast-needle Survey without Loose-needle Base—Dial with Outside Vernier.—Another method of proceeding with the fast-needle survey is to fix the dial in the road which it is desired to survey, notwithstanding that there is attraction, and turn the sights towards a mark in the centre of the shaft or other station forming the beginning of the survey. The vernier circle being clamped at 0°, this line is booked as due north, or 0°. The forward sight is taken to a lamp fixed in a cup on the tripod stand. To take this sight, the vertical axis having been first clamped upon the back sight, the vernier circle is unclamped and the sights turned upon the light; the angle is then read on the outside circle, say N.E. 350° or N. 10° E., and this bearing is booked. The vernier circle is then clamped, and vertical axis unclamped, and the dial moved forward and the sights fixed again in the line of the last sight, reading, of course, the same bearing. The vertical axis is now clamped, the vernier screw unclamped, and the sights turned upon the forward light, the bearing reading on the outside circle say N.E. 340° or N. 20° E. The survey is continued in the same way, the first sight that was observed being taken as the meridian line.

When some portion of the mine is reached where there is no attraction, the sights are fixed upon the back sight, which has been recorded say N.W. 20°; the needle is released, and the real bearing of the line in which the sights are clamped is shown by the needle-point; thus the bearing, as read on the vernier circle, is N.W. 20°, whereas the actual bearing as shown by the needle is S.W. 50°, or 130° on the circle, showing a difference between the real bearing and the bearing so far recorded in the survey, of 110°. The bearings hitherto recorded in the note-book may now be all corrected by the addition of 110°; thus the first bearing, instead of being N. or 0°, is really S.W. 70° in the quadrant, or 110° on the circle; the next bearing, instead of being N.E. 10°, or 350° in the quadrant, is S.W. 80°, or 100° on the circle; the next bearing, instead of being N.E. 20°, or 340° on the circle, is due W., or 90° on the circle.

Having now got the true bearing, the vernier circle is adjusted to it, and set at S. 50° W. or 130°; the forward sight can then be read say 140°, or S. 40° W.; the same bearing, of course, will be given by the loose needle. The dial is now moved to the forward station, where there is attraction; the vernier plate has been clamped at S. 40° W.; the sights are now fixed on the tripod previously occupied by the dial, and the forward bearing read with the vernier, say S. 30° W., or 150°, and the survey continued in the manner described for fast-needle dialling (p. 133, Fig. 93).

This process is sometimes modified as follows: The whole survey is made with the fast needle, using the first sight as a base-line, and calling that north, without any reference to the actual direction, as in the instance above given, no loose-needle sight being taken until the end of the survey (loose-needle sights may be taken during the progress of the survey at places where there is no attraction, to obtain the bearing; for this purpose a diversion may be made into some place where there is no iron), which is, say, at the face of the workings or the end [illegible] rails or other iron have been removed. [illegible] and the true bearing of the last [illegible] say N. 80° E. in the quadrant, or 330, [illegible] whereas the nominal bearing, as recorded by [illegible] of the same line, was N. 50° W., or 50° in [illegible] of 80° between the two bearings, [illegible] graduations on the circle. The first [illegible] survey may now be corrected to that extent, [illegible] now read 280° in the circle, or [illegible] bearing.

[illegible] the bearings will be plotted as [illegible] direction of the first sight as the [illegible] meridian will now be ruled [illegible] starting-point, which is say the centre [illegible] ruled at an angle of 80° (or [illegible] meridian. A careful tracing [illegible] the plan of the estate, the centre [illegible] coincident, placing the real [illegible] line of the plan. In a [illegible] from the intermediate [illegible]

[illegible]—Dial with Inside Vernier. [illegible] inside graduated circle, [illegible] is recorded as north, [illegible] axis is now clamped, the [illegible] upon the forward [illegible] dial was read with [illegible] with this dial N.W. [illegible] N. 10° E. The vernier [illegible] unclamped, and the dial [illegible] sights directed back to [illegible] fixed: the vertical [illegible] still reading N.W. [illegible] and the sights fixed [illegible] N.W. [illegible] in the circle, [illegible] N.E. [illegible] and this bearing is [illegible] in this way, the [illegible] E. when the vernier reads W., and *vice* [illegible] is no attraction. [illegible] fixed at this place, and the bearing as [illegible] observed with great accuracy, the vernier is turned to 0° and there clamped; the vertical axis being unclamped, the sights are now turned upon the back light, and the actual bearing with the loose needle is observed,—the actual bearing is say, as in the instance given on p. 139, S. 50° W., or 130° in the circle; whereas the bearing, as recorded by the vernier, was N. 20° E., and the bearing as booked N. 20° W., or 20° in the circle. There is thus a difference of 110° in the readings, and the readings hitherto taken may now be corrected by adding 110°. The vernier is now unclamped, and is fixed at S.E. 50°, or 230° on the graduated circle; the sights are now turned again upon the back sight, when the loose needle should point to the north, or 0° on the graduated circle; the vertical axis is now again clamped, the vernier circle unclamped, and the forward sight taken; the loose needle still points to the north; the bearing is read with the vernier, and the survey is continued as in the method given on p. 138.

Large Surveys.—It is frequently the case that the survey of a mine or district of a mine has to be interrupted, and recommenced the next day or after an interval of days or weeks. If the survey is made on the loose-needle plan, there is no difficulty or disadvantage attending the interruption; the place where the survey ends may be marked by means of a hole drilled or cut in the side or in the roof, or may be simply recorded by measurements from some fixed place, such as branch roads, and the exact position of the light taken by offsets, as shown on the sketch (see Fig. 94). The survey can be continued at any time by placing a light at this place, which can be refound by measurements, and then proceeding to observe the bearing of the forward lines.

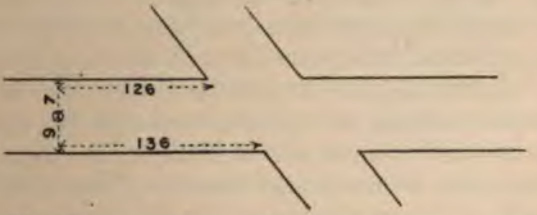


FIG. 94.—Station in underground survey fixed by measurements.

In the case of a fast-needle survey, however, the last line of which the bearing has been noted being the base-line from which the bearings of the continued survey have to be taken, it is essential that the exact position of the instrument and of the lamp last observed should be marked with great care. This, however, is rather a difficult and unsatisfactory operation. The ordinary workings and roadways of a mine are not suitable places for accurate and permanent marks; the roof, floor, sides, and timber are liable to continuous movement, and might move an inch or two in the night; the probability or otherwise of such a movement may, however, be known to those who are constantly in the mine, and know whether that part is quiet or subject to movement. In order to diminish and to discover errors due to inaccurate marks, at least three places on the line of survey should be marked. As long as these three marks preserve their original relative positions, the chances are very much against any error due to the movement or inaccurate placing of the marks. The distances from mark to mark should also be as long as possible; thus if the distance were 100 links, an error of 1 inch in the position of a mark would amount to 1 in 792, or about 4⅓ minutes, whereas if the distance were 5 chains, the error would be proportionally less, or about 1 minute.

A common plan of fixing a mark is to drill a hole in the roof; into this a wooden plug is driven, and into the wooden plug is driven a nail or hook, from which a lamp may be hung by a string, care being taken to see that the lamp-flame is vertically below the hook. Three marks may all be fixed in the same line, and the distance from the dial measured. On restarting the survey, lamps are hung from each of the three marks, and if they are in one straight line as originally fixed, the surveyor may have confidence that there has been no disturbance. He then fixes the dial under the forward mark, and adjusts the vernier to the reading of the bearing as recorded in his note-book. He then clamps the vernier, unclamps the vertical axis, and turns the sights on to the back light; then clamping the vertical axis, he unclamps the vernier circle, and takes the forward sight in the ordinary manner, continuing the survey as if there had been no interruption.

Theodolite.—Where extreme accuracy is required (and in every large mine it is required), the theodolite is often substituted for the dial. The process of surveying is the same as that used with the Hedley dial with outside vernier, and the booking is done in the manner shown in Fig. 93. In the theodolite as generally made the compass needle cannot be conveniently read, and is therefore only used for obtaining the meridian. Where a trough or tubular compass is used, the needle only swings freely when in the magnetic meridian. If the survey is begun at some place where there is no attraction, the telescope is turned towards the magnetic north, the utmost care being taken to see that the needle is swinging freely, and that the direction of the telescope is parallel to the meridian line; the vertical axis of the theodolite is then clamped, the vernier plate is unclamped, and the telescope directed towards the light on the line of survey of which the bearing has to be observed, whether that is a backward or a forward light. The bearing is now read from the vernier, and is recorded both as the bearing of a quadrant and as the degree of the circle, thus: N.E. 40°, or 40° on the graduated circle.

It will be noted that the graduated circle of the theodolite reads clockwise, and that, therefore, when the telescope is turned from north eastwardly, the bearings as read advance from 0° to 40°, 50°, and upwards; whereas on the outside circle of the dial, shown in Fig. 24, the graduations read the reverse of clockwise, and when the dial is turned N.E., the figures read from 360° backwards, as 350°, 340°, etc. On the other hand, when the sights are turned W., the figures advance, as 10°, 20°, 30°, etc.; the reverse of this being the case with the theodolite. Some confusion is therefore apt to arise in the mind of the surveyor who first uses a dial of which the vernier circle is graduated the reverse of clockwise, and then uses a theodolite graduated clockwise. The remedy for this appears to be that the mining surveyor using an outside-circle dial should have the inner circle read from the needle, graduated the reverse of clockwise, and the outside circle graduated clockwise.

The advantages to be gained by the use of a theodolite, as compared with an ordinary dial, are as follows: (1) More accurate sighting of the stations, owing to the use of a telescope; (2) more accurate reading of the angles owing to the use of a more finely graduated circle and vernier, read by means of a microscope; (3) longer sights, due to the use of a telescope; (4) greater accuracy in fixing the marks, also due to the use of a telescope; (5) greater accuracy in observing the inclination, due to the long level on the telescope, and to the finely graduated vertical circle and vernier read with the aid of microscopes; (6) use of the theodolite for levelling, either as an ordinary level, the vernier fixed on the vertical circle at 0°, or by taking angles, the latter process being sufficiently accurate for most mining purposes, and very much more rapid than the ordinary process of levelling; (7) the measurement of lengths by using the instrument as a tacheometer; (8) the possibility of taking sights upwards at any degree of elevation, and downwards with a depression of 60°.

A special eye-piece is supplied with the instrument, to be used when taking sights vertically upwards, or nearly vertical; this enables the theodolite to be used for sighting up vertical shafts, and marks can be placed on the surface or at some intermediate level above the theodolite in the same vertical plane as some line of underground survey.

Surveying with Prismatic Compass.—This instrument, shown in Fig. 23, may be used instead of the ordinary miner's dial for loose-needle surveys. Of course, for work having any pretence of accuracy, it must be fixed on a tripod stand.

Surveying with Henderson's Rapid Traverser.—This instrument, shown in Figs. 43 and 44, has one notable convenience, which is that the survey can be just as conveniently started where there is attraction and the needle cannot be used, as where there is no attraction. Referring to Fig. 95, the instrument is fixed up at A, and levelled, and the sights turned to the centre of the shaft O and clamped. By means of a pencil the line of the fiducial edge is marked in two places on the fifth ring, and the direction of the survey indicated by an arrow-head; No. 1 is written in the corresponding notch of the alidade. The alidade is now unclamped and the sights turned towards the forward light at B and clamped; the line of the sight is again marked on the fifth ring and marked No. 2. If, however, No. 2 line should nearly coincide

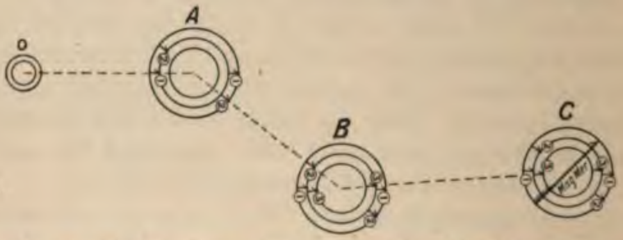


FIG. 95.—Method of surveying with Henderson's rapid traverser.

with No. 1 line, then it should be marked on the fourth ring. The fiducial edge being clamped on this sight, the instrument is lifted off the tripod, a lamp and lamp-cup are substituted for it, and the instrument is placed on the forward tripod **B**, in place of the lamp and cup previously there. The sights are turned on to the back light at **A**; the vertical axis being then clamped, the sights are now unclamped, and turned on the forward light **C** and again clamped, and the direction of the sight marked on the fifth or fourth ring, or, in case the direction should be nearly the same as in the lines 1 and 2, on the third ring, so as to avoid confusion; this line is marked No. 3. The instrument is now moved to the forward tripod at **C**, and here, as there is no attraction, the bearings can be taken. The sights are turned upon the back light **B**; the vertical axis is again securely clamped, the sights are then unclamped, and a trough compass is placed on the disc beside the alidade. The trough compass (see Fig. 38) is a compass needle in an elongated rectangular box, the sides of which are parallel to the meridian on the graduated arcs. One of these parallel sides is accurately placed against the thick side of the alidade, which is then turned until the needle of the compass points exactly in the meridian line; the alidade, of course, is then in the same line, and this line is ruled with a fine-pointed pencil across the whole width of the disc and by the thick side of the alidade. All the bearings previously drawn on the disc now appear in their correct relation to the meridian line. The survey may be continued in the same way as it was begun, and all the bearings afterwards marked will also be in correct relation to the meridian line. If any other place is met with where there is no attraction, the compass can be again applied, and if the meridian first marked on the disc was accurately shown, and the survey has since proceeded with accuracy, the second meridian line will correspond with the first.

This method of surveying is similar to the fast needle in this respect, that the instrument is placed at each end of each line, the first line being used as a base from which to measure the angle made by it and the second line.

The instrument may be used to make a loose-needle survey in the following manner (see Fig. 96): The instrument is set up as before at **A**, and the vertical axis clamped; the sights are then unclamped, and by means of the trough compass the meridian is marked on the disc; the sights are then turned on the back light at **O**, and the bearing No. 1 ruled by means of the fiducial edge, as in the preceding example; the sights are then turned on the forward light **I**, and the bearing No. 2 marked as before. The instrument and tripod may now be lifted up and carried forward beyond the light **I**, and fixed at the place **B**. The sights are then unclamped and moved till the alidade is parallel with the meridian, as marked on the disc, and clamped; the trough compass is now placed on the disc with its side against the alidade, and the vertical axis is then turned until the needle points in the meridian; the vertical axis is then clamped, the sights unclamped and turned on the light **I**, and the bearing No. 3 marked in pencil against the fiducial edge. The sights are next turned on the forward light **2**, and the bearing No. 4 marked. The survey may be continued in the same way. By this method the instrument is only set up once for two bearings.

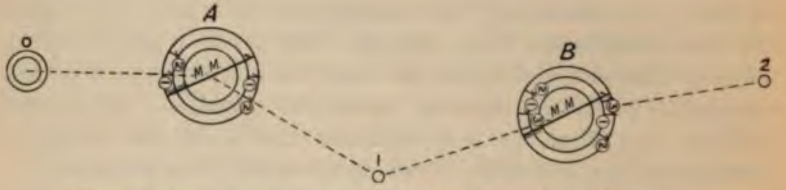


Fig. 96.—Loose-needle surveying with Henderson's rapid traverser.

The reader will notice that in using this instrument no bearings are recorded in the note-book, only lengths corresponding to the numbers of the sights, and, with regard to the booking of the numbers and lengths, he may adopt either of the three methods used for the dial, that is, the graphic, the written, or the tabular.

The Henderson traverser may have as a separate attachment a vertical semicircle with small telescope, by which inclinations can be read and sights taken vertically. There is also an arrangement by which moderate inclinations can be read without the use of the graduated semicircle; this consists of a slide *h*, which can be moved up or down the vertical limb through the openings of which the sights are taken, the eye being fixed at an opening at the top of the other vertical limb as shown in Fig. 97; the slide is moved up or down till it becomes in line with the light that is being observed. The position of this slide marks the angle which the line of sight makes with the horizontal line.

For the purpose of levelling the disc, a loose spirit-level is used, which may be carried in the waistcoat pocket. In plotting the survey, the celluloid disc is removed from the instrument and placed on the paper, where it serves as a protractor; the directions, being already marked on it, have only to be ruled off by means of a good metal parallel ruler. An example of an actual survey is given in Chapter IX. on "Methods of Plotting."

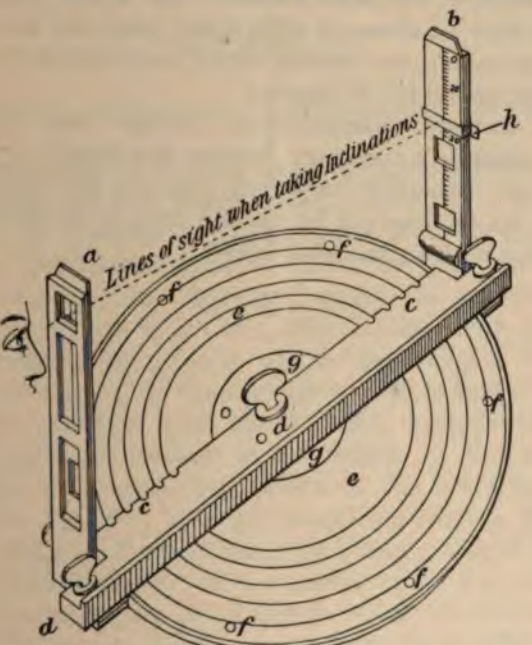


FIG. 97.—Method of measuring inclinations with Henderson's rapid traverser.

Surveying with Suspended Dial.—In some mines, especially in metal-mines, many of the passages, whether called shafts, drifts, rises, or winzes, are so highly inclined that an ordinary dial can scarcely be fixed. If the passage is so short and straight that a sight can be taken through from one level to another, the steepness of the road constitutes no difficulty, at least it does not when working with the theodolite or dial, unless the inclination is more than 60°; but where the passage is crooked so that it cannot be surveyed without placing the instrument in it, the suspended dial is often used. A strong linen cord is attached to a bar or prop fixed at either end of the length to be surveyed, and upon this the dial is suspended by two hooks, as shown in Fig. 36. The dial hangs level, and the needle shows the bearing of the cord. The length is then measured with a chain or tape, and the next length above or below is then observed in the same manner; the vertical angle must at the same time be observed with equal care, and this is accomplished by having the vertical circle round which the compass box rotates graduated in degrees.

It is, however, comparatively seldom that this method becomes absolutely necessary, because the length of these steep roads is not generally very long between the levels, and the direction can be observed by looking down from the level above, and looking up from the level below.

CHAPTER IX.

METHODS OF PLOTTING AN UNDERGROUND SURVEY.

THE usual method of plotting an underground survey is with the protractor, parallel ruler, scale, needle-point, and pencil. Protractors are described in Chap. V. and shown in Figs. 60–62.

Plotting with Metal Protractor.—The surveyor, having drawn a line to represent the meridian, places the protractor upon some part of the line which is a little distance from the part of the plan on which he wishes to plot the beginning of his survey. By means of weights, he fixes the protractor so that 0° and 180° are on the meridian line, the 0° being towards the north. Having made a prick-mark at the centre of the protractor, he takes the needle-point and pricks off No. 1 bearing against the edge (see Fig. 98); with his pencil he draws a dotted line away from this prick-mark, being the prolongation of an imaginary line from the centre of the protractor to the prick-mark. At the end of this short dotted line he writes the number of the sight and the bearing; he then pricks off No. 2 sight, marking the paper in a similar manner, and so on till he has pricked off all the sights of the survey or of that portion of the survey which falls within a convenient distance of where the protractor is placed. If the survey is extensive, he will rule another meridian line, exactly parallel to the first, on a portion of the paper over which the survey will extend. He then fixes the protractor on this new meridian line, and pricks off the remaining bearings, or as many as relate to that portion of the survey which lies near the protractor. If necessary, he may rule a third and fourth meridian, and mark off the bearings in a similar manner. He now takes the parallel ruler and, placing it on bearing No. 1, moves the ruler to that part of the paper on which he wishes to commence plotting, and rules a line; the beginning of it is marked with a prick-mark, and the end of it is pricked off on the line by means of a scale; he now fixes the parallel ruler in the direction of bearing No. 2 as pricked off from the protractor, and, rolling the ruler to the end of line No. 1, he draws line No. 2 from the prick-mark at the end of No. 1 line, and marks off the length with a scale and pricks it

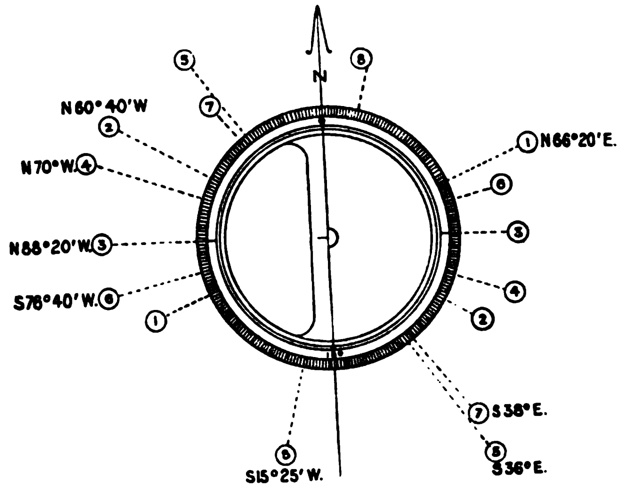


Fig. 98.—Method of plotting with brass protractor.

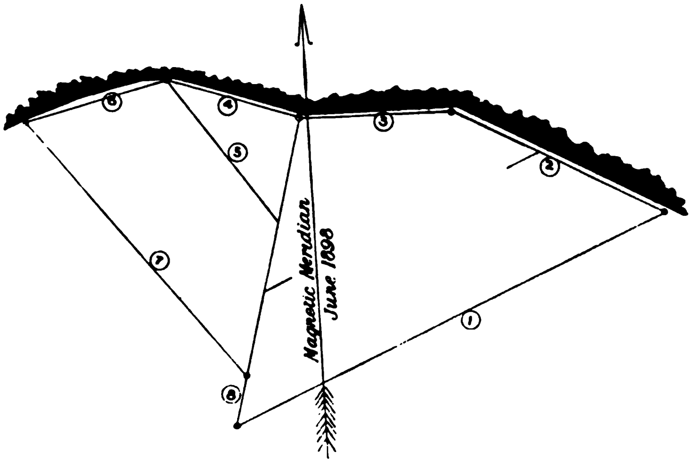


Fig. 99.—Draft of survey plotted from the bearings given in Fig. 98.

off; and so on through the whole of the survey. The draft of the plotted survey is shown in Fig. 99, and the finished plan in Fig. 100.

In marking off the bearings with the protractor, it is a common plan to make a mark on each side of the protractor; thus, if the bearing was N. 50° W., it would be numbered, and the direction N.W. written in pencil at the end of the dotted line.

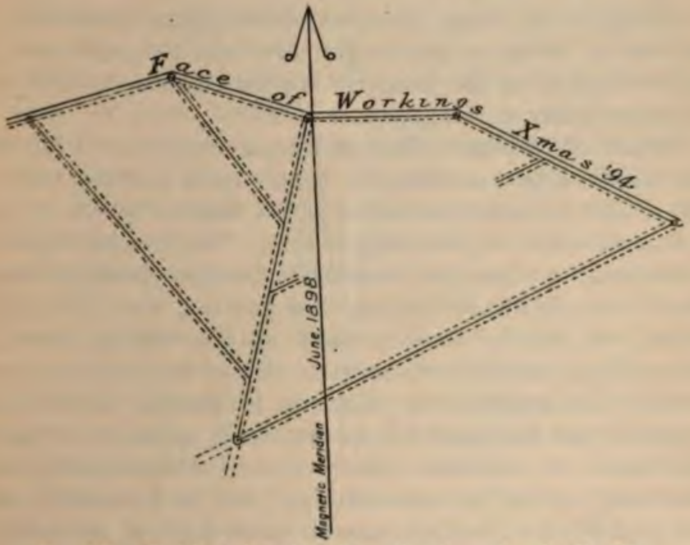


FIG. 100.—Finished plan plotted from bearings given in Fig. 98.

Another prick-mark would then be put opposite to it at S. 50° E., and the same number attached to it as to the first prick-mark. Making these two marks gives a longer base by which to set the parallel ruler, and the written bearing on the N.W. side reminds the surveyor of the direction in which the line is proceeding.

Plotting with Cardboard Protractor.—The cardboard protractor (as shown in Fig. 62) is often preferred to the metal protractor. This is fixed upon a meridian with the zero towards the north and 180° towards the south; a line is then ruled across the meridian in the direction east to west, that is to say, from 90° to 270°, so fixing the centre of the circle. The parallel ruler is then placed with one edge at the centre-mark and the same edge at the degree of the bearing, say N. 50° W., and is then rolled to the required position. When using the cardboard protractor the meridian is ruled on that part of the plan on which the plottings are to be made, so that the lines may all be laid down within the circle. If the parallel ruler is sufficiently long, it may be stretched right across the circle, say from N. 50° W. to S. 50° E., and this is the best plan and the one most commonly adopted. As the plotting proceeds the protractor can be moved from time to time along the meridian or to a fresh line ruled parallel to the first.

Owing to the large diameter of the paper protractor, the fractions of a degree are easily observed, and, with care, this method of plotting the bearings is very accurate, and no prick-marks are made on the paper.

Vernier Protractor.—The protractor shown in Fig. 61 is used where minute accuracy is necessary in plotting the bearings, as, for instance, in setting out a bearing, which proceeds for a great length in one straight line. The method of plotting is the same as that just described with the metal protractor. The vernier is set to the required bearing, and then this is pricked off by the needles fixed in the folding arms, the bearing being pricked off on each side of the centre so as to increase the length over which it is marked on the plan. Supposing the instrument to be accurately adjusted, so that the prick-marks on each side, when united by a line drawn through the centre, are in the same straight line (a test which can be easily made), the bearings can be marked off as accurately as they can be read by the theodolite vernier, but the points of the needle by which they are pricked must, of course, be fine for accurate work.

Errors of Plotting.—In plotting by the methods just described, the errors that may creep in are of a very obvious kind. With an 8 inch protractor the size of a prick-mark with an ordinary needle varies from $\frac{1}{8}^\circ$, which is very small, to $\frac{1}{6}^\circ$, which is an ordinary size; a pencil-line may be drawn much finer, and, if a hard and carefully sharpened pencil is used, may be drawn to about $\frac{1}{30}^\circ$ in thickness. Roughly speaking, however, it may be said that with an 8-inch protractor the bearing cannot be pricked off with a mark less than $\frac{1}{8}^\circ$ in width, and that even with the utmost care there may be an error of half that, or $\frac{1}{16}^\circ$.

Great care is required to fix the parallel ruler over the centre of the prick-marks, and the draughtsman is generally sufficiently satisfied if he can be sure that the parallel ruler is over both prick-marks without using a magnifying-glass to ascertain that it is over the centre of the prick-marks. In rolling the ruler the pressure must be applied midway between the two rollers, so as to prevent any slipping of one roller. If the rollers have exactly the same diameter, the ruler will keep its edge parallel to the line from which it started; if one roller is a little larger than the other, or has upon its circumference any dirt accidentally increasing its diameter, the ruler will not keep its edge parallel to the starting-line. The accuracy of the rolling may be tested by ruling two lines, the second line 12 inches or more distant from the first; then turning the ruler end for end, set it parallel to the first line, and roll it to the second line; if the edge of the ruler exactly coincides with this line, it shows that the rollers are each of the same size, and also that they are fixed concentrically on the axis. It is, however, difficult to get a parallel ruler that is perfectly accurate, and it is not uncommon to find that in rolling a distance of 8 inches it changes its direction to the extent of 4′, and of course such a ruler is no use for accurate work. The error may be reduced, however, by setting up on the plan, by means of scales, a number of parallel meridians not more than 12 inches apart, so that the ruler will not have to be moved any great distance; and the error can be still further eliminated by ruling the bearings first with one edge of the ruler and then reversing it and ruling the bearings with the other edge, and taking the mean.

The length of the lines is also subject to errors due to the practical difficulty of correctly marking off the distance. The diameter of a fine prick-mark on a 2-chain scale is about 1½ links, and the diameter of a clear prick-mark on a 2-chain scale is about 2 links. It is thus evident that two different draughtsmen may plot the same survey so as to show a considerable difference at the end; and if, after plotting the survey, the surveyor finds that it does not tie in, it may be quite easy for him, knowing in which direction lies the apparent error, by going over his plotting, to eliminate it.

All these errors may be reduced in amount in the following way: By using (1) larger protractors or a vernier protractor; (2) a very fine needle-point; (3) a very finely pointed pencil; (4) an accurately rolling parallel ruler. If the scale to which the plan is plotted is a large one, the measurements will be plotted more accurately; but any errors made in marking off the bearings from the protractor will be increased.

In making a plan, the surveyor first plots the skeleton outline as shown in Fig. 99. When satisfied with that, he rules in the details as shown in Fig. 100; this gives the width of the gate-roads, strait-work, banks, and, if desired, the position of overcasts, stoppings, and other ventilating arrangements, though these are not usually shown on the working plan, but are put on another plan kept especially for ventilation, the arrangements for which, except in the case of permanent overcasts and some of the stoppings and separation doors, are liable to continual alteration.

Ogle's Protractor.—Where it is possible to fix the paper on to a drawing-board and to use a T-square, the protractor shown in

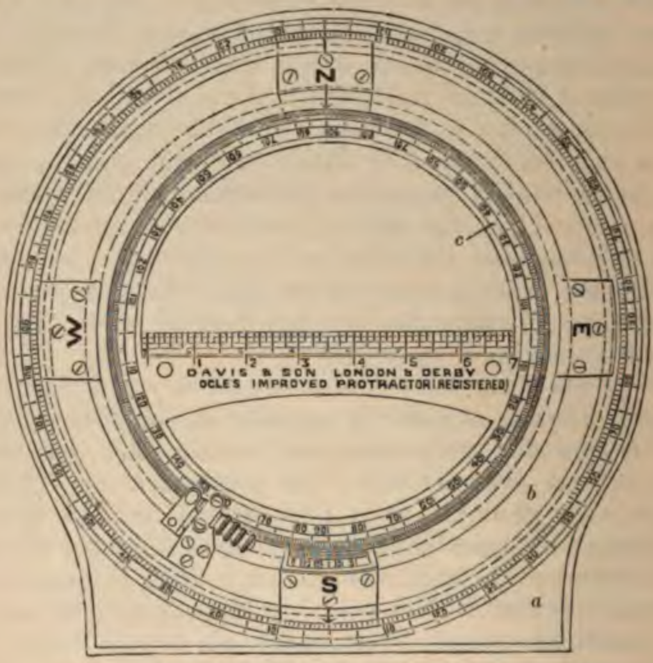

N
W
E
S
c
b
a
1
2
3
4
5
6
7
DAVIS & SON LONDON & DERBY
OGLE'S IMPROVED PROTRACTOR (REGISTERED)

FIG. 100A.—Ogle's form of protractor.

Fig. 100A can be advantageously employed. It consists of an outer frame, *a*, with a true edge to work on the T-square; inside this frame is a graduated ring, *b*, capable of being rotated; and inside this is another ring, *c*, also free to rotate. To use the protractor, the N and S marks on the ring *b* are placed parallel with the meridian line on the plan, and the ring is then clamped; the required bearing can then be set off by moving the inner ring *c* to the required angle.

Trigonometrical Plotting.[1]—The mechanical errors of plotting may be altogether eliminated by adopting a system of trigonometrical computation, by which the latitude and longitude of every station in the mine are found, and recorded in a survey-book. The positions on the plan may be sketched in by hand or put on by scale, according to circumstances, and the distance between any two parts of the plan may be calculated from the information contained in the survey-book, and also the bearing of any proposed new road between any two places on the survey. To facilitate the drawing of the plan, it is made on paper ruled in squares, thus forming lines of latitude and longitude. In France it is a common thing to have the plan made upon a number of separate pieces of paper or cardboard, each piece say about 2 feet square; these can be pieced together, as shown in Fig. 101, as required. In England, however, the practice is almost universal of having the whole of the survey on one large piece of paper. If the size of this becomes unwieldy, the plan is divided into several districts; in this case a smaller scale plan is used, containing the whole of the mine for occasional reference, so that the engineer may see at a glance the relative positions of different parts of the mine, whilst using the large scale plan for details. The trigonometrical system of computation, where used in England, is generally used for checking some main stations when, owing to particular circumstances, greater accuracy than usual is necessary. The system, however, of ascertaining the latitude and longitude of every station has many advantages, especially where the area under one management covers a large extent of country, and in fixing the boundaries between different concerns. Wherever there is a Government survey the lines of latitude and longitude shown on the Ordnance maps should be adopted, the measurement to the shaft being taken from three or four of the nearest station marks.

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[1] A short but excellent treatise on this subject, entitled, "Practice in Underground Surveying, etc.," by the late Mr. W. F. Howard, A.I.C.E., of Chesterfield, is contained in the *Proceedings of the North of England Institute*, vol. xx., and in the *Chesterfield and Derbyshire Institute*, April 13, 1878.

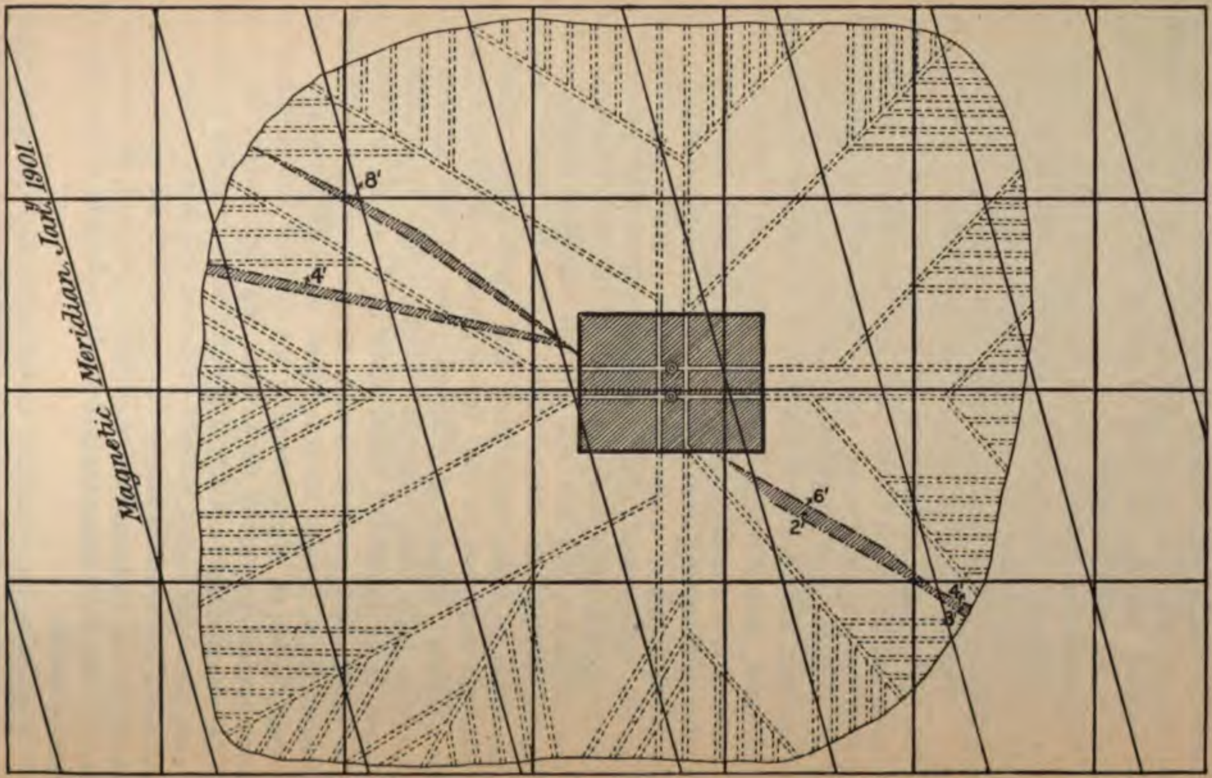

Fig. 101.—Colliery plan, showing lines of latitude and longitude.

This method of plotting is applicable equally to surface and underground surveying. It is usual, in England, to calculate the position of every station in links and to two decimal places. If the calculations are properly checked, there can be no error, and the relative positions of any two places on the surface, or any two underground places, or of one place on the surface and another place underground, can be stated to two decimal places of a link for distance, and with equal accuracy for bearing, always supposing, of course, that the measurements taken in the survey and the angles observed are perfectly accurate. By this system, therefore, the errors of plotting are entirely eliminated.

On reference to Fig. 102 the method of computation will be explained. Five points on the survey are **A**, **B**, **C**, **D**, and **E**, of which **A** is the beginning. The bearing **AB** is N. 50° W., the length 850 links; the bearing **BC** is N. 33° 20′ W., and the length 731; the bearing **CD** is N. 41° 35′ 20″ E., and the distance 762·2; and the bearing **DE**, S. 38° 30′ E., and the distance 280. If we assume that the point **A** is the point of origin, and has 0° longitude and 0° latitude, what are the positions of **B**, **C**, **D**, and **E**?

In ordinary technical parlance in England it is usual to speak of distances measured from longitude to longitude as "departures," and of distances measured from latitude to latitude as "latitudes." In France the geographical terminology is maintained, and the distances measured from longitude to longitude are referred to as "longitudes;" but as in English books the word "departure" is constantly substituted for "longitude," the student must understand that they are convertible terms: the "latitude" means the distance measured N. or S. along the meridian, and the "departure" means the distance measured E. or W. at right angles to the meridian.

To ascertain the latitude and longitude of **B**, the distance **AB** may be regarded as the radius of a circle of which a portion is shown, xyz; the meridian line **AM** is drawn through another radial line, and from **B** a perpendicular is let fall on to the meridian at s. The line **B**s is the departure of the line **AB**, or distance measured between lines of longitude, and is the sine of the angle at **A** to radius **AB**. The line **A**s is classed under the title of latitudes, and is the distance from latitude to latitude of the line **AB**, which is the cosine of the angle at **A**

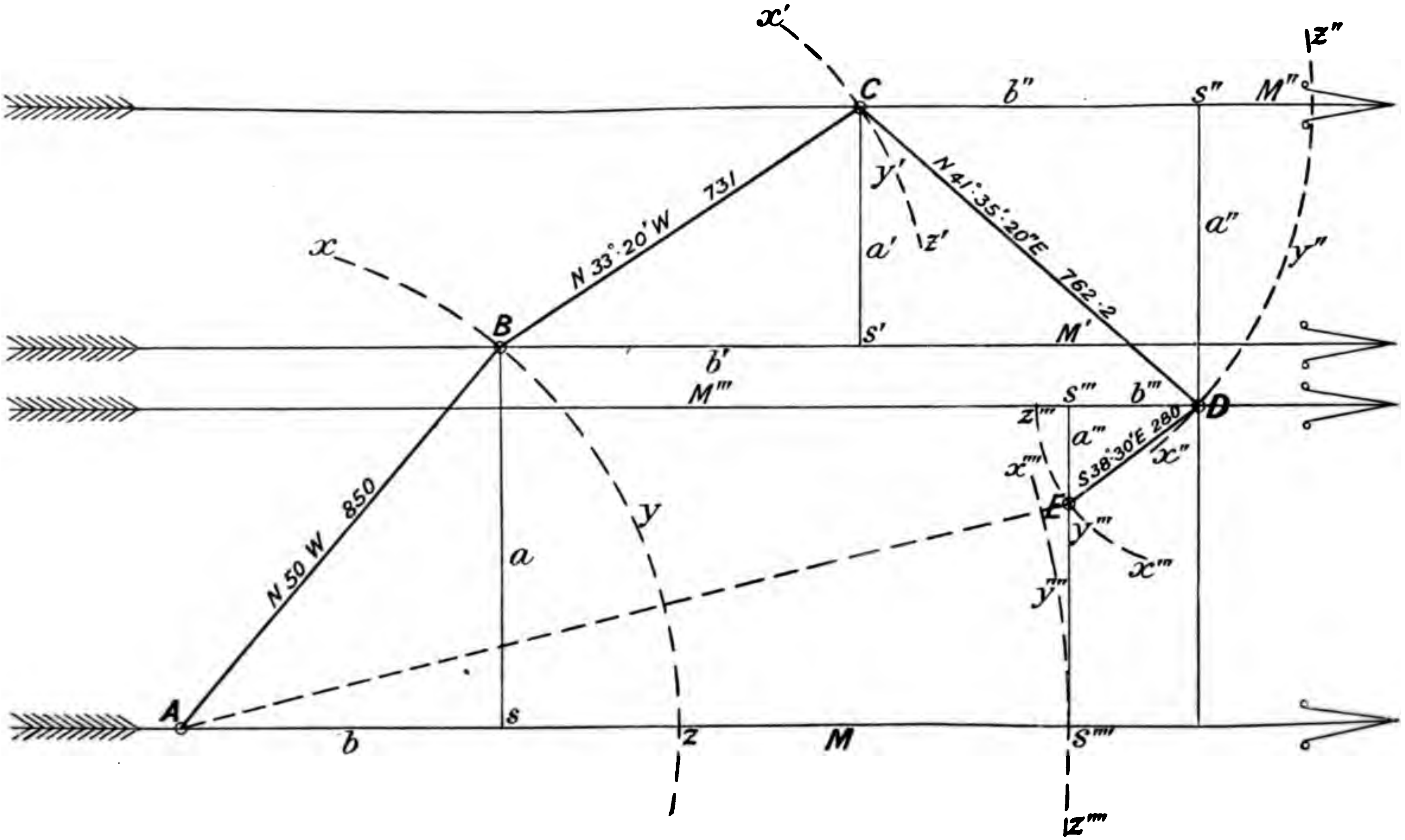

Fig. 102.—Trigonometrical plotting. Scale, three chains to an inch.

to the radius **AB**. The position **B** is obtained as follows by the use of a table of natural sines and cosines: The sine of 50° to radius 1 is 0·76604; this, multiplied by the actual radius, which is 850, gives the actual length of the sine **B***s*, 0·76604 × 850 = (*a*) 651·13, and the cosine of 50° to radius 1 = 0·64278, and to radius 850 = 0·64278 × 850 = (*b*) 546·37; therefore the longitude or departure of **B** is a = 651·13, and the latitude is b = 546·37.

In the same way we proceed to calculate the longitude and latitude of **C**. The line $x'y'z'$ is the arc of a circle with radius **BC**, **C**s' is a perpendicular let fall from **C** to the meridian **BM**′, the line **C**s' is the sine of the angle at **B**, and the line **B**s' is the cosine. The natural sine of 33° 20′ to radius 1 is 0·54951, and to radius 731 is 0·54951 × 731 = (a') 401·69. The natural cosine to radius 1 is 0·83548, and to radius 731 is 0·83548 × 731 = (b') 610·74. Then the distance a' is the departure or longitude of **C**, and the distance b' is the latitude of **C**, taking **B** as the point of origin.

We now rule the meridian **CM**″, and let fall the perpendicular **D**s'', and draw the arc $x''y''z''$. **D**s'' is the sine of the angle at **C**, and **C**s'' is the cosine. The natural sine of the angle 41° 35′ 20″ to radius 1 is found—if using Chambers's Tables, in which the natural sines are only calculated to minutes—in the following manner:—

Natural sine 41° 36′ is 0·6639262	
,, 41° 35′ is 0·6637087	0·6637087
60 | 0·0002175	0·0000724
0·00000362	0·6637811
20	
0·0000724	

Natural sine 41° 35′ 20″ to radius 1 = 0·6637811

It will be seen that the natural sine of 41° 35′ is 0·6637087. To this there has to be an addition for the 20″; the amount of this addition is found by taking the proportional part of the difference between the sine of 41° 35′ and the sine of 41° 36′. The sine of 41° 36′, as shown above, is 0·6639262, and the difference is 0·0002175; this, divided by 60, gives the addition for 1″ = 0·00000362, and this again, multiplied by 20, gives the addition for 20″, which is 0·0000724; this, added to the fraction already found for 41° 35′, gives the exact natural sine of 41° 35′ 20″

= 0·6637811. The greater the number of degrees in the arc of a quadrant, the greater the sine and the less the cosine.

To find the cosine for the above angle, we proceed as follows:—

$$\begin{array}{rrl}
\text{Natural cosine } 41^\circ\ 35' = & 0{\cdot}7479912 & \quad 0{\cdot}7479912 \\
\text{,, }\quad 41^\circ\ 36' = & 0{\cdot}7477981 & \quad \underline{0{\cdot}0000642} \\
60\ | & \underline{0{\cdot}0001931} & \quad 0{\cdot}7479270 \\
 & 0{\cdot}00000321 & \\
 & \underline{20} & \\
 & 0{\cdot}0000642 & \\
\end{array}$$

Natural cosine 41° 35′ 20″ to radius 1 = 0·7479270

In the above sum the natural cosine of 41° 35′ is first found as shown above, then the natural cosine of 41° 36′; this is subtracted from the first figure, and is the difference for 1′. Dividing this by 60, we have 0·00000321, the subtraction for 1″; multiplying this by 20, we have 0·0000642, the subtraction for 20″; subtracting this from the fraction for 41° 35″, we have the natural cosine of 41° 35′ 20″ to radius 1 = 0·7479270.

Multiplying the sine above found by the actual radius 762·2, we have 0·6637811 × 762·2 = a'', 505·93; and multiplying the natural cosine above found for radius 1 by the actual radius, we have 0·7479270 × 762·2 = actual cosine b'', 570·07. The departure of **D** is thus $a'' = 505{\cdot}93$, and the latitude $b'' = 570{\cdot}07$, taking **C** as the point of origin.

Applying the same method again to the line **DE**, we rule the meridian **DM**‴, and draw the arc $x'''y'''z'''$ with radius **DE**. Let fall the perpendicular **E**s'''; then **E**s''' is the sine of the angle 38° 30′, and **D**s''' is the cosine. The natural sine of 38° 30′ to radius 1 is 0·62251, and the natural cosine is 0·78261, and the natural sine to radius **DE** is 0·62251 × 280 = a''', 174·3, and the natural cosine is 0·78261 × 280 = b''', 219·13. Therefore the departure or longitude of point **E** is $a''' = 174{\cdot}3$, and the latitude is $b''' = 219{\cdot}13$, taking the point **D** as the point of origin.

If these points **B**, **C**, **D**, **E** are all referred to the starting-point **A**, their positions can be shown in the tabular form given on p. 161.

In that table the position north of each of the stations **B**, **C**, **D**, and **E** is shown in the total column under the letter N.

TABLE I.

SHOWING BOOKING OF SURVEY GIVEN IN FIG. 103, WORKED OUT BY MEANS OF NATURAL SINES AND COSINES.

Station.	Bearing.	Length.	Sine.	Cosine.	Departure or longitude.		Total departure or longitude.		Latitude.		Total latitude.	
					E.	W.	E.	W.	N.	S.	N.	S.
A	—	—	—	—	—	—	—	—	—	—	—	—
B	N. 50° W.	850	·7660444	·6427876	—	651·13	—	651·13	546·37	—	546·37	—
C	N. 33° 20′ W.	731	·5495090	·8354878	—	401·69	—	1052·82	610·74	—	1157·11	—
D	N. 41° 35′ 20″ E.	762·2	·6637811	·7479270	505·93	—	—	546·89	570·07	—	1727·18	—
E	S. 38° 30′ E.	280	·6225146	·7826082	174·30	—	—	372·59	—	219·13	1508·05	—

The position west of each of the above stations is shown in the total column under the letter W., and the amount that any one station is further north or south from any other station can easily be obtained by comparison with the figures, and the amount that any one station is east or west from any other station can also be obtained in the same way.

In ascertaining the relative positions of any given station and the starting-point, the following method can be pursued: All the latitudes or distances measured on lines parallel to the meridian going north as far as the station whose position has to be found are added together; all the distances going south are also added together; that total which is less is subtracted from the larger total, and the position of the station is thus found either north or south of the starting-point. Similarly, all the distances between the starting-point and the station whose position has to be found which have been measured at right angles to the meridian, called departures, or lengths going in a westerly direction, are totalled; all those going in an easterly direction are totalled, and the less sum subtracted from the larger; the distance of the station east or west of the starting-point is thus found. These positions are shown in the total columns, Table I., the figures in which have been obtained by means of this process of addition and subtraction. It is there seen, for instance, that station **D** is 1727·18 links north of **A**, and 546·89 links west of **A**.

Suppose that it is desired to know the distance in a straight line from **A** to **E**, and the bearing of the line. Fig. 102 shows that **E** is $b + b' + b'' - b'''$ north of **A**, and is $a + a' - a'' - a'''$ west of **A**. Then, for the sake of clearness, make a sketch as shown in Fig. 102, draw the line **AE**, from **E** drop the perpendicular **E**s''''; then **A**s'''' may be considered the radius of a circle $x''''y''''z''''$. **AE** is the secant of the angle **EAM**, and **E**s'''' is the tangent of the same angle—

$$\frac{\text{the actual tangent}}{\text{the actual radius}} = \text{the tangent of radius 1}$$

$$\therefore \frac{\mathbf{E}s''''}{\mathbf{A}s''''} = \text{natural tangent of radius 1}$$

$$\therefore \frac{a + a' - a'' - a'''}{b + b' + b'' - b'''} = \text{natural tangent of the angle } \mathbf{EAM}$$

$$= \frac{372{\cdot}59}{1508{\cdot}05} = 0{\cdot}247067$$

Looking down the table of natural tangents, this is found to correspond with the angle 13° 52′ 40″, and the bearing **AE** is therefore N. 13° 52′ 40″ W. The natural secant of the angle 13° 52′ 40″ to radius 1 is 1·0300681. Multiplying this by the radius $\mathsf{A}s''''$, we have the actual secant $1{\cdot}0300681 \times b + b' + b'' - b''' = 1{\cdot}0300681 \times 1508{\cdot}05$. Therefore the distance **AE** is 1553·39.

When the student has gone over the above figures with his Mathematical Tables, and has also—to satisfy himself that the calculations are correct—drawn out the measurements to scale and the angles with the protractor, and has repeated the operation several times, he will have mastered the elements of trigonometrical plotting.

If paper ruled in square sections is used, no scale is required for plotting the latitudes and departures. Where the survey is made with a Gunter's chain, the paper should be divided into squares the side of which equals 1 chain on the scale to be adopted in plotting the survey; if the scale is 2 chains to 1 inch, the squares must each measure half an inch (or 1 chain) on each side; these squares are again subdivided into 100 smaller squares, measuring 10 links on each side. This subdivision, however, is rather small, and the surveyor may have to be content with squares measuring 20 links on each side, and must measure the subdivisions, as required, with a scale. The divisions of the chain-squares should be in stronger lines than the subdivisions.

The survey shown in Fig. 102 and given in the above table is shown again in Fig. 103, plotted on sectional paper, scale 4 chains to an inch. In actual practice, however, the lines on the sectional paper are lithographed in some light colour, say brown or yellow, which is not likely to be confused with any part of the plan.

Another method of plotting is by means of a drawing-board and T-square, or by a straight-edge and set-square. The meridian is ruled on the paper by means of the straight-edge. The straight-edge is fixed on the paper by weights, and the latitudes are pricked off on the line from the starting-point or origin, and the departures are ruled off by means of the set-square. The set-square is moved along the straight-edge to the required distance or latitude; the departure is then ruled, and the distance pricked off with the scale. For departures on the other side of the meridian, the straight-edge is moved to the other side of the meridian line, and the process repeated.

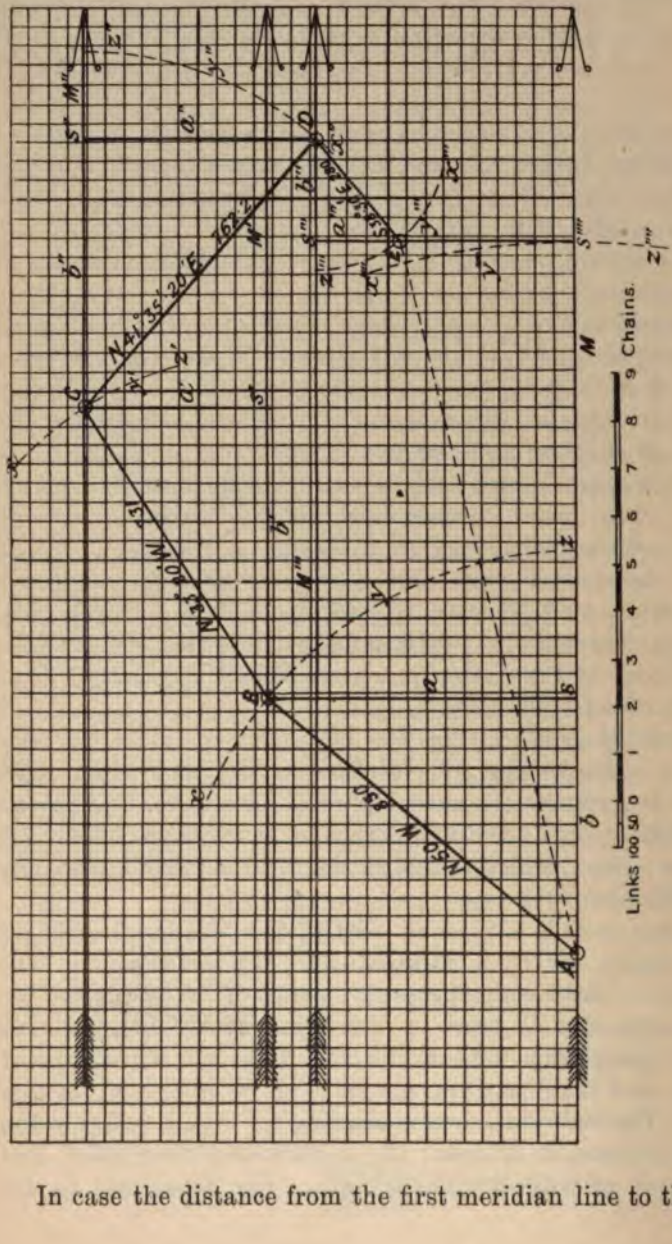

FIG. 103.—Trigonometrical plotting on squared paper. Scale, 4 chains to an inch.
(In the above diagram the small squares have sides $\frac{1}{10}$ inch long, which equals 40 links on a scale of 4 chains to the inch.)

In case the distance from the first meridian line to the place on the survey is longer than the set-square, a new meridian can be ruled parallel to the first by means of a set-square or parallel ruler, the parallelism of the two meridians being tested by measurements with the scale. In this way a more accurate drawing is made than by using the ordinary sectional paper.

A sketch made upon ordinary sectional paper is sufficiently accurate for most purposes, and is perfectly accurate for all lengths measured in the meridian and at right angles to the meridian, because the lengths can be measured off the plan by counting the divisions on the paper, which, by the assumption made in plotting, are the correct length, so that all lengths measured in these directions are perfectly accurate, except such errors as may arise in scaling between the divisions ruled on the paper. If, however, the length of a diagonal is required, some error in this length may arise from unevenness in the ruling.

The way to measure any length not in the meridian or at right angles to it is to take the distance with a pair of dividers, and then mark that distance on the paper in a line parallel to the meridian, and count the divisions. If, however, the divisions as ruled are not exactly square, or if the squares are not all the same size, this measurement of the diagonal will not give the exact result, and the exact length would have to be obtained by calculation, which can easily be done, as the latitude and longitude of each of the two points are known, but will take some time.

If, however, the lengths are accurately set out by scaling in the second method of plotting just described, the diagonals can be accurately scaled; and, indeed, the scaling of the line connecting any two points is often coincident with an actual survey-line, and the agreement between the two measurements is a check upon the accuracy of the plan.

Logarithmic Computation.—Instead of using natural sines and cosines and ordinary numbers for multiplication, surveyors commonly prefer to adopt the aid of logarithms. A short explanation of the nature of logarithms has already been given, and, in addition to this, a sufficient explanation is generally given at the beginning of a book of Mathematical Tables to enable the student to make use of the logarithmic system, even without understanding it.

The method of using logarithms is shown in Table II.

TABLE II.

SHOWING BOOKING OF SURVEY GIVEN IN FIG. 103, WORKED OUT BY MEANS OF LOGARITHMIC SINES AND COSINES.

Station.	Bearing.	Length.	Log. length.	Log. sine of angle of bearing.	Log. cosine of angle of bearing.	Departure or longitude.		Total departure or longitude.		Latitude.		Total latitude.	
						E.	W.	E.	W.	N.	S.	N.	S.
A	—	—	—	—	—	—	—	—	—	—	—	—	—
B	N. 50° W.	850	2·9294189	9·8842540	9·8080675	—	651·13	—	651·13	546·37	—	546·37	—
C	N. 33° 20′ W.	731	2·8639174	9·7399748	9·9219401	—	401·69	—	1052·82	610 74	—	1157·11	—
D	N. 41° 35′ 20″ E.	762·2	2·8820689	9·8220249	9·8738592	505·93	—	—	546·89	570·07	—	1727·18	—
E	S. 38° 30′ E.	280	2·4471580	9·7941496	9·8935444	174·30	—	—	372·59	—	219·13	1508·05	—

In this table the fourth column contains the logarithms of the distance. Thus No. 1 distance (**AB**, Fig. 103) is 850 links. On referring to a table of logarithms of numbers, and under the column headed "number," the figure 850 will be found, and opposite to that will be found the logarithm, which is 9294189. As in this particular survey extreme accuracy is not required, it will be sufficient to take the first five figures, 92941; but taking into account the remaining figures, it will be more exact to call the fifth figure 2, and the logarithm may therefore be written 92942. This logarithm is the same for 850 and 8500, as can be seen by looking for the log. of 8500; and it would be the same for 85,000, 850,000, or any higher sum the result of multiplication by a power of 10; it is also the same for 85, 8·5, 0·85, 0·085, 0·0085, or any smaller sum obtained by the division by any power of 10. In order that the logarithm shown in the table may be distinguished as the logarithm of 850, it is necessary to add a figure which is called the characteristic. For 850, the characteristic is 2, and the log. of 850 is 2·92942; the log. of 8500 would be 3·92942; of 85,000, 4·92942, and so on. The number in the characteristic is one less than the number of figures before the decimal point of the natural number. If the length had been 85, the log. would be 1·92942; if the number were 8·5, the log. would be 0·92942; if it were 0·85, the log. would be $\bar{1}$·92942; if it were 0·085, the log. would be $\bar{2}$·92942; if it were 0·0085, the log. would be $\bar{3}$·92942. In every case where the number consists of integral figures, the characteristic of the logarithm represents one less figure; and where the number is a decimal fraction, the characteristic has the sign − written over it, and is one more than the number of cyphers after the decimal point in the number.

If it is desired to multiply two numbers together, this can be done by adding their logarithms, the number corresponding to the logarithm so found is the number that would have been obtained by multiplication in the ordinary way. Thus to multiply 850 by 769, add the two logarithms 2·9294189 and 2·8859263; the addition gives 5·8153452. The table of logarithms is then referred to, to find the decimal part of the logarithm. Taking for the present no account of the characteristic, 8153453 is found, which is near enough for all ordinary purposes, and take the number corresponding, which is 65365, the last figure being the number at the head of the column.

The characteristic, 5, of the logarithm shows that there are six figures before the decimal point, and the answer is therefore 653650.

If, instead of multiplying 850, we were to divide it by 769, the process would be to subtract one logarithm from the other; thus 2·9294189 – 2·8859263 leaves 0·0434926. To find the number corresponding to this logarithm, we look down the columns for the decimal portion; we find the logarithm 0434802, and the number corresponding to this is 11053. Subtracting the logarithm so found from that which is given, we have a difference of 124; in the table of differences under 394 (the difference for one) we find the number 118 (the nearest number lower than 124) and opposite to this the figure 3, and that gives us the sixth figure; the difference between 118 (the figure in the column of differences) and 124 is 6; multiplying this by 10, and again looking in the column of differences, we find 1 as the figure opposite 39; this is the seventh figure. The number now found is 1105331. On reference to the logarithm, it is seen that the figure of the characteristic is 0, the number corresponding to that logarithm has therefore one integral figure; therefore the decimal point comes after the first figure, and the actual number is 1·105331.

Continuing the description of Table II.; having written down the logarithms of the lengths, the logarithmic sines and cosines of the angles are written down in the next column. Thus the log. sin of 50° is 9·8842540, and the log. cos 9·8080675. It is not always necessary to write out the decimals to seven places; for small surveys five places are generally sufficient. The student will notice the enormous number represented by the characteristic 9; this is because the logarithmic sines are calculated to an assumed radius of 10000000000.

The length of the sine is found by adding the logarithm of the length to the logarithmic sine of the angle; thus 9·88425 +2·92942=12·81367. It is evident that this represents a number vastly in excess of the real length. Whenever logarithmic sines and cosines are used, it is necessary, before the logarithms so found can be reduced to natural numbers, to subtract 10 from the characteristic. Subtracting 10 from the above figure, we have the logarithm of the sine 2·81367; on referring to the table of logarithms, we find the number corresponding to this is 651·13; and therefore the actual length of the sine or departure is 651·13, which is placed under the column of departures under the letter W., as the direction is westward.

The latitude is found by adding the logarithm of the cosine to the logarithm of the length; thus 9·80806 + 2·92942 = 12·73748. Subtracting 10 from this, we have the logarithm 2·73748; the corresponding number is 54637, and the decimal point comes after the third figure; the cosine is therefore 546·37, which is placed under the column of latitudes under the letter N., as the direction is northward.

It must be noted that in Chambers's Tables the logarithm is not given for any variation in the angle of less than 1′; in Babbage and Callet's Tables the logarithms are given for all angles to 10″, the sine and tangent are also given for 1″ up to 5°, and the cosine and cotangent for 1″ between 85° and 90°.

A great deal of time spent in calculating may be saved by the use of traverse tables. These are tables in which the latitude and departure (longitude) have been already calculated out for certain lengths. Suppose, for instance, that the lengths for which the calculations are made are from 1 to 100 inclusive, then, if the actual length is less than 100, the latitude and departure can be read off the table; if the length is more than 100 and less than 200, the latitude and departure for 100 are taken from the table, then the latitude and departure for the remainder also taken and added to the other figures; if the distance is several hundreds, then the latitude and departure as found for 100 must be multiplied by the number of hundreds, and the latitude and departure for the remaining part of the length less than 100 taken from the table. Traverse tables, however, are not much use to the surveyor unless they are calculated to angles of 1′; this has been done by R. L. Gurden.[1]

The following example shows the mode of using these tables. Bearing N. 20° 10′ E., distance 164:—

Bearing.	Distance.	Latitude.	Departure (or longitude).
N. 20° 10′ E.	100	93·87	34·48
	64	60·076	22·06
	164	153·946	56·54

Traverse tables are, however, sometimes used which are not calculated for every length up to 100. For instance, in Mr. H. T. Hoskold's *Treatise on Surveying*, the latitude and departure are calculated for 1, 2, 3, 4, and 5, or for any of these figures multiplied by 10, 100, 1000, 10,000, or 100,000. Using such tables as these, the above latitude and departure is set out as follows :—

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[1] *Traverse Tables for the use of Surveyors and Engineers*, by Richard Lloyd Gurden, 3rd edit. (Chas. Griffin and Co., Ltd., London).

Bearing.	Distance.	Latitude.	Departure (or longitude).
N. 20° 10′ E.	100	93·87	34·475
	50	46·934	17·237
	10	9·387	3·447
	4	3·755	1·378
	164	153·946	56·537

Table III. (p. 171) shows the survey given in Tables I. and II. worked out by means of traverse tables.

Inclination and Reduction of Lengths.—In the preceding examples of underground surveying, booking, and plotting, no notice has been taken of the inclination of the mine. It is, of course, obvious that this is of the utmost importance; where minute accuracy is required, the inclination of every bearing must be observed. Where good and carefully adjusted levels are attached to the instrument, these observations of inclination serve two purposes—first, that of ascertaining the proper reduction of length for the plan, and second, that of ascertaining the levels in all parts of the mine. The accuracy of this levelling process, of course, depends upon the nature of the instrument used and the care exercised by the observer. With a good theodolite sufficient accuracy may be obtained for most practical purposes, but not for all purposes. With a 5-inch theodolite, which only reads to minutes, an error of 3 in 10,000 may be expected, and this error would be too much for many purposes, for instance, such as setting out a water-level; but for the ordinary contour of a mine and setting out roads for the purposes of haulage, the accuracy attainable with the theodolite would be quite sufficient, and for rough approximations careful levelling with a good dial is very useful.

For the purposes of reducing the length, minute accuracy in reading the vertical angle is not generally required. For the angle of 1° the natural cosine is 0·9998477; if the measured length was 10,000, the cosine or reduced length would be 9998·477. It is, however, seldom that a length of 10,000 is dealt with in one measurement in a mining survey, as the minor inequalities are of more importance in considering reductions of length, as already explained in p. 128, so that it is very seldom

TABLE III.

BOOKING OF SURVEY GIVEN IN FIG. 103, WORKED OUT BY MEANS OF TRAVERSE TABLES.

Station.	Bearing.	Length.	Latitude.	Departure.	Latitude.		Departure.	
					N.	S.	E.	W.
A	—	—	—	—	—	—	—	—
B	N. 50° W.	850	514·23 32·139	612·836 38·302				
			546·369	651·138	546·369	—	—	651·138
C	N. 33° 20′ W.	731	584·84 25·90	384·66 17·035				
			610·74	401·695	610·74	—	—	401·69
D	N. 41° 35′ 20″ E.	762·2	523·59 46·37	464·60 41·15				
	In this case, as the tables only read to minutes, the 20″ has been neglected, which accounts for the discrepancy in the figures.		569·96	505·75	569·96	—	505·75	—
E	S. 38° 30′ E.	280	156·52 62·608	124·50 49·80				
			219·128	174·30	—	219·128	174·3	—

that an inclination of less than 1° involves an appreciable reduction in the measured length.

The steeper the inclination, however, the more important it is to observe the inclination with accuracy; for instance, referring to Table IV., if the length of the slope (or radius) is 1000 and the inclination 1°, the reduced length (or cosine) is 999·84, and for an inclination of 1½°, the reduced length (or cosine) is 999·65, or a difference of 0·19, while for 70° the reduced length is 342·02, and for 70½°, 333·80, showing a difference of 8·22. Therefore, whilst at moderate inclinations it may be sufficiently near to read the vertical angle to ½°, at steep inclinations it is necessary to read to minutes in order to obtain the reduced length correctly.

TABLE IV.

Reduction of Length measured on the Slope to Horizontal Distance for Angles from 1° to 70°, and the Difference for ½°.

Length measured on slope.	Angle of inclination.	Reduced length (cosine).	Difference for ½°.
1000	1°	999·84	0·19
1000	1½°	999·65	
1000	10°	984·80	1·55
1000	10½°	983·25	
1000	20°	939·69	3·02
1000	20½°	936·67	
1000	30°	866·02	4·39
1000	30½°	861·63	
1000	40°	766·04	5·64
1000	40½°	760·40	
1000	50°	642·78	6·71
1000	50½°	636·07	
1000	60°	500·00	7·58
1000	60½°	492·42	
1000	70°	342·02	8·22
1000	70½°	333·80	

For the purpose, however, of obtaining the variation in level with precision, the less the inclination the greater the accuracy with which the angle must be read. Whilst the reduced length is represented by the natural cosine, the variation in level is represented by the natural sine. Taking the length of slope as before at 1000, and the angle at 1°, the altitude or depression (sine) for 1° is 17·45, and for 1° 30′, 26·17, showing a difference in level of 8·72 feet for a variation of ½° (see Table V.); at 70° the altitude is 939·6926; at 70½°, 942·1550, showing a variation of 2·4624.

TABLE V.

VERTICAL RISE FOR A CONSTANT LENGTH MEASURED ON THE SLOPE WITH ANGLES VARYING FROM 1° TO 70°, AND THE DIFFERENCE FOR ½°.

Length measured on slope.	Angle of inclination.	Vertical rise (sine).	Difference for ½°.
1000	1°	17·45	8·72
1000	1½°	26·17	
1000	10°	173·64	8·59
1000	10½°	182·23	
1000	20°	342·02	8·18
1000	20½°	350·20	
1000	30°	500·00	7·53
1000	30½°	507·53	
1000	40°	642·78	6·66
1000	40½°	649·44	
1000	50°	766·04	5·58
1000	50½°	771·62	
1000	60°	866·02	4·33
1000	60½°	870·35	
1000	70°	939·69	2·95
1000	70½°	942·64	

In taking his observations, the surveyor, of course, will bear in mind what part of the mine it is which he desires to delineate on the plan, and of which he desires to show the relative levels. As a general rule, the part shown on the plan is the floor or rail-level, and he must take care that all his observations to obtain the inclination or level must be made to marks equidistant from the floor; thus, if the level of the eye-piece of the theodolite or dial is 4 feet from the floor, he must take care that the mark to which he directs the sight is at precisely the same altitude above the floor, otherwise he will be led into error. For this purpose, when surveying with three tripods, it is better that they should each be of the same height, and that the lamp or mark-holder should be of such a height above the tripod as to bring the flame or other mark to the same height above the tripod as the centre of the telescope of the theodolite or the cross-hairs of the dial. When in the course of surveying an assistant is sent forward to fix a mark on a tripod, the exact height the mark will be above the ground cannot be known with certainty, as the legs may be extended to suit irregularities in the surface, so that the level of the lamp may vary a few inches above or below the average height. In most cases this is immaterial, but when the lamp is set over some permanent fixed station, the exact altitude of which has to be determined, then the height from the lamp or other mark to the ground-level should be measured and compared with the height of the mark above the ground level of the other stations in the survey.

Table VI. (see p. 175) shows the survey made to ascertain the inclination of a road in the mine, and the relative level of different stations.

The plotted section is shown in Fig. 104; if it is desired to show the roof of the road on the section, the height must be measured at each station.

In the above table, the measured lengths are under column 1 the angles of inclination under columns 2 and 3. When the angle observed shows that the roadway is rising in the direction in which the observer is proceeding, the angle is entered in column 2, under the heading "elevation;" and when the angle observed shows that the roadway is falling in the direction in which the surveyor proceeds, the angle is entered under column 3, under the heading "depression." Assuming that the surveyor prefers the use of logarithms for his calculations, column 4 (a) shows the logarithms of the lengths; column 5, the logarithmic sine (b); column 6, the log. cosine (c); column 7 is the reduced length obtained, $a + c - 10 = f$; column 8 is the elevation, $a + b - 10$; column 9, the depression, $a + b - 10$; column 10 is the elevation in links; column 11, the depression in links; column 12 is the total elevation above datum, or total depression below; and column 13, the reduced length in links.

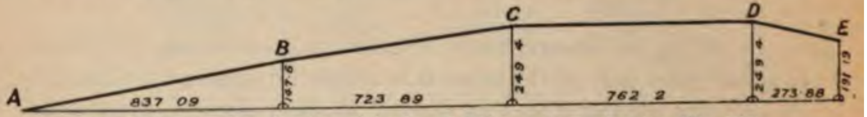


FIG. 104.—Section showing altitudes of stations shown in plan, Fig. 102 (see Table VI.).

It is convenient to measure all heights above some fixed or imaginary level. For surface-work surveyors commonly take Ordnance datum, which is the level of the old Docks Sill at Liverpool; for underground work it is frequently necessary to take another datum, which the surveyor will choose according to circumstances. Suppose, for instance, that the bottom of the shaft is 1000 feet below Ordnance datum, and some of the workings are 300 feet below the bottom of the shaft, his datum might be 2000 feet below sea-level, in which case the plates at the pit-bottom from which he began his survey would be 1000 feet above

TABLE VI.

SURVEY NOTES FROM WHICH THE SECTION GIVEN IN FIG. 104 HAS BEEN PLOTTED, SHOWING LOGARITHMIC CALCULATIONS.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Measured length (links).	Angle of inclination.		Logarithms.						Elevation (links).	Depression (links).	Altitude above datum (starting-point).	Reduced length.
	Elevation.	Depression.	Length.	Sine of angle of inclination.	Cosine of angle of inclination.	Reduced length.	Elevation.	Depression.				
			a	b	c	$a+c-10=f$	$a+b-10$	$a+b-10$				
850	10°	—	2·9294189	9·2396702	9·9933515	2·9227704	2·1690891	—	147·6	—	147·6	837·09
731	8°	—	2·8639174	9·1435553	9·9957528	2·8596702	2·0074727	—	101·8	—	249·4	723·89
762·2	Level		2·8820689	—	—	—	—	—	—	—	249·4	762·20
280	—	12°	2·4471580	9·3178789	9·9904044	2·4375624	—	1·7650369	—	58·21	191·19	273·88

Date of survey.	Colliery.	Seam of coal.	District.	Station where survey begins.

Number of set.	Bearing angle of quadrant.	Degrees. 0° to 360°.	Length hypothenuse.	Angle of inclination.		Reduction length per chain.	Offsets.				Remarks.
				Dip.	Rise.		Right.	Left.	Above.	Below.	

FIG. 105.—Trigonometrical plotting: method of keeping the survey-book.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)
								Logarithms.												Links.										
									Inclination.		Bearing.									Dep. or Long.		Latitude.		Dep. from origin.		Lat. from origin.				
Date.	Starting-point.	Number of sight.	Distance (links).	Angle of inclination. Elevation.	Angle of inclination. Depression.	Horizontal angle. Bearing.	Horizontal angle. Degrees from 0°.	Lgth. (a)	Sine. (b)	Cos. (c)	Sin. (d)	Cos (e)	Reduced length. $a + c - 10 = f$	Elevation. $a + b - 10$	Depression. $a + b - 10$	Dep. or Longitude. $f + d - 10$	Latitude. $f + e - 10$	Elevation.	Depression.	E.	W.	N.	S.	E.	W.	N.	S.	Altitude above datum.	Number of sight.	Remarks.

Fig. 106.—Trigonometrical plotting: office survey-book.

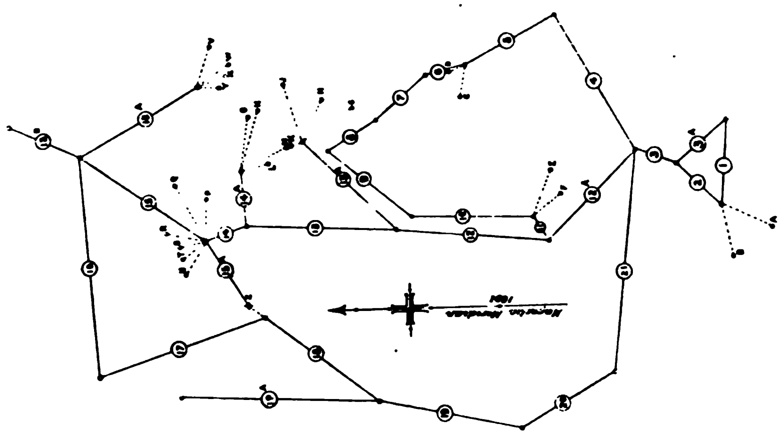


FIG. 108.—Plan plotted from Fig. 107.

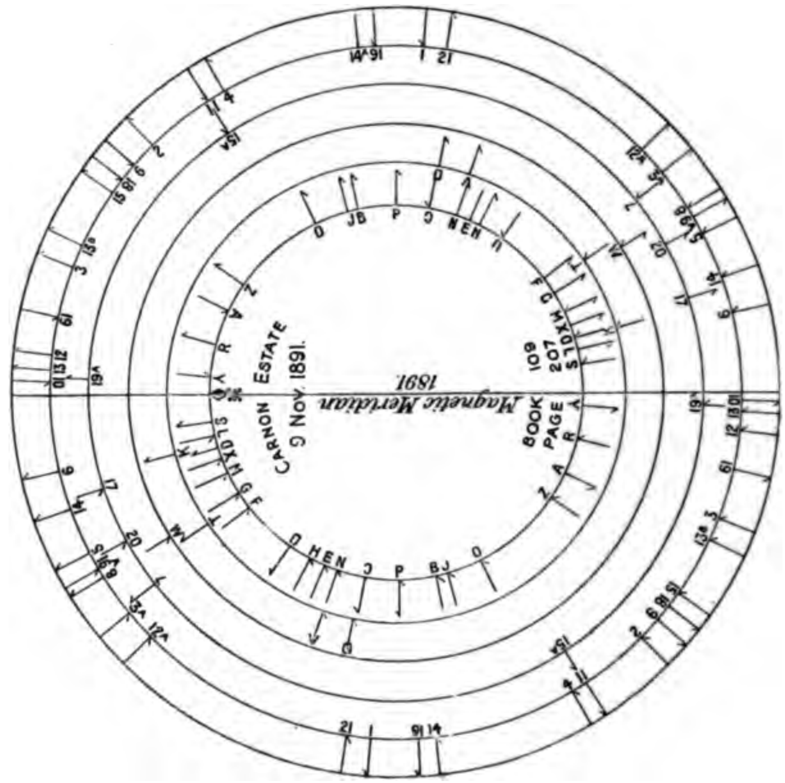


FIG. 107.—Disc of Henderson's rapid traverser, showing survey.

datum, and all the other stations would be more or less than 1000 feet. In plotting the survey, all the altitudes will be measured upwards from a datum line drawn at the bottom of the paper. But it might be equally convenient to use the Ordnance datum; then, the bottom of the shaft being 1000 feet, all the other stations will be more or less than 1000 feet below Ordnance datum.

In preparing the section shown in Fig. 104, the reduced lengths are marked off on a horizontal line, at each station a vertical line is ruled, and the altitudes above or below datum marked off.

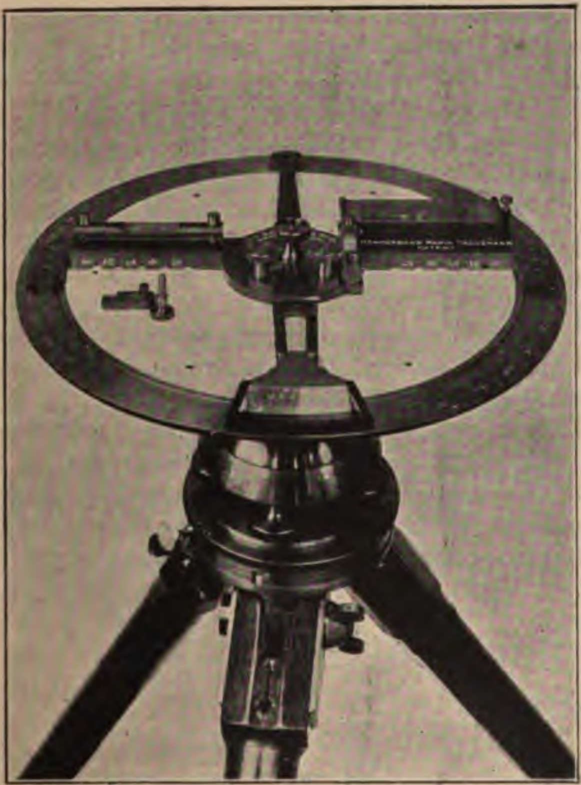


FIG. 109.—Disc of Henderson's rapid traverser, with protractor and vernier for reading off bearings.

Horizontal and Vertical Angles measured at the same Time.—Of course, when the surveyor has fixed his instrument, he will take the horizontal and vertical angles at the same time, and thus make sure that the stations observed for the plan and section coincide, and he will therefore make a note in his book of the vertical angle of elevation or depression. He may keep his book either in the graphic, written, or tabular form. In case he adopts the latter mode, a convenient form of book is shown in Fig. 105. From this note-book the office survey-book (shown in Fig. 106) is filled up: the sheet to carry these thirty-one columns to be 2 feet to 2 feet 6 inches, otherwise the written figures will be too small.

Method of plotting Survey made with Henderson's Rapid Traverser.—Fig. 107 shows the disc of the traverser, which has been removed from the table of the instrument. A meridian line having been drawn on the plan, the disc is placed on the paper over this line, and held in position by weights. With the aid of a rolling parallel ruler, the sight numbered 1 is now ruled off, reference being made to the note-book to find the length. No. 2 sight is then ruled from the end of No. 1, the arrow-heads giving the direction in which the line has to be drawn. When the lines have all been plotted, and proof obtained that the survey has been accurately made, the offsets and other measurements can be filled in.

One objection raised to the Henderson rapid traverser is that it is necessary to keep the disc, because it is the only record of the survey. In Fig. 109, however, is shown an arrangement by which the bearings of the lines can be read off by means of a protractor, and filled into the note-book.

CHAPTER X.

METALLIFEROUS MINE SURVEYING.

The surveying of metalliferous mines is conducted with similar instruments and on the same principles as the surveying of coal-mines. In metalliferous mines, however, the workings more commonly lie in seams of steep inclination, so that the cross-cuts which in a coal-mine are nearly level, in a tin-mine are nearly vertical. The shafts are generally inclined, and the inclination is not regular, following the vein. This causes a special difficulty in the preparation of an accurate plan.

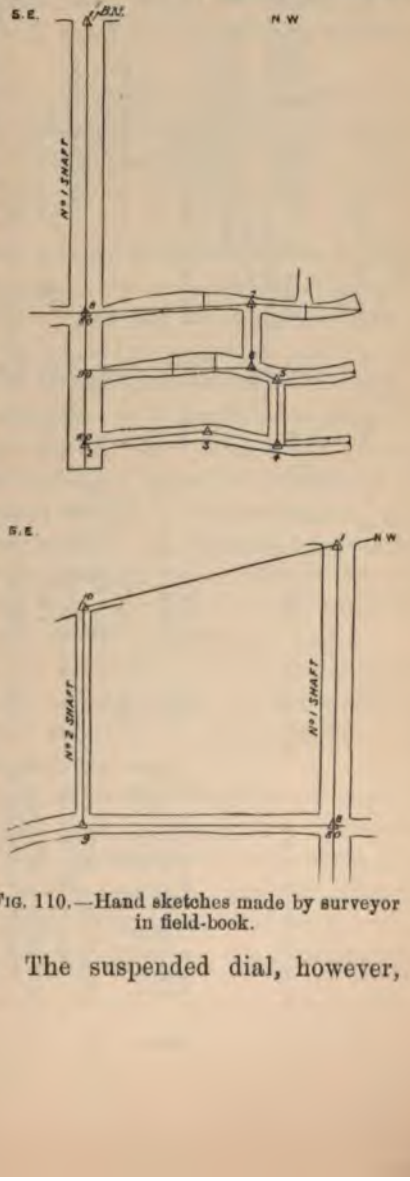


Fig. 110.—Hand sketches made by surveyor in field-book.

Instruments.—For ascertaining the bearing of these inclined shafts, it is necessary to have an instrument capable of reading any desired angle in a vertical plane, and for this purpose a transit theodolite is commonly used. Where the angle of inclination, however, does not exceed 60°, a Hedley dial may serve the purpose, and under circumstances explained in p. 65 the suspended dial may be used. The suspended dial, however,

TABLE VII.—FIELD-BOOK.

Station.		Horizontal angles.		Vertical angles.		Measured distances.				Bearing.	Reduced measurement.	
No.	Location.	B.S.	F.S.	Elev.	Depr.	Lines.	Offsets.		Notes.		Vertical.	Horizontal.
							Hanging wall.	Foot wall.				
						Feet.						
1	Top of No. 1 shaft	0°=due N.	45°	—	60°	480	Shaft	8'×20'	Peg opp. 80-fath. lev. = station 8	N. 45° E.	415·7	240
						540	Survey	line =	Peg opp. 90-fath. lev.	—	467·6	270
						600	Centre	line				
							of	shaft	Peg opp. 100-fath. lev. = station 2	—	519·6	300
						625	—	—	Bottom of sump.	—	541·3	312·5
2	No. 1 shaft 100-fath. lev.	45°	313°	—	—	120	Drives	6' wide	Station 3	N. 47° W.	—	120
3	100-fath. lev. N.W. at 120	313°	322°	—	—	80	—	—	Station 4	N. 38° W.	—	80
4	100-fath. lev. N.W. bot. of rise No. 1	322°	314°	—	—	40	—	—	Face of drive	N. 46° W.	—	40
			225° 35'	58° 21'	—	61	—	—	Station 5 at top of rise	345° W.	52	32
5	90-fath. lev. N.W. top of winze to 100-fath. lev.	225° 35'	315° 30'	—	—	62	—	—	Face of drive	N. 44° 30' W.	—	62
			142°	—	—	51	3½	2½	Station 6	S. 38° E.	—	51 off
6	90-fath. lev. N.W. bot. of rise No. 1 to 80-fath. lev.	142°	133° 28'	—	—	20	½	5½	—	S. 41° 32' E.	—	20 off
						50	3	3	—	—	—	50 off
						140	5	1	Edge of shaft No. 1	—	—	140 off
						150	—	—	Peg in centre of shaft = 540 line 1	—	—	150
			225° 30'	59°	—	60½	4	2	Rise to 80-fath. lev. station 7	S. 45° 30' W.	52	31
7	80-fath. lev. N.W. top of rise from 90-fath. lev.	225° 30'	320° 41'	—	—	50	5½	½	Bottom rise to 70-fath. lev.	N. 39° 19' W.	—	50 off
						135	0	6	Face of drive	—	—	135 off
			134° 30'	—	—	20	1	5	—	S. 45° 30' E.	—	20 off
						140	4	2	—	—	—	140 off
						150½	—	—	Station 8 = 480 line 1	—	—	150½
8	No. 1 shaft 80-fath. lev.	134° 34'	45°	—	—	—	—	—	Check sight to station 2	N. 45° E.	—	—
			225°	—	—	—	—	—	Check sight to station 1	S. 45° W.	—	—
			135°	—	—	300	—	—	Along 80-fath. lev. S.E. to station 9	S. 45° E.	—	300
9	80-fath. lev. S.E. bot. of shaft No. 2	135°	133° 42'	—	—	—	—	—	S.E. continuation of 80-fath. lev.	S. 46° 18' E.	—	—
			225°	60°	—	422	—	—	Station 10 at top of shaft No. 2	S. 45° W.	365·5	211
10	Top of shaft No. 2	225°	309° 15'	9° 34'	—	305	—	—	Station 1	N. 50° 45' W.	50	301
No. of col.: 1	2	3	4	5	6	7	8	9	10	11	12	13
				Observed.							Calculated.	

is only useful where there is no attraction, and therefore is very often inapplicable.

The measurements are usually taken with a 100-ft. chain. Formerly a chain 10 fathoms in length was used; every fathom being marked, and the links 6 inches long.

Table VII. shows the note-book of a survey in a metalliferous mine, commencing at the surface and proceeding down an inclined shaft to the 100-fathom level, along the 100-fathom level, and up a rise to the 90-fathom level, and on this level back to the shaft; then up a rise to the 80-fathom level, and on this level back to the shaft, and across the shaft to the bottom of No. 2 shaft, up No. 2 shaft to the surface, and back to the starting-point.

Method of Surveying.—The survey is usually made by the "fast-needle method," and before commencing, the true bearing of a line from the peg or B.M. at the top of the shaft to some distant object is obtained with great accuracy, and forms a base for all future surveys.

In starting the survey, the vernier of the theodolite is set to the bearing of this line and clamped; the telescope is then directed so as to sight the distant object, and the whole instrument is then clamped on this line. The zero line is now N. and S., and the survey is proceeded with in the manner already described.

Referring to Table VII., columns 1 to 10 are filled up from the observations made in the mine during the course of the survey; the columns headed "Horizontal angles" give the direction of the lines of survey, and the columns headed "Vertical angles" give their inclination from the horizontal. At necessary points the surveyor will take offsets to the hanging wall (which is the wall above the vein) and to the foot wall (which is the wall below the vein); these measurements are shown in columns 8 and 9.

In addition to filling up the columns 1 to 10, the surveyor will make sketches showing the numbers and relative positions of the different stations; these sketches are shown in Fig. 110, and are used to facilitate the plotting.

The reader will by this time understand that the principle of plan-making is to represent all measurements in a horizontal plane; thus if a shaft is inclined, its length as shown on the plan will be less than the length actually measured, and will be equal to the cosine of the angle of inclination multiplied by the measured length. Similarly, in making a section, the measurement on a vertical plane will be less than that measured on the slope, and will be equal to the sine of the angle of inclination

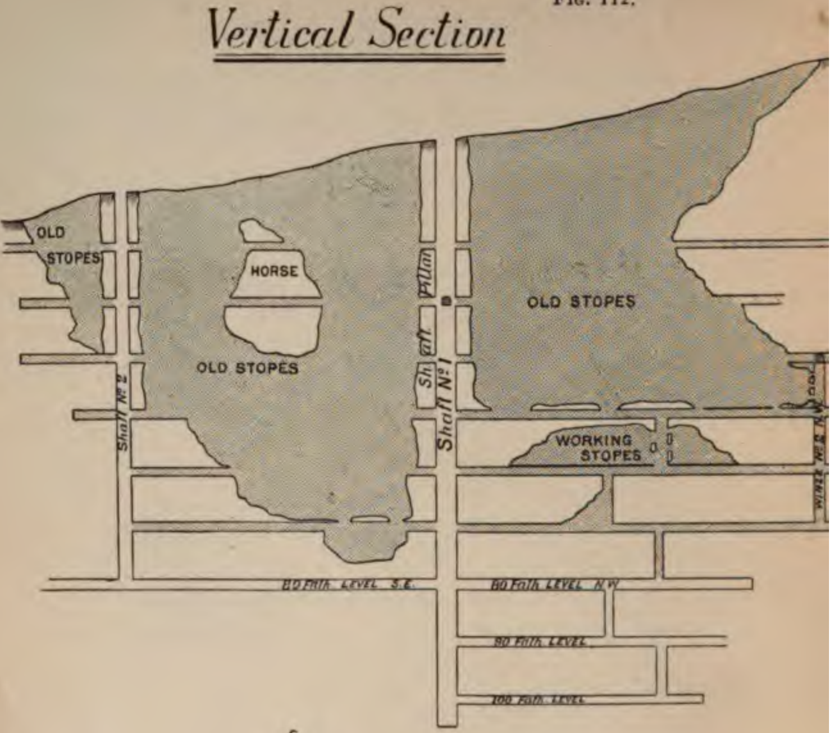
TABLE VIII.

METHOD OF OBTAINING REDUCED LENGTHS FOR PLAN BY MEANS OF LOGARITHMS.

(a) No. of sight.	(b) Measured length. Feet.	(c) Vertical angles.		(d) Log. cosine of angle of inclination.	(e) Log. of measured length.	(f) Log. of reduced length. (d) + (e) − 10	(g) Reduced length for plan.
		Elevation.	Depression.				
From station 1 to station 2	600	—	60°	9·6989700	2·7781513	2·4771213	300
„ 2 „ 3	120	on 100-fathom level		—	—	—	120
„ 3 „ 4	80	on ditto		—	—	—	80
„ 4 „ 5	61	58° 21′	—	9·7199350	1·7853298	1·5052648	32
„ 5 „ 6	51	on 90-fathom level		—	—	—	51
„ 6 „ 7	60·5	59°	—	9·7118393	1·7817554	1·4935947	31
„ 7 „ 8	150·5	on 80 fathom level		—	—	—	150·5
„ 8 „ 9	300	on ditto		—	—	—	300
„ 9 „ 10	422	60°	—	9·6989700	2·6253125	2·3242825	211
„ 10 „ 1	305	9° 34′	—	9·9939178	2·4842998	2·4782176	301

multiplied by the measured length. Columns 12 and 13 of Table VII. give these reduced measurements as obtained from

Fig. 112.



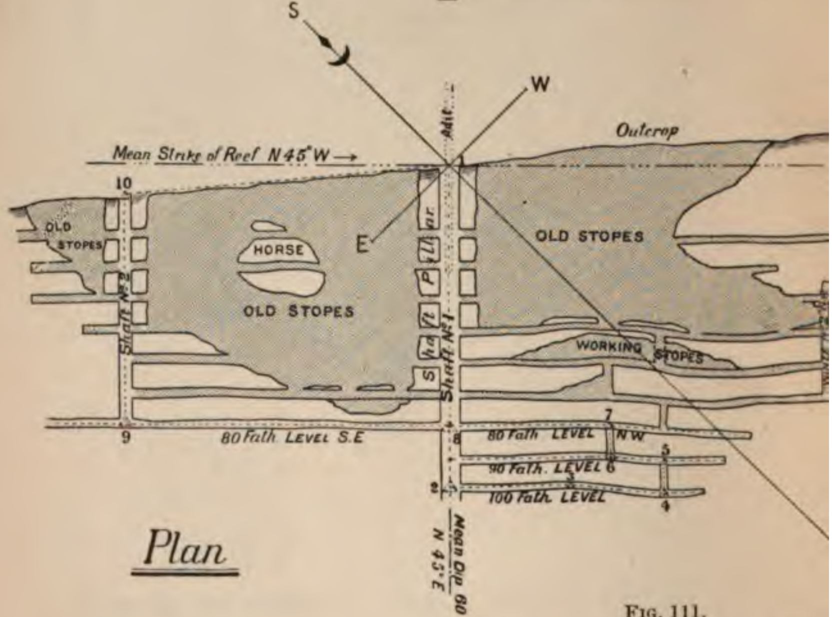
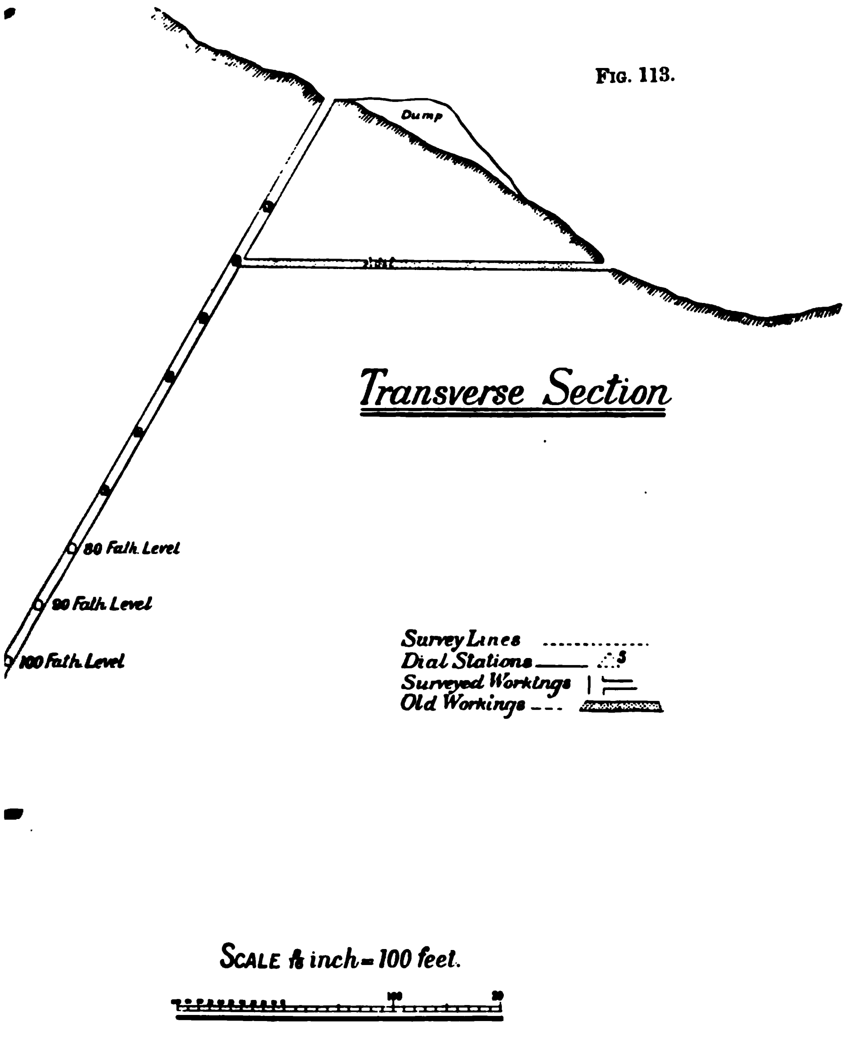


Fig. 111.

Fig. 111.—Plan of workings in metallifer
Fig. 112.—Vertical section of ditto.
Fig. 113.—Transverse section of ditto.

FIG. 113.



ine plotted from the bookings in Table VII.

Traverse Tables; the method of calculating them by logarithms is shown in Tables VIII. and IX.

TABLE IX.

Method of obtaining Reduced Lengths for Vertical Section by means of Logarithms.

(*a*) No. of sight.	(*b*) Measured length. Feet.	(*c*) Vertical angles		(*d*) Log. sine of angle of inclination.	(*e*) Log. of measured length.	(*f*) Log. of reduced length. (*d*) + (*e*) − 10	(*g*) Reduced length for vertical section.
		Elevation.	Depression.				
From station 1 to station 2	600	—	60°	9·9375306	2·7781513	2·7156819	519·6
„ 2 „ 3	120	on 100-fathom level		—	—	—	120
„ 3 „ 4	80	on ditto		—	—	—	80
„ 4 „ 5	61	58° 21′	—	9·9300670	1·7853298	1·7153968	52
„ 5 „ 6	51	on 90-fathom level		—	—	—	51
„ 6 „ 7	60·5	59°	—	9·9330656	1·7817554	1·7148210	52
„ 7 „ 8	150·5	on 80-fathom level		—	—	—	150·5
„ 8 „ 9	300	on ditto		—	—	—	300
„ 9 „ 10	422	60°	—	9·9375306	2·6253125	2·5628431	365·5
„ 10 „ 1	305	9° 34′	—	9·2206182	2·4842998	1·7049180	51

Fig. 111 is the underground plan of the above survey, showing the top of the shafts and the direction of the levels.

Fig. 112 is a longitudinal section of the same mine plotted as if all the workings were in one vertical plane, all the inclined lengths being reduced to vertical distances, in the same way as in a plan the length measured on the slope is reduced to a horizontal distance.

Fig. 113 is a transverse section of the mine across the No. 1 shaft, showing the surface and entrance to the levels.

These three figures include more than is contained in the survey, as there are old workings which are taken from another plan.

Fig. 114 is a longitudinal section in which the lengths measured on the slope of the vein, and also the lengths measured along the levels, are drawn without reduction; so that if the plan were, so to speak, made to natural scale, it could be laid down upon the vein following the sinuosities of the levels. This section is useful as a kind of working plan on which the lengths of level or shaft driven by the workmen can be shown exactly as paid for, although its use might lead to confusion with regard to the relative positions of the stations shown on this working section, and the stations in some other vein or adjoining mine. Practically all four drawings are necessary.

Method of Plotting.—The plan is first plotted at the lower edge of the paper, from the horizontal angles in column 4 or the bearings in column 11, using the horizontal measurements given in column 13 (calculated in the shafts and winzes, and measured in the levels), the measured offsets given in columns 8 and 9 also being plotted. The directions of dip and mean strike of the vein are at right angles to one another, and the plan is usually plotted with the mean strike line approximately parallel to the bottom edge of the paper. To obtain the vertical projection (Fig. 112), station 1 is projected from the plan at right angles to the mean strike; No. 1 shaft is then plotted according to the measurements given in column 12. At the points determined for the respective levels, horizontal lines are drawn, and the ends of the levels and positions of the winzes are projected from the plan. Shaft No. 2 is plotted upwards from station 9, according to figures calculated and observed.

The transverse section is plotted from the observed figures only. Station 1 is plotted at the same height as station 1 in the vertical section; the angles of inclination of the vein are then marked off with the protractor, and the length as measured on the slope is marked off, and if the work is correctly done the corresponding levels in both vertical and transverse sections should be on the same horizontal line.

In the longitudinal section (Fig. 114) the observed figures are also used. The levels are ruled horizontally, and their lengths laid off as the sum of the measurements made along them. It will be seen in the plan that the levels curve about a little, and, owing to this fact, the winze No. 2 N.W. appears distorted in the longitudinal section.

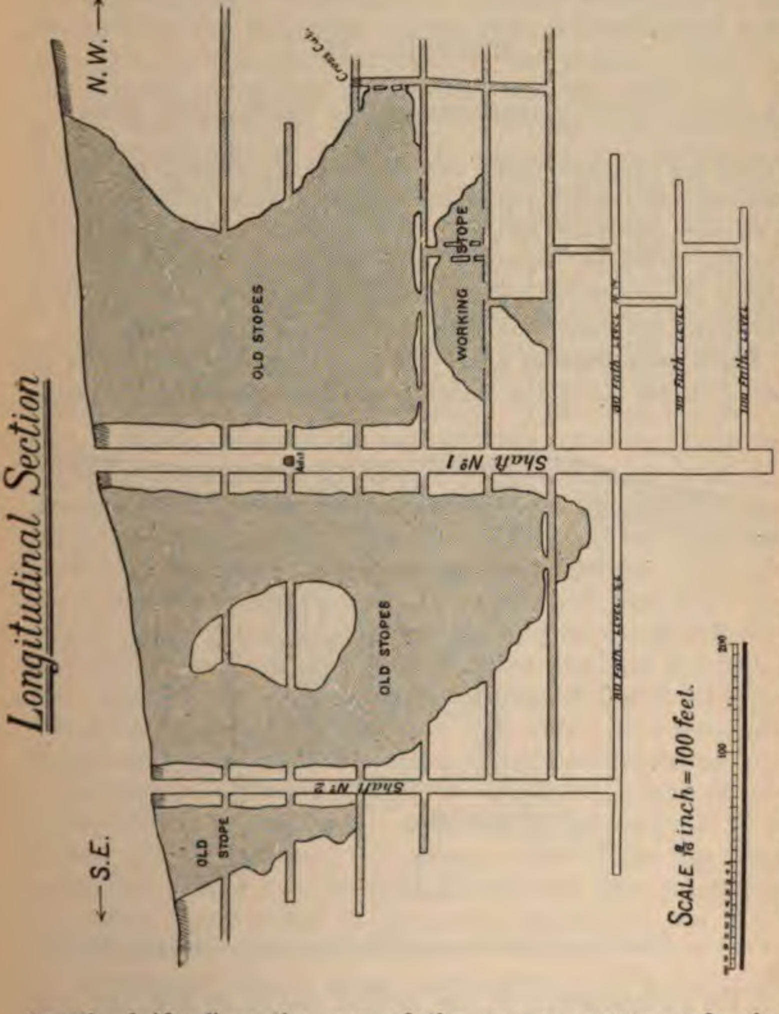


FIG. 114.—Longitudinal section of metalliferous mine along the vein, plotted from the notes given in Table VII. (7).

CHAPTER XI.

METHODS OF CONNECTING SURFACE AND UNDERGROUND SURVEY.

Where the compass needle can be used, the general method of connecting the underground and surface surveys is by its means as already described, and for most purposes this is sufficiently accurate. Where, however, there is attraction, or in case there is no attraction, but the magnetic needle is not considered sufficiently reliable, some other method has to be devised.

Shafts some Distance apart.—Where there are two shafts at some distance apart, the underground survey may be connected with the surface survey in the method shown in Fig. 115. In this case the centres of the shafts have been accurately fixed on the surface survey, and by means of a plumb-line have been accurately transferred to the bottom of the shaft and carefully marked. A fast-needle survey is then made of the

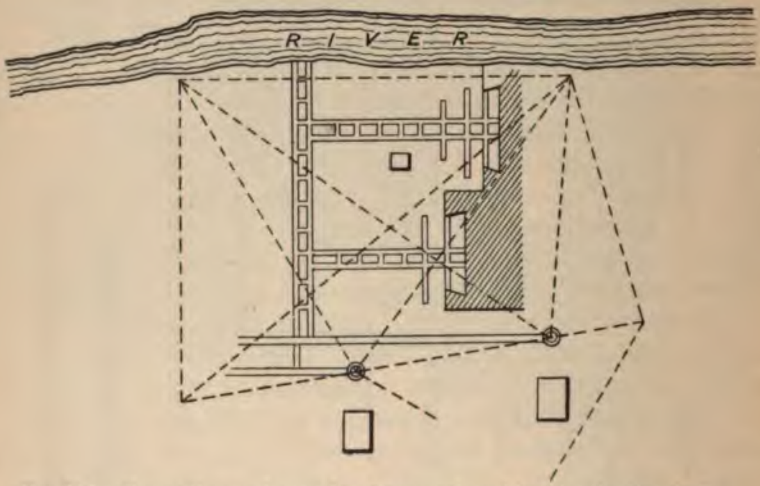


Fig. 115.—Connecting surface and underground workings by the two shafts.

mine, beginning at the centre of one shaft and ending at the centre of the second shaft. This is plotted, then carefully traced on stiff but transparent paper or cloth. This tracing is then fixed on the surface plan by means of the two shafts, and the workings, as shown on the tracing, transferred by a style and marking-paper to the plan. Another method is to plot the underground fast-needle survey from a hypothetical meridian line, such as the first sight in the survey. When the underground survey has been plotted and the position of the second shaft fixed, the bearing of a straight line connecting the two shafts may be calculated by trigonometric computation, as shown on p. 158, Fig. 102, line connecting the stations A and E; but the actual bearing of a straight line connecting the two shafts has been ascertained by observation of the magnetic needle on the surface; this actual bearing differs, say 100°, from the bearing as calculated from the hypothetical meridian. All the other bearings obtained by the fast-needle survey can now be corrected to the same extent, and the survey plotted as if it had been made by the loose needle. In case, however, the magnetic meridian has not been ascertained on the surface, either by reason of attraction or want of a compass, or because it is considered to be too variable, then the geographical meridian may be set out, and the bearing of a straight line between the shafts ascertained by means of a theodolite or circumferentor. The bearing of this same line, as calculated on the underground survey from the hypothetical meridian, is then compared with the true line, and is found to vary, say 100°. The hypothetical bearings of the underground survey can now be corrected to a like extent, and the underground survey plotted in its correct position on the surface plan.

In case these shafts are, as above suggested, a long distance apart—that is to say, long in reference to the total size of the survey, as, for instance, the distance between the two shafts being half a mile and the furthest workings from either shaft two miles—this method of connecting the surface and underground presents no special difficulty.

Underground Survey Shafts near together.—If, however, the shafts are close together, say from 10 yards to 60 yards apart, as is commonly the case in coal-mines, then it is evident that a base-line drawn through these shafts is a very short one upon which to erect the whole of a large survey, and special care has to be taken to avoid mistakes. For instance, the bearing of a line drawn through the centre of the shafts cannot be ascertained by means of a pencil, protractor, and parallel ruler from the plan; but it is necessary to carefully set up marks over the centre of the shafts, and to produce this line across the estate, or a considerable part of it, and to connect the line so produced with a number of the principal stations of the surface survey, carefully ascertaining the bearings of various lines. It will, of course, be unnecessary to repeat this operation if it has been already done in the construction of the plan and the bearing of a line through the centre of the shafts recorded, the line being set out with the most absolute precision, and the angle made by this line and the other main surface-lines noted and recorded. In setting out this line an ordinary wooden staff is much too thick to sight to, and it is better that a wire centre-line should be suspended from a frame above each shaft. The theodolite, set in a line with these wires, will connect their direction with the surface survey; at the same time, these wires hang down to the bottom of the shaft, and by means of a theodolite the underground survey is made connecting the two wires.

The survey connecting these two centre-lines should take the best and shortest road, so as to reduce the possibility of error. Where there is no road straight from one shaft to the other, it generally happens that there is a cross-road connecting the main intake and return at no great distance from the shafts, and in this case the survey may only require the setting up of the instrument two or three times. Not only must the angles be read with the most minute accuracy, so as to ensure that no error exceeding a quarter of a minute shall creep in, but the lengths must be measured with a carefully checked steel tape, and the measurements recorded to the decimal of an inch. The relative positions of the two shafts may then be accurately calculated from the hypothetical meridian, both as regards hypothetical bearing and distance. If this calculated length agrees with the actual distance as measured on the surface, the accuracy, to some extent, of the underground survey is confirmed. The hypothetical bearing between the shafts is now compared with the actual magnetic bearing as observed on the surface, or with the actual geographical bearing; the hypothetical meridian can thus be corrected, and the workings plotted upon the plan of the surface in their true position. It must, however, be impressed on the student that this calculation merely eliminates errors of plotting, and does not eliminate any error arising from want of accuracy either in marking out the line drawn through the centres of the shafts on the surface, or in connecting the plumb-lines by the underground survey.

Where Two Shafts are connected by a Straight Level on the same Level as the Workings.—Where the two shafts are connected by a straight and level passage, it is comparatively easy to ascertain the bearing of an underground road. Referring to Fig. 116, we will assume, as in the previous cases, that

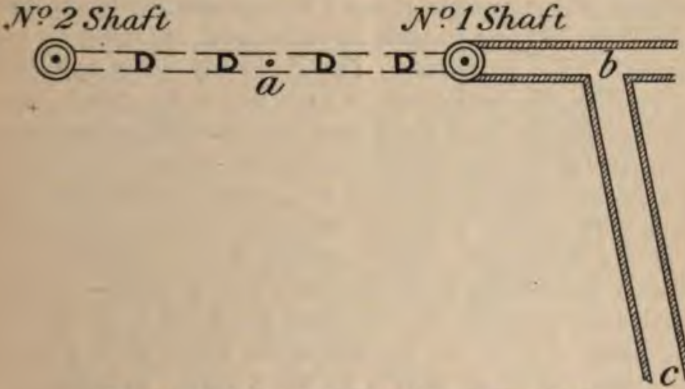


FIG. 116.—Taking an observation between two shafts.

a wire with a heavy plumb-bob is suspended in each shaft, and that the direction of the line connecting these wires is accurately shown on the surface-plan and the bearing ascertained. The theodolite is now placed at *a*, Fig. 116, midway between the two shafts, and in a straight line between the two wires. Since all four doors cannot be opened at once, the position *a* has either to be guessed at or ascertained by preliminary survey. When the theodolite has been set up, it is turned upon the wire in No. 1 shaft, the intermediate doors being open; these doors are then shut, the telescope reversed and turned towards No. 2 shaft, the doors being open; the distance of the line of sight from the suspended wire is then measured. If this is 1 foot, and the theodolite is exactly half-way between the two wires, it follows that the position of the theodolite is exactly 6 inches out of the centre-line. The theodolite is then moved approximately 6 inches, the telescope is then set upon the wire in No. 1 shaft, then reversed. If in looking towards No. 2 wire the line of sight is found to be not straight for No. 2 wire—say $\frac{1}{8}$ inch out—the theodolite must be moved $\frac{1}{16}$ inch, and then carefully fixed upon No. 1 wire. The telescope is again reversed, and this time the line of sight should exactly hit No. 2 wire. If, however, it does not hit No. 2 wire, the operation must be repeated until the position *a* is fixed exactly in a straight line between No. 1 and No. 2 shafts. In order that the theodolite may be fixed with the required accuracy, it is necessary that it should be on a stage where it can be moved by means of screws. A movable stage permitting of a movement of say 1 inch is sold by optical instrument makers, but one suitable for occasional use could be easily constructed by a carpenter. The theodolite being now in the right position, the tripod can be fixed at *b* in the porch beyond the plumb-bob in No. 1 shaft, and very carefully adjusted so as to be exactly in the line connecting the two wires; the theodolite is now moved from *a* and set up at *b*, a mark having been left at *a* made with precise accuracy under the centre of the vertical axis of the theodolite. The telescope is now directed to this mark at *a*, and should be in the same line as before. In order to make sure, however, that no error has crept in in the fixing of the mark *a*, or in the erection of the tripod at *b*, a through sight should be taken past *a* to No. 2 wire. In order to get this sight, it is necessary that there should be an opening through all four doors at the same time. This, however, is impracticable as a general rule, on account of the ventilation, especially if the colliery is ventilated with a furnace, or if the upcast is heated with steam, because, even if the furnace were put out, the heat of the shaft would remain some days. If the colliery, however, is ventilated with a fan, there would not be so much wind when the fan was standing. Nevertheless, the current due to natural ventilation would probably be such as to shake the wires, so that it is in the highest degree inadvisable to open all four doors at the same time. The best plan would be, having clamped the telescope in the right direction, to proceed to cut holes in each of the four doors in succession sufficiently large for the line of sight, the diameter of these holes being say 2 inches to 3 inches, the correct position of the holes being fixed by the surveyor looking through the telescope so that all four holes are in precisely the same line. In order that there may be no difficulty in cutting these holes, the theodolite stand at *b* has been fixed at such an altitude that the line of sight will not cross any iron bands or stiffening bars on the doors, and a drill suitable for boring a hole of the required diameter made beforehand. When all the four doors are closed, the current of air through the holes will not be excessive, and they can be screwed up as soon as the operation of the surveyors is completed.

In case it should be difficult to fix a tripod at *b*, it may be necessary to erect a timber platform to receive the instrument, with a traversing-table capable of movement by screws to the extent of an inch or two, on which in the first instance the mark, and in the second instance the theodolite, can be fixed. Having thus fixed the theodolite at *b* in the direction *ba*, the mark in the main tunnel at *c* can then be sighted, and the angle *abc* observed. Since the bearing of the line *ab* is known, the bearing of the line *bc* can be easily calculated. The theodolite is then transferred to *c*, and the survey continued from the base *bc* in the ordinary manner, and plotted from the meridian, either magnetic or geographical, marked on the plan. Permanent marks should be made in the lines *ab* and *bc*.

In the Case of only One Shaft.—It sometimes happens that there is only one shaft. In coal-mines, of course, this is only the case when opening out a new mine. It also sometimes happens that where there are two shafts, one of them is not available for the surveyor's use, either by reason of a furnace or buildings over the pit-top making it difficult and expensive to fix a centre-line visible by the surveyor on the surface and at the pit-bottom. In this case the line of survey has to be connected with the surface by observations made in one shaft only.

Wires in One Shaft.—As in the case of two shafts, so in the case of one shaft, the line may be transferred to the bottom by means of wires. Where the distance between the wires is small, as in the case of one shaft, it is essential that these wires should be thin and perfectly steady. As steel is the strongest material, it is best to use a thin steel wire, and to hang a heavy plumb-bob at the bottom. The diameter of this wire should be, say $\frac{1}{32}$ inch, and the weight of the plumb-bob at the bottom should be, say from 10 lbs. to 40 lbs. according to the quality of the steel. The plumb-bob should be attached to the wire when at the bottom of the shaft, so that the breakage of the wire will entail no danger, because the wires are not qualified to hold this heavy load for long, and would break with a little jerk. By the use of a heavy weight, the wire is stretched perfectly straight, and is able to resist the effect of the air-current; the weights themselves must be protected from the air-current by being placed in buckets of water, oil, or tar, or in a box, the wires passing upwards through a hole sufficiently small to prevent an air-current getting into the box. The wires above the pit-top must be securely fastened to a steady frame, and, if there is a wind, must be protected from this as much as possible.

Before the position of the wires at the bottom of the shaft can be observed, it is necessary to wait until the plumb-bobs have finished swinging. The wire and plumb-bob may be likened to a long pendulum. When a pendulum in the latitude of London swings from left to right, if it is 39·1383 inches in length, the swing will occupy exactly one second; thus, if the pendulum is lifted by hand and then allowed to drop, it will be 1 second proceeding away from the hand and 1 second coming back, or 2 seconds in making the return journey. The time required for a swing is proportional to the square root of the length of the pendulum; thus if, instead of being 39·1383 inches in length, it were 391·383, the period of the swing would be multiplied by the square root of 10; if the length of the pendulum were 39138·3 inches, the duration of each swing would be 1 second $\times \sqrt{1000}$, or 31·623 seconds, or the swing and return would occupy 63·246 seconds, or a little more than a minute.

Taking the case of a mine 1000 feet in depth, or 12,000 inches, we should find the duration of the swing as follows: $\sqrt{39 \cdot 1383} : \sqrt{12000} :: 1 \text{ second} : x$, and the period of the swing would be 17·5 seconds, and the return swing 35 seconds. When a pendulum is first started, it is easy to notice that it is not stationary because of the length of the swing; as, however, the length of the swing decreases, it is not so easy to notice whether it is swinging at all, and the longer the pendulum is the greater the care required to make sure that the pendulum is stationary. This can only be ascertained by placing a scale beside the pendulum when it appears to be quite still, and then to observe whether the pendulum increases or decreases its distance from the fixed scale and moves along the scale or keeps at one fixed point. The surveyor, of course, will satisfy himself that the plumb-bob and wire are swinging quite clear of impediment. In order that the plumb-bob may be stationary, it is essential that there should be no vibration in the frame at the surface to which it is attached, therefore the machinery at the pit-bank must not be working; also it must not be too windy, or else the exposed framework, though solid in appearance, will slightly rock with the wind; if there is any continuously moving machinery in the shafts, such as pumps, care must be taken that the frame suspending the wires is not in any way subject to vibration from this machinery, but takes its support independently from the solid ground. A very rapid current of air may also cause the wires to swing, and it may, therefore, be necessary to slacken the ventilation at the time when the observation is being made.

If the circumstances are such as to permit the above-named conditions to be fulfilled, the surveyor has now got a means of connecting the underground and surface surveys with sufficient accuracy for most purposes. The greater the distance between the wires, the greater the accuracy; but there are often serious practical difficulties in the way of utilizing the entire width of the shaft for this purpose. Suppose, however, that the distance between the wires is 100 inches, and the thickness of the wires is $\frac{1}{32}$ inch, if sufficient care is used, a surveyor may fix his theodolite on the surface in a line with these two wires in the following manner: Firstly without any instrument he looks past the wires and fixes a mark in some convenient place in a line with these wires, and as near to them as possible without being too near to focus, say 30 feet from the nearest wire, the instrument is then fixed over this mark on a traversing-table, so that it can be brought exactly into the line of the two wires. The surveyor can now range a line of poles in the same direction as these wires, and connect this line with the rest of the survey, and carefully establish its bearing in regard to the geographical or magnetic meridian. The degree of accuracy with which this can be done may be estimated in the following manner. If the theodolite on the traversing-table, being at a distance of 30 feet from the front wire, is so fixed that the second wire is completely hidden behind the first wire, then it might be possible to move the second wire to the extent of 0·00431 inch before it became visible on either side; this divergence in a length of 100 inches is equal to 1 inch in 23,202 inches—a degree of accuracy which is sufficient for the most part. But this error may be eliminated by placing the theodolite on the other side of the wires and repeating the observation. The line as now poled out on the surface should agree with that poled out by the first fixing of the instrument. In the same way, the underground survey may be connected to these wires; by sighting past the wires the theodolite may be placed with approximate accuracy in the same line. By means of the screw, the traversing-table can then be moved until the theodolite is precisely in the line of these wires. Having clamped the instrument on this line, the vernier clamp can be loosened and the telescope turned in the direction of the next sight, the angle read, and the survey proceeded with as usual, and plotted from the meridian. If, when the telescope is first set up at the pit-bottom in line with the wires, the vernier is set at the angle of the bearing as observed on the surface, then all the subsequent readings will be correct readings, and will not require further correction. In case, however, the correct bearing has not been ascertained, the bearings recorded can afterwards be corrected. The surveyor will, of course, make permanent marks at the pit-bottom showing the direction of the line through the wires.

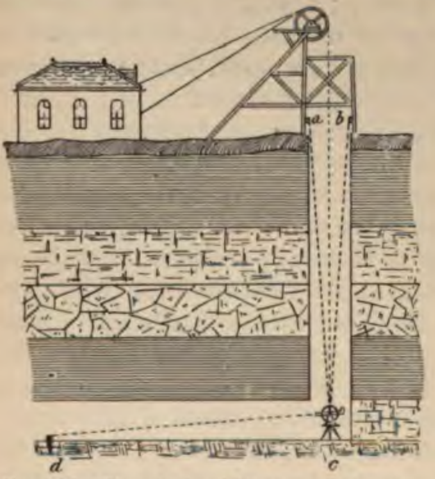
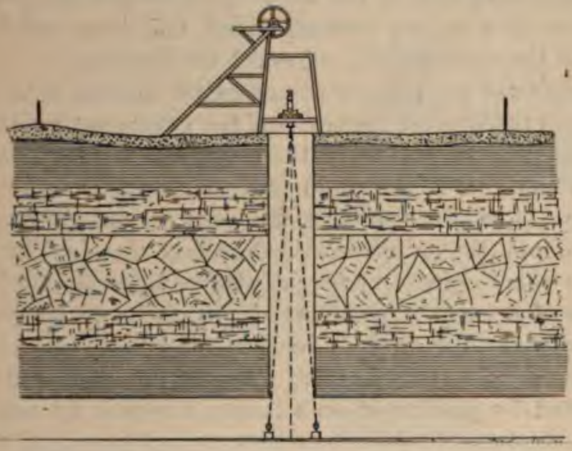


Fig. 117.—Transit theodolite transferring line of sight underground to surface.

By Means of Transit Instrument.—In view of the difficulty in steadying the wires, many surveyors prefer to use a theodolite or other transit instrument for transferring a line of survey from the shaft-bottom to the surface, or *vice versâ*. Taking the former case, the surveyor finishes his survey at the bottom of the shaft as shown in Fig. 117. Placing his theodolite over the last mark at the bottom of the shaft, he clamps the vertical axis with the telescope in the line of the last sight *cd*; he now points the telescope to the top of the shaft, using the right-angle eye-piece for convenience. He may now place a mark as at *a* upon some framework, either in or over the shaft, and another mark at *b*; these two marks will then be in the same vertical plane as the last line of sight in the pit, and a line may afterwards be produced from these marks by means of a theodolite, or a fine wire stretched over them, or otherwise.

Another method, shown in Fig. 118, is to place the theodolite



The original diagram of the one on page 196 of this book, exhibiting a mode of connecting underground workings to the surface by sighting with a Transit Theodolite up a shaft, is to be found at page 84 of the work upon Mine Surveying, published by Mr. H. D. Hoskold, in 1863.

two things is, therefore, necessary: the theodolite must be constructed with a telescope at the outer end of the horizontal axis, so that its plane of rotation is outside the graduated circle, or else the telescope may be taken off the bearings of the theodolite and placed on other bearings carried on a plate with a circular hole in the centre, through which the surveyor can look downwards. The instrument the surveyor is then using inches is equal to 1 inch in 23,202 inches—a degree of accuracy which is sufficient for the most part. But this error may be eliminated by placing the theodolite on the other side of the wires and repeating the observation. The line as now poled out on the surface should agree with that poled out by the first fixing of the instrument. In the same way, the underground survey may be connected to these wires; by sighting past the wires the theodolite may be placed with approximate accuracy in the same line. By means of the screw, the traversing-table can then be moved until the theodolite is precisely in the line of these wires. Having clamped the instrument on this line, the vernier clamp can be loosened and the telescope turned in the direction of the next sight, the angle read, and the survey proceeded with as usual, and plotted from the meridian. If, when the telescope is first set up at the pit-bottom in line with the wires, the vernier is set at the angle of the bearing as observed on the surface, then all the subsequent readings will be correct readings, and will not require further correction. In case, however, the correct bearing has not been ascertained, the bearings recorded can afterwards be corrected. The surveyor will, of course, make

FIG. 117.—Transit theodolite transferring line of sight underground to surface.

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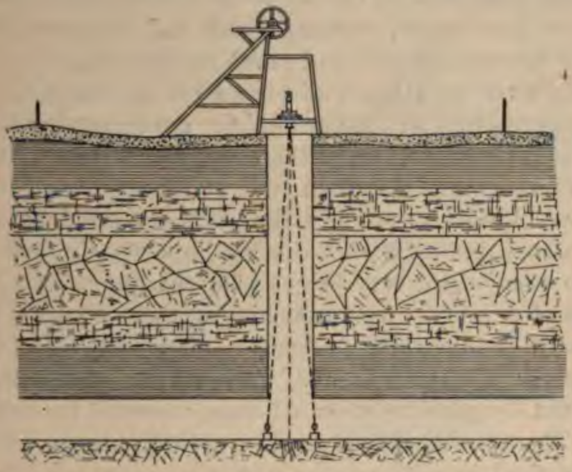

FIG. 118.—Transit theodolite transferring line of sight on surface underground.

on a frame over the pit-top and fix the telescope in some convenient direction, as nearly as can be judged in the direction of the underground level or tunnel along which the survey will have to be continued. A line is poled out on the surface in this direction and connected with other stations on the survey with every accuracy; the theodolite is now turned so as to look straight down the shaft. With an ordinary theodolite and ordinary tripod stand this cannot be done, because the vertical axis of the instrument is precisely under the telescope. One of two things is, therefore, necessary: the theodolite must be constructed with a telescope at the outer end of the horizontal axis, so that its plane of rotation is outside the graduated circle, or else the telescope may be taken off the bearings of the theodolite and placed on other bearings carried on a plate with a circular hole in the centre, through which the surveyor can look downwards. The instrument the surveyor is then using is not a theodolite at all, but simply a transit telescope, and this is all that is required. If this telescope revolves in a truly vertical plane, or revolves truly on the centre-line of its axis in any plane, whether strictly vertical or not, a line drawn through any two stations in that plane will have the same bearing as a line drawn through any other two stations in the same plane, providing both the stations (if the plane is not vertical) are on the same level in each case. It will, however, be well to take care to level the axis of the telescope very carefully, so that it will revolve in a strictly vertical plane, and then any difference in level of the stations will not affect the bearing.

Having fixed the telescope in the most suitable direction, and marked out the line on the surface, the surveyor now directs his line of sight to the bottom of the shaft, and fixes two marks in the same direction as the line marked out on the surface. With a telescope of ordinary power and a shaft several hundred feet in depth or more, the only mark that could be clearly distinguished would be some point very brightly illuminated, as, for instance, the flame of a lamp. A special lamp may be constructed, with a narrow wick-tube only $\frac{1}{16}$ inch in diameter, and, of course, a correspondingly small flame. The position of this lamp may be adjusted by placing it on a frame, movable by means of a screw. There must, of course, be two such lamps. When the lamps are fixed in position, the surveyor can take his theodolite down the pit, and fix it in a line with these two flames, and continue the survey in the way previously described when using wires.

The accuracy with which this work can be done depends to a great extent on the power of the telescope used, and the accuracy with which the telescope revolves in its bearings. To ensure sufficient accuracy, a special transit instrument should be used in the case of deep mines and very important surveys. When a powerful telescope is used, it is not necessary to direct it on to a lamp-flame; a brightly illuminated mark may permit of more accurate adjustment, and will not be moved by the wind; this mark should be a dark line on a white board, or a white line on a black board. The limelight, or other bright light, might be used to throw a brilliant illumination on to these marks, whilst every other part is sheltered from the illumination. In this way a survey of the deepest mine may be accurately connected with the surface.

In an interesting paper contributed to the Federated Institute of Mining Engineers, Professor Liveing gives the result of his experience with the transit method.[1]

He usually found it advisable to make the connection observation in the downcast shaft, because the air is there clear, and has a fairly uniform temperature, which is a matter of great importance, as avoiding irregular refraction.

At the shaft-bottom he fixes two short battens, **A** and **B** (Fig. 118A), and upon these is placed a horizontal bar of pine. The middle point of this bar should be placed perpendicularly below the centre of the transit instrument, and the bar is placed in the direction of the main road from the shaft-bottom.

Upon this bar, at equal distances from the centre-points, are screwed two small wooden boxes, the top of each is inclined at 45°, and has a square opening across which two fine copper wires are stretched diagonally forming a cross. The thickness of these wires must vary with the depth of the shaft and the power of telescope employed. With a telescope with a 2¼-inch aperture, wires of No. 20 B.W.G., about 0·035 inch thick, answer well for pits not exceeding 200 yards in depth.

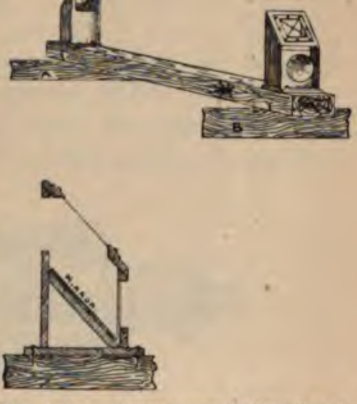

FIG. 118A.—Arrangement of marks at bottom of shaft.

In the lower part of this box is fixed a mirror, which reflects in an upward direction the light of a lamp placed opposite an opening in the side of the box. These two boxes are placed upon the bar as far apart as the width of the shaft will permit. The observation from the surface to those marks should be repeated at least six times, and the mean of the results taken.

The wire crosses can be observed both from the surface and from the underground level.

Professor Liveing records an instance in which he had to make the connection observation in the case of a shaft 500 yards deep, and where the cross-wires could not be placed more than 8 feet apart. In this case he replaced the cross-wires by two small electric lamps, each having small arched filaments, and these were so fixed that the planes of the filaments were in the vertical plane joining them, and their axes were inclined at an angle of 45°, so that observations could be taken to them both from the surface and from the main road underground.

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[1] *Transactions Federated Institute of Mining Engineers*, vol. xviii. p. 65.

CHAPTER XII.

LEVELLING.

Dumpy Level.—To ascertain the relative levels of different points, the instrument usually employed, called a level, is a telescope, on the top of which are fixed two ordinary spirit-levels (see Fig. 119), one long level, *b*, fixed parallel to the axis of the telescope, and also a cross-level, *a*, at right angles to it. When the instrument is in correct adjustment, and the spirit-levels have been adjusted by the levelling-screws so as to bring the bubbles into the centre, the line of sight past the crossing of the hairs is a level line.

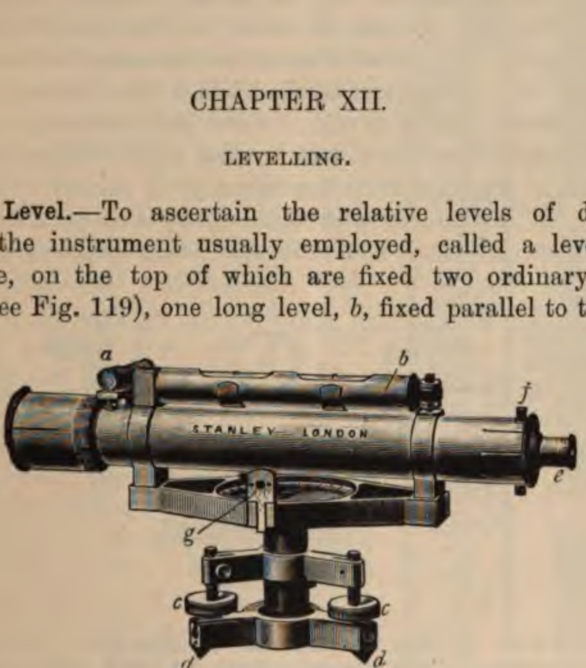


Fig. 119.—Dumpy level.
(Kindly lent by Messrs. W. F. Stanley and Co., Ltd.)

At one end of the telescope is a diaphragm, *f*, containing the webs, and an eye-piece, *e*, which can be drawn out of the tube so as to focus on to the hairs in the diaphragm.

Underneath the telescope a compass box and needle are sometimes placed, provided with a prismatic reader, *g*, so that the bearing of lines being levelled may be taken.

The level is screwed on to a tripod similar to that of the theodolite; when circumstances do not permit the use of a tripod, the level may be placed on a piece of wood or any other flat surface, feet (*d*, *d*) being often provided on the base plate for this purpose. The level shown in Fig. 119 is the type known as the "Dumpy."

The levelling-screws may be either three in number, as shown in Fig. 119, or four, as shown in Fig. 120. In levelling an instrument with three screws, the telescope is first placed parallel to two of them and levelled. It is then turned a quarter of a revolution; that is to say, one end of the telescope is placed over the other screw, and it is levelled in this direction. The operation is repeated until the telescope is perfectly level. In levelling an instrument with four screws and parallel plates, the telescope is placed in a line with two opposite screws and levelled. It is then turned in line with the other two screws and levelled. The operation of levelling is performed by turning opposite screws in opposite directions to each other. It is generally considered that the three-screw arrangement is quicker.

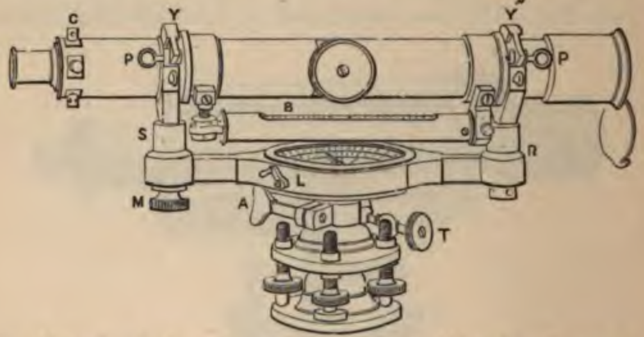


FIG. 120.—**Y**-level. A, clamp for vertical axis; B, bubble-tube; C, screw to adjust the diaphragm; L, clamp for compass needle; M, milled head screw to adjust the limb carrying the telescope; PP, pins to secure telescope in the **Y**'s; T, tangent screw for accurate fixing of the telescope.

If the cross-level already referred to is in accurate adjustment, and is a good length, it is, of course, unnecessary to turn the level; but the usual practice is as described.

Y-Level.—Another form of level, known as the **Y**-level, is shown in Fig. 120. In this type of instrument the telescope is supported in **Y** bearings, and can be taken out and reversed. There is only one level tube, which is parallel to the length of the instrument, and for this reason it is not so quickly levelled as the dumpy level, as it has to be first levelled in one direction, and then turned a quarter of a revolution and levelled in this direction, to get the instrument truly horizontal.

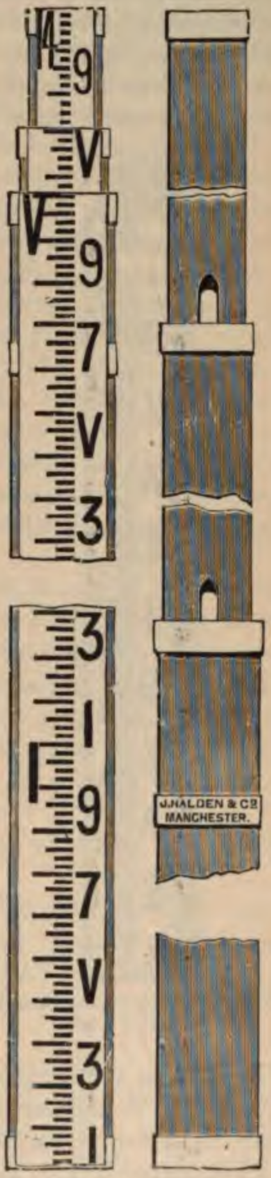


FIG. 121.—Levelling-staff.

The advantage of the **Y**-level, however, lies in the fact that, as will be seen later on, it is capable of being easily and accurately adjusted.

Levelling-staff.—The level is used in conjunction with the levelling-staff, which is generally about 16 feet in length for surface work, and of much shorter length for underground work. The staff, as used on the surface, is generally a wooden tube (see Fig. 121) about 3 inches wide and 2 inches thick, inside which slides a similar but smaller tube, inside which again slides a wooden lath. On the face of the staff is painted a scale of feet, tenths and hundredths, the measurements starting from the lower end. Sometimes the figures on the scale are reversed, as shown in Fig. 122. By this means the reading on the staff is seen in its natural position (this can also be accomplished with the ordinary staff by the use of a special eye-piece, but this reduces the light).

Most surveyors prefer the staff with the figures the correct way up; a little practice soon enables the surveyor to see them correctly although really inverted by the telescope.

Pit Levelling-staff.—The staff used in the mine may be made of special length to suit some particular seam. A staff has been made by Linsley, of Newcastle, consisting of a piece of wood 3 feet long and about

$2\frac{1}{2}$ inches wide by $\frac{7}{8}$ inch thick, at the back of which slides a lath which, when drawn out, gives a staff 6 feet high. The lower part of the staff has the scale painted on, but the sliding lath carries at the top a roller, on which wraps a tape $1\frac{3}{4}$ inch in width, on which the scale is painted. The roller contains a spring, which rolls up the tape. As the lath is extended, the tape is unwound and the scale is always in its correct position. A similar staff, called Jee's pit levelling-staff, is manufactured by Messrs. John Davis and Son, of Derby, and is illustrated in Fig. 123.

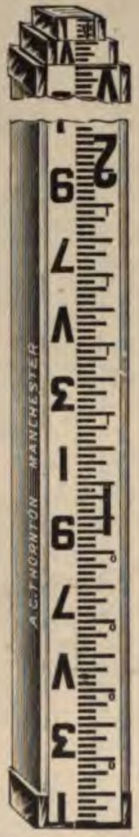


Fig. 122.—Levelling-staff with figures reversed.

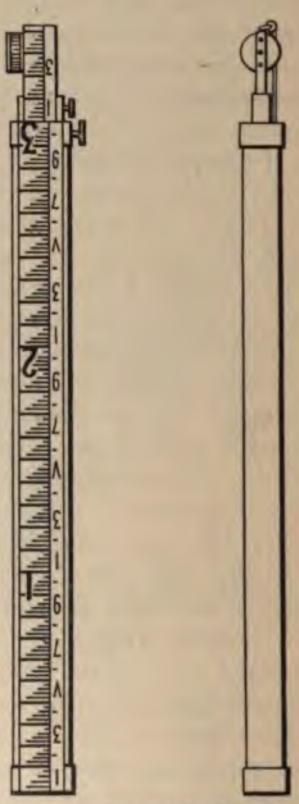


Fig. 123.—Jee's pit levelling-staff.

Mode of using the Level.—The instrument is fixed on a tripod stand and carefully levelled, so that it may revolve in a horizontal plane; the staff is held on some mark which is the base or starting-point of the survey. The level cross-hair of the telescope appears to cut this staff at some mark which reads say 1·27 feet; the staff is then taken forward to another station, and the telescope again directed upon it, when the cross-hair appears to cut the staff at say 14·56 feet, showing a difference in level of 13·29. It is thus obvious that the ground at the first station was 1·27 feet below the level of the cross-hair of the telescope, and the ground at the second station was 14·56 below the level of the cross-hair, and therefore the ground at the second station is 13·29 feet below the ground at the first station. The staff is left standing at the second station, and the level is moved forward beyond the staff; it is then levelled and directed upon the staff, when the reading is say 0·74; the staff is then moved to the third station, and read, say 15·62. It is again obvious that the ground at the second station was 0·74 below the level of the cross-hairs, and that the ground at the third station was 15·62 below the level of the cross-hairs, the difference being 14·88. The staff is again left standing whilst the level is moved forward to the next station, and so on.

When once the telescope has been fixed, it may be convenient to take the relative levels of a number of stations within view. The first sight that is taken is called the back sight, and the last sight that is taken before moving the level is called the fore sight, and all the other sights are called intermediate sights. The staff is always left fixed at the station to which the last fore sight was taken, whilst the level is being moved and fixed ready to take another back sight.

Tables X. and XI. are pages from the surveyor's note-book, showing levellings.

The first column in Tables X. and XI. shows the distance from the starting-point (that is to say, the place where the staff was first set) to each station; the measurement in each case is to where the staff is placed, as that is the point of which the level is observed.

In the second column the back sights are placed. The first position of the staff is always a back sight, and therefore the first back sight is at the starting-point, or at 0 in the distance column. The fore sight is the last position of the staff before the level is moved to another place, and the second back sight is taken to the staff, where it stood when the first fore sight

TABLE X.

Levels taken at ——— for New Road, Oct. 4, 1898.

Total distance.	Back sight.	Intermediate sight.	Fore sight.	Rise.	Fall.	Reduced level.	Remarks.
Links.							
0	13·27	—	—	—	—	100·00	Back sight on peg A. See survey-book No. 9, p.15, 24/Sept./1898.
55	—	8·15	—	5·12	—	105·12	
72	—	8·97	—	4·30	—	104·30	
127	9·20	—	1·09	12·18	—	112·18	
163	—	6·37	—	2·83	—	115·01	
249	—	4·85	—	4·35	—	116·53	
308	—	10·26	—	—	1·06	111·12	Same level as door-sill of woodman's cottage, 10 yards west of proposed road.
354	1·87	—	14·75	—	5·55	106·63	
396	—	4·15	—	—	2·28	104·35	
430	—	4·66	—	—	2·79	103·84	
467	—	9·39	—	—	7·52	99·11	Same level as top of lower hinge hook of gate in quarry-field.
553	—	13·24	—	—	11·37	95·26	
624	—	—	13·87	—	12·00	94·63	Fore sight on peg B (see above).
	24·34		29·71 24·34				
			5·37				

TABLE XI.

Levels taken at ——— for New Road, Oct. 4, 1898.

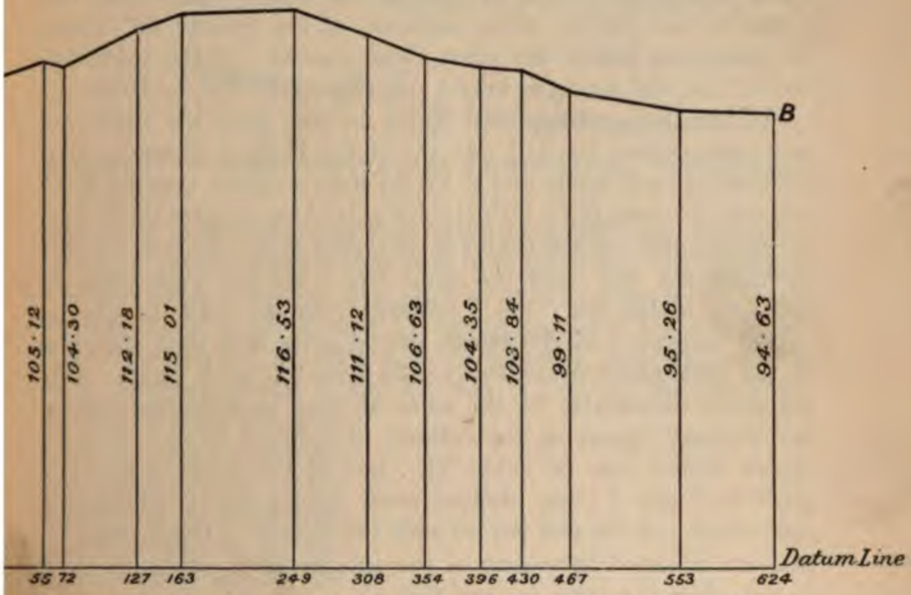
Total distance.	Back sight.	Intermediate sight.	Fore sight.	Rise.	Fall.	Reduced level.	Remarks.
Links.							
0	13·27	—	—	—	—	100·00	Back sight on peg A.
55	—	8·15	—	5·12	—	105·12	
72	—	8·97	—	—	0·82	104·30	
127	9·20	—	1·09	7·88	—	112·18	
163	—	6·37	—	2·83	—	115·01	
249	—	4·85	—	1·52	—	116·53	
308	—	10·26	—	—	5·41	111·12	
354	1·87	—	14·75	—	4·49	106·63	
396	—	4·15	—	—	2·28	104·35	
430	—	4·66	—	—	0·51	103·84	
467	—	9·39	—	—	4·73	99·11	
553	—	13·24	—	—	3·85	95·26	
624	—	—	13·87	—	0·63	94·63	Fore sight on peg B.
	24·34		29·71 24·34	17·35	22·72 17·35		
			5·37		5·37		

Total fall from A to B, 5·37 feet.

Fig. 124 is a section plotted from this page.

was taken; and for that reason the back sight of the second position of the level is shown in the tables on the same line, and opposite the same distance, as the fore sight of the first position. Similarly the subsequent back sights are placed in the same line as the distance and fore sight of the preceding position.

The student will see, on reference to Table X., that after the three columns of staff-readings come two columns, one headed "rise," the other "fall;" if the back sight reading is a larger figure than the fore sight, the difference comes in the rise column; if the fore sight reading is a larger figure than the back sight reading, the difference comes in the fall column; the readings in the intermediate column are compared with the back sight. In Table XI. the calculations are done in a different manner, the intermediate sights being treated as back sights and fore sights. The next column is headed "reduced level," which shows the relative altitudes of the various stations, and also their altitude as compared with the datum; at the head of this column is placed a figure at the discretion of the surveyor, the



Vertical scale 40 feet to 1 inch. Horizontal scale 1½ chains to 1 inch.

FIG. 124.—Section plotted from levels given in Tables X. and XI.

figure to be sufficiently large so that the fall below that level shall not reduce the original figure to less than 1; thus if the surveyor thinks that the total fall of the section he is about to level will not be more than 70 or 80 feet, he would put at the head of the column 100; then if the levelling shows that the ground is falling, the amount of fall is deducted from 100; but if the observations show that the ground is rising, the amount of rise is added to 100. By putting this figure 100 at the head of the column, the surveyor is relieved from the necessity he would otherwise be under of putting the minus sign before the figures in this column when the level of the ground fell below the level from which the survey was started. If the total fall is over 100, the surveyor might substitute 200, 300, or 1000.

To test the accuracy with which he has done the additions and subtractions, he will add up at the bottom of each page the total of back sights and the total of fore sights, also the total rise and the total fall; if the back sights show a larger total than the fore sights, he will subtract the latter from the former, and the difference will show the total rise. He will then subtract the total under the "fall" column from the total under the "rise" column, and the result should agree with that obtained by the subtraction of the fore sights from the back sights; this difference should also be the same as that between the bottom and the top figures in the column of reduced levels. This is shown in the case of Table XI. But in the form of booking given in Table X., the student must remember, in adding up the columns of rise and fall, to omit the figures in these columns that are obtained by means of the intermediate sights, as they do not affect the total rise and fall.

Elimination of Errors of Adjustment.—Whilst the surveyor should always use instruments that are in correct adjustment so far as he knows, it is usual, in levelling, so far as possible to use the instrument in such a manner as to eliminate errors that might arise owing to the spirit-level not being precisely parallel to the line of sight or axis of the telescope; thus if the position of the level is midway between the back sight and fore sight, then the error in one reading is corrected by a precisely similar error in the other reading (see Fig. 125). In this case the level is out of adjustment, so that the back sight reads 3·64, whereas the correct reading should have been 3·45; the fore sight reads 6·31, whereas the correct reading should have been

6·12; but the difference between the two incorrect readings, 2·67 feet, is the same as that between the two correct readings, so that there is no error in the result. When, however, it is remembered that the height of the telescope, as commonly used on the surface, is only about 4 feet, and the length of the staff is 16 feet, it is obvious that, when surveying on an incline, the fore sight may be four times as long as the back sight, and that if the length of the fore sight is restricted in order to keep the two sights of equal length, a great deal of time will be lost. The length of the sights, however, may be nearly equalized by putting the instrument on one side of the line of survey, as shown in Fig. 126. Here the slope of the hill **AB** is 1 in 20, and therefore, if there is a difference of level of 14 feet between the upper position of the staff **A** and the lower position of the staff **B**, the distance on the line of survey will be 280 feet. If the level were put on this line, it would only be about 60 feet from the upper station, and about 220 feet from the lower station. If, however, the position of the level is moved to a distance of 240 feet to one side of the line, the length of the back sight will then be about 250 feet, and that of the fore sight about 320 feet, so that the length of the back sight to the fore sight is about as 3 to 4, and the error that might have occurred had the telescope been in the line of survey is reduced

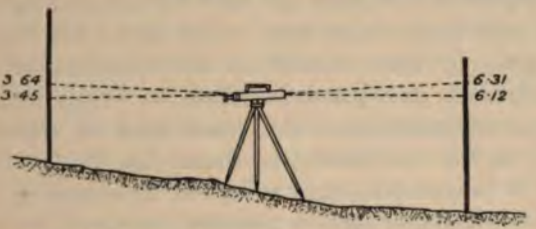


FIG. 125.—Level out of adjustment; errors eliminated by fixing level equidistant.

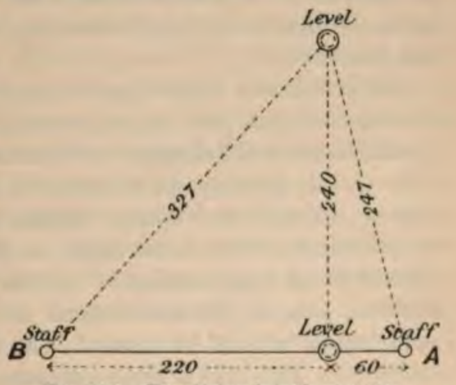


FIG. 126.—Equalizing length of sights.

to one-third, and therefore will be insignificant, unless the instrument is very seriously out of adjustment, which ought not to be the case.

Bench Marks.—The student will remember that the staff, when once placed for the fore sight and the reading taken, has not to be moved until after the level has been again fixed and the reading taken; should the staff be accidentally moved, the surveyor will have to go back to his last fixed station. It is a good plan to leave marks at convenient places, such as walls, flagstones, gate-posts, buildings, or bridges, which he will be able to recognize, and the exact level of which has been recorded in his note-book, as shown in Tables X. and XI. In case of any error, or having to continue an unfinished survey, these marks are convenient stations from which to start another series of levels, and also in case the line of survey should return to the starting-point or cross itself, the accuracy of the levels can be checked if a second reading is taken at one of the original marks.

As in surveying, so in levelling, the surveyor cannot consider his work complete until he has proved the accuracy of it, either by levelling the same line twice, or by levelling in a circuit returning to his original station, or by levelling from one mark the altitude of which has been fixed by a previous survey (such as the Ordnance Survey) to another mark of which the altitude has been fixed.

Adjustments.—From time to time the surveyor should test his level, to see that it is in proper adjustment.

Adjustment of Telescope to Vertical Axis.—The line of collimation of the telescope of a level or theodolite is the line which passes through the optical centre of the object-glass and the intersection of the cross-hairs in the diaphragm. This line should be at right angles to the vertical axis, so that when the vertical axis of the instrument is truly vertical, the line of collimation will be horizontal. In the case of the dumpy level (shown in Fig. 119), the adjustment of the telescope at right angles to the vertical axis is made by the maker, the telescope being rigidly attached to the vertical axis.

In order to test the accuracy of the adjustment, make or find two firm places at a convenient distance apart (say 200 feet), each exactly level with the other. Place the telescope half-way between these places, and in a straight line with them, and level it. Take the reading of the staff at each of these marks. If the vertical axis is at right angles to the telescope, then the two readings will be the same; if the two readings differ, the vertical axis is out of adjustment, and this will involve some loss of time in levelling the bubble-tube for each sight instead of only once for each time the level is set up.

In the case of the **Y**-level (Fig. 120), where the telescope is movable and the supports are capable of adjustment by means of screws, the adjustment is made in the following manner: A distant object (such as a levelling-staff) is sighted, and the position of the cross-hairs upon this object marked precisely; the telescope is then taken out of its **Y**'s and turned end for end, and replaced and rotated through an arc of 180° on the vertical axis. If the horizontal cross-hair is still on the same mark, the telescope and vertical axis are at right angles; if not, the attachment to the vertical axis must be adjusted by means of the screw M (Fig. 120), so as to move the telescope to bring the cross-hairs half the distance between the two marks on the staff.

To make the Spirit-level Parallel to the Line of Collimation.—The spirit-level is attached to the telescope by means of screws at each end, or sometimes by means of a hinge at one end and screws at the other. By moving these screws, one end of the bubble-tube may be raised or lowered in relation to the other end. The level should be strictly parallel to the line of collimation. This parallelism may be tested in the following manner: The telescope is fixed at *a*, half-way between the stations *b* and *c*, the distance from *a* to *b* being say 60 yards, and the staff is first read at *b* and then at *c*, the position of the staff at each reading being carefully noted and being on some hard and immovable place. The level is now moved to the place *e*, about six or seven yards from the station *b*; the staff is then read again at the two former stations *b* and *c*. If the difference in the readings is the same as when the level was half-way between the two stations, it is a sign that the bubble tube is parallel to the line of sight; if it is not the same, it is a sign that the bubble tube is not parallel. Suppose that in the first reading, when the level was equidistant between *b* and *c*, the difference in levels of the two stations appeared to be 1·2 foot, and that if when the level was at *e*, the difference in the readings of the two stations was 1·4, then the bubble-tube must be adjusted so that upon again bringing the bubble into the centre of the tube with the levelling-screws, the difference between the readings *b* and *c* is precisely 1·2, which is the real difference in level, because, in the first observation, the level being midway between the two stations, errors due to the bubble-tube not being parallel to the telescope were eliminated, because the error was equal in both readings.

The method of adjusting the bubble-tube is first of all to alter the inclination of the telescope as it stands at station *e* by means of the levelling-screws till the readings of the staff in a line with it show a difference between the two stations *b* and *c* of 1·20. We know that the line of sight is then level. The bubble-tube must now be adjusted by the screw or hinge attachment to the telescope till it appears level.

Adjustments Preliminary to taking Sights with the Level.—In using the level there are some adjustments which have to be made continually. In the first place, the tripod must be firmly fixed, and, if the ground is soft, the legs pushed down so that they are not likely to sink before the observations are completed; the brass head of the tripod must be fixed by the eye approximately level; the levelling-screws should be all about the same length through the upper plate, in order not to put a strain on the threads of the screws.

The telescope is now directed upon the staff, and the adjustment for focus and parallax is made. The object-glass is on a sliding tube, which can be moved in and out by means of a rack and pinion. The nearer the object is to the telescope, the further the object-glass has to be moved outwards (this is the adjustment for focus). In order that the cross-hairs may be distinctly seen, the eye-piece, which slides in and out, has to be adjusted (this is the adjustment for parallax). When both the eye-piece and the object-glass are correctly adjusted, the cross-hairs seem clearly and steadily fixed upon the staff in one place, even though the observer's eye may be moved from side to side or up and down; unless this is so, different readings will be taken according to the position of the eye, and consequently errors may be made. The bubble must now be brought to the centre of the tube by means of the levelling-screws, so that the telescope can be turned in any direction without moving the bubble, in the manner described at the beginning of this chapter.

Correct Method of holding the Staff.—It is important that the staff should be held in a strictly vertical line, otherwise there is an error in the apparent altitude proportionate to the difference between the radius and the cosine of the angle from the vertical at which the staff is held; thus, if the staff is sloping backwards at an angle of 10°, and the reading is 10 feet, the real height will be 9·848, or an error of $\frac{15}{100}$ of a foot; a staff, however, that is 10° out of truth is obviously not vertical to the most inexperienced eye. If the inclination of the staff is 5° from the vertical, and the reading is 10 feet, the real height would be 9·96, or $\frac{4}{100}$ of a foot less. Any person standing on one side of a staff can see at once if it is more than 2° out of the vertical, and that amount of inclination will not seriously affect the accuracy of the reading. Sometimes a little spirit-level is fixed to the back of the staff, so that the man holding the staff may see by the position of the bubble when he is holding it vertically. The surveyor can tell by the vertical cross-hair if the staff is leaning to the right or left, but he cannot tell whether it is leaning to or from him. Sometimes the man holding the staff is instructed to swing the upper end of it to and from the surveyor. The vertical position of the staff is given by the lowest reading.

It is exceedingly important that the man holding the staff should not carelessly move it after the reading of the fore sight has been taken. A plan adopted by an old railway surveyor to impress this on the mind of the labourer holding the staff, was to give the man half a crown to place on the ground at the station, the staff on the top of it; as he is not likely to leave the half-crown behind, he is not likely to move the staff without picking up the coin—a movement which would probably be observed.

In taking sights of a chain and upwards in length, the surveyor can read the height of the cross-hairs on the staff in feet and decimals. If, however, the staff is close to the level, the length of the staff within focus may be too short for him to read the feet, and he may require the staff lifted up to enable him to see the number of feet below the decimal which he has already read. He ought to carefully note the decimal before the staff is lifted, if it is the back sight; and, if it is the fore sight, *after* the staff has been replaced on the ground. There is no difficulty about instructing the man to lift the staff, as a rule, because the surveyor is close to the staff. In the case, however, of a mine where there is no free space above the staff to permit of its being lifted, it is common for the man holding the staff to put his hand about the level of the cross-hairs, and then to read the red figure, which is the number of feet below his hand, or, what is better still, to keep his hand on the staff until the surveyor can come to read the figure himself, which involves very little loss of time, as the staff is only a few yards away from the level. With the staff shown in Fig. 127 this procedure would be unnecessary, as the feet-reading is shown three times in every foot-length.

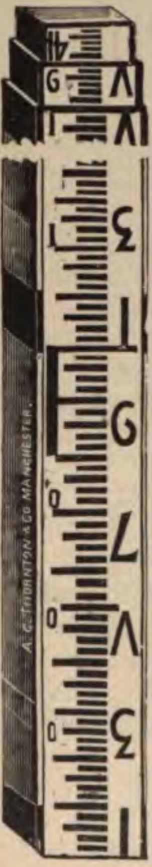


FIG. 127.—Staff with intermediate foot readings.

Curvature of the Earth.—Owing to the curvature of the earth's surface, objects as viewed through the telescope of a level, appear lower than they really are. For instance, if we suppose that the earth is perfectly circular (a perfect sphere without any hills or valleys), and that we have a level carefully adjusted so as to give a level or horizontal line of sight, then, if a staff is held some distance away from the level, it will give a higher reading than if held near the level, making it appear that there was a fall. As a matter of fact, both the points at which the staff was held would have the same level, as they would be the same distance from the earth's centre.

Thus the greater the length of the sight, the greater the correction to be added to the apparent level to obtain the actual level.

Refraction caused by the Atmosphere.—Light always travels in a straight line unless diverted by the media through which it passes. In passing through the atmosphere, the ray of light is refracted (or bent) in such a manner that the object viewed appears higher than it really is.

Correction for Curvature and Refraction.—Molesworth's pocket-book gives the following useful rule and table :—

D = distance in statute miles.
C = curvature in feet = $\frac{2}{3}D^2$ (approximately).
C − R = curvature less refraction = $\frac{4}{7}D^2$ (approximately).

D.	C.	C − R.	D.	C.	C − R.	D.	C.	C − R.
	Feet.	Feet.		Feet.	Feet.		Feet.	Feet.
1	0·66	0·57	6	24·00	20·57	12	96·0	82·0
2	2·67	2·29	7	32·67	28·00	14	130·0	112·0
3	6·00	5·14	8	42·67	36·57	16	170·0	146·0
4	10·67	9·14	9	54·00	46·30	18	216·0	185·0
5	16·67	14·29	10	66·67	57·14	20	266·7	228·6

The feet given in the above table (under the heading C − R) have to be added to each sight which is long enough to require correction.

A little reflection will make the effect of the curvature of the earth perfectly clear. When the levelling-instrument is erected on the tripod stand ready for use, with the proper adjustments made and the bubble-tube perfectly level, the vertical axis of the instrument is then a continuation of a radial line drawn from the centre of the earth to the circumference. The line of sight of the telescope is a line at right angles to this vertical axis, and is therefore comparable to the tangent of a circle of which the radius is the distance from the centre of the instrument to the centre of the earth. The levelling-staff, which is sighted through the telescope, being held vertically, is pointing towards the centre of the earth, and is really a continuation of a straight line drawn from the centre of the earth through the circumference to where it meets the line of sight of the telescope. This straight line from the centre of the earth is comparable to the secant of the angle at the earth's centre. That part of the staff which is between the surface of the earth and the line of sight is that part of the line which, added to the radius, makes up the full length of the secant. This part of the secant, however, in dealing with arcs of only a few seconds or even a few degrees in size, is practically identical in length with the versed sine of the arc, and the length of the tangent in arcs up to 3° differs by only a small fraction from the length of the sine. So for the purpose of considering the effects of curvature, the length from the level to the staff may be taken as either the tangent or the sine of the angle subtended at the earth's centre, whichever is more convenient, and the distance as measured on the levelling-staff from the line of sight to the ground, may be recorded as the versed sine. For arcs of less than 10° the versed sine varies approximately as the square of the number of degrees, or of the number of minutes or seconds in the arc respectively. The distance from the level to the staff is proportional to the size of the angle it subtends at the earth's centre, and that is why the correction for curvature varies as the square of the distance. It is easy for the student to satisfy himself what the correction should be. For example, assume the radius of the earth to be 4000 miles, and the length of sight to be 10 miles, say from the side of one mountain and across a valley to the side of another mountain. Then the sine or tangent is $\frac{10}{4000} = 0·0025$, which is the natural sine of an arc of about 8½ minutes. The versed sine of this arc is 0·000003, that is to say, the versed sine is 0·000003 of the radius of the earth (4000 miles), and is equal to 63·36 feet; so that the mark sighted to on the mountain, at a distance of 10 miles, is 63 feet further from the centre of the earth, or, in common parlance, 63 feet higher than the eye of the observer at the telescope.

The above calculation has to be corrected for refraction, as previously mentioned. The refraction of light rays is the change in their direction when they pass from one medium to another, as, for instance, when they pass from air into water, or when they pass from a layer of air of one density to another layer of different density. The effect of refraction is to make bodies appear higher than they really are, and the effect of the earth's curvature is to make them appear lower than they really are. The effect of refraction is much less than that of curvature. Refraction varies as the square of the distance, and for 1 mile is 0·105 foot. Referring to the table previously given, it will be seen that in the case of a sight 1 mile in length, the correction for curvature and refraction is 0·57 feet. For ½ mile the correction would be about one-fourth of the above figure, and for ⅛ mile about one sixty-fourth of the above figure, or rather less than 0·01 foot. With an ordinary 14-inch level it is difficult to read the staff with accuracy at a greater distance than 220 yards, at which distance the correction would be insignificant. For this reason, when levelling in the ordinary way, any correction of this kind can be entirely disregarded. But supposing that the surveyor could read the staff with a powerful telescope with minute accuracy, the amount of the correction is so small that it would be a waste of time under ordinary circumstances to take any notice of curvature. Suppose, for instance, that a line was levelled continuously downhill for 15 miles, with the back sights about 26 feet long, and the fore sights about 126 feet long, the total correction would only amount to about 1 foot. The surveyor will, of course, bear in mind that if he takes back sights and fore sights of equal length, the error due to curvature and refraction will correct itself, and, if the surface is undulating, the errors due to the uneven lengths of the sights will correct themselves.

Confusion has often arisen in the minds of people from the natural and prevailing idea that a line at right angles to a vertical line, and which for ordinary practical purposes is a level line, is necessarily a level line for geographical purposes.

When the sea is calm, the surface is level; but it follows the curve of the earth. In the same way, the surface of a canal is level; but it also follows the curve of the earth, so that any number of lines drawn from the surface of the canal to the centre of the earth at any points in the canal, are all of the same length, the water being held in this position by the attraction of gravitation.

Borchers' Vane Rod.—In some mines the vertical measurements are taken from the roof, and not from the floor. Mr. B. H. Brough, in his book on *Mine Surveying*, describes a staff known as Borchers' vane rod, which is a steel rod having a hook at the upper end. The rod is suspended from hooks fixed in the roof, and hangs vertically by its own weight; it is graduated, measuring from the centre of the hook at the top downwards. A circular disc or target of sheet iron slides up and down this staff; a line is scratched across the centre of the target at right angles to the staff, and on this line are three holes; two of the holes are 0·4 inch diameter, and in one of these is fixed a piece of ground glass; the other hole is 0·07 inch. The surveyor sights to a lamp held behind the disc opposite the small hole if he is near, and opposite one of the larger holes if he is some distance off. The assistant moves the disc up and down the staff, until the illuminated opening is bisected by the cross-hairs of the level, when the assistant reads the height below the roof. By this system of suspending the staff, it is always vertical, and where the road happens to be level and free from smoke or vapour, long sights may be taken, owing to the clearness with which the illuminated hole can be seen.

Levelling over Rough Ground by Means of Straight-edge and Spirit-level.—For rough work, or very awkward ground such as a thin seam, the telescope is sometimes dispensed with, and the surveyor uses an ordinary mason's spirit-level, with a straight-edge of convenient length, say from 6 to 12 feet, according to the steepness of the incline (see Fig. 144). One end of the straight-edge rests on the ground, rail, or sleeper; the other end is lifted up in contact with a vertical graduated rule or staff, until the bubble comes to the centre of the spirit-level on the straight-edge, when the altitude of the end of the straight-edge above the ground is read on the foot rule or other vertical graduated staff. The straight-edge is then moved down the hill, and one end placed on the spot where the vertical staff was held, and the operation repeated. This method of work admits of great accuracy, and it is obvious that the accuracy attained is in exact ratio to the care employed, providing the straight-edge is really straight, and the spirit-level is accurately constructed.

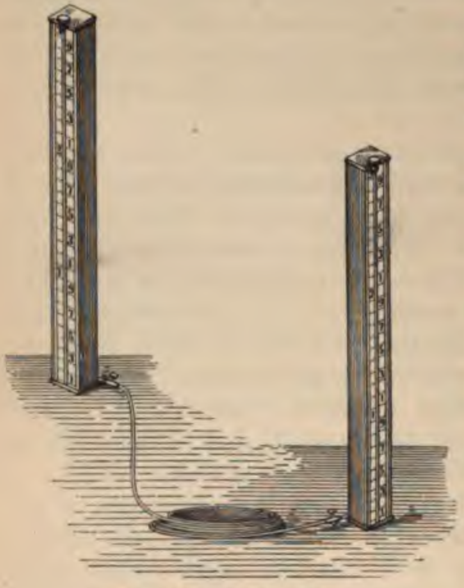


Fig. 128.—Water-level.

Water-level.—A water-level, designed by Messrs. T. L. Galloway and C. Z. Bunning, has been used.[1] This is a modification of instruments of considerable antiquity. The apparatus, as constructed by these gentlemen, shown in Fig. 128, consists of two glass tubes connected by an indiarubber tube of any convenient length, according to the steepness of the roads it is intended to level, say 10 to 20 yards. Each glass tube is fixed in a stand, and has attached to it a scale graduated into feet and hundredths; coloured water is put into the tubes, so that when the stands are both on the same level the water will be half-way up each glass tube; if one stand is now placed at a lower level, the water will rise in that tube and sink in the upper one, and the difference between the height of the water in the tubes shows the difference in level; thus if the water in the upper glass tube reads 0·54, and that in the lower glass tube reads 3·72, the difference in level is 3·18. A stop-cock is provided near the bottom of each glass tube, which is shut whilst the apparatus is being carried from station to station.

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[1] *North of England Inst. M.E.*, vol. xxvii. p. 23.

When carefully used, this instrument is capable of giving accurate results, and it is handy for getting over rough places.

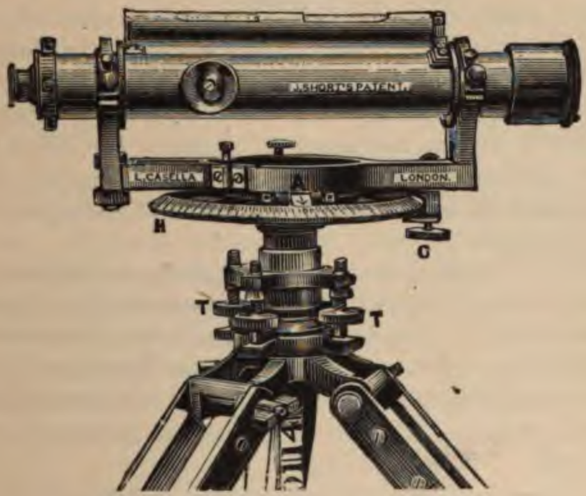

FIG. 128A.—The gradient telemeter level.

The Gradient Telemeter Level.—This ingenious instrument is intended to act both as a telemeter and as a level. Its construction may be gathered by reference to Fig. 128A. The gradient of any piece of land is ascertained by inclining the telescope to the same slope as the ground, and the distance is measured with the aid of a levelling-staff. The angle of inclination, covered by a given length of staff when observed twice, gives a measure of the distance.

The value of the instrument lies in the exceedingly ingenious manner in which these operations are performed, and in the rules and tables which are supplied with the instrument.

On reference to the drawing, it will be seen that the instrument is carried on a tripod, has three levelling-screws, and a horizontal graduated plate.

This horizontal plate revolves on a vertical axis, and carries with it the whole of the instrument. Inside this vertical axis is a second axis, which is attached to that part of the instrument which is above the horizontal plate. This upper part carries a compass by which the direction of the telescope can be ascertained; it also carries an ordinary **Y**-level. Attached to the compass-box is an index or arrow-head, **A**; and by means of the screw **G** the upper part of the instrument can be clamped to the horizontal plate **H**. When proceeding to use the instrument, the arrow **A** is clamped at zero, and the telescope is levelled in the ordinary manner, and turned in the direction of the staff. If the staff is within sight, the level of the station can be read in the ordinary way. If the staff is too low or too high, then it will be necessary to unclamp the screw **G**, and turn the horizontal plate round until the telescope is inclined (being carried on an inclined axis) to such an angle that the staff can be read.

If the horizontal plate is turned until the reading on the staff is the same as the height of the telescope above the ground, then the figure on the horizontal plate opposite the arrow-head is the gradient of the slope. If by turning the horizontal plate two observations are taken to the staff, so that the readings differ by 5 or 6 feet, a simple calculation enables the distance of the staff to be found.

The instrument that will measure the distance in this simple manner, and also give the gradient, and can be used as an ordinary level and give compass-bearings, seems to be very useful.[1]

Levelling by Angles.—As already stated in Chapter IX., pp. 170 to 180, the relative altitudes of the stations in the line of survey can be ascertained by reading the vertical angle with the theodolite, or, more roughly, with the dial or clinometer. The method may be understood by reference to Fig. 129. Here the theodolite is fixed at the top of a long but moderate slope, and at the foot of a steep incline. The line of collimation is fixed perfectly level with the vernier of the vertical circle at zero; the telescope is now directed to a staff at the bottom of the hill, on which is a cross-bar the same height above the ground as the telescope; the angle of depression read, 4° 34′, is the slope of the incline, and the distance measured is the hypothenuse of a triangle of which the horizontal distance is the base and the vertical elevation is the perpendicular, or the measured distance may be considered the radius of the arc, and the vertical distance is the sine and the horizontal distance the cosine. Half-way between the bottom of the hill and the instrument is a depression; the levelling-staff is held in this depression, and the height read, which is 12·56; the height of the theodolite was 4·4, therefore the depression is 8·16 below the line of sight. This depression can, therefore, be drawn on the plotted section. The telescope is now directed to the staff higher up the hill, and the angle read, which is 15° 14′. There are several knobs and depressions of the ground on this line which

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[1] Further particulars can be ascertained from the pamphlet, which can be obtained from the maker, Mr. L. Casella, 147, Holborn Bars, London, E.C.

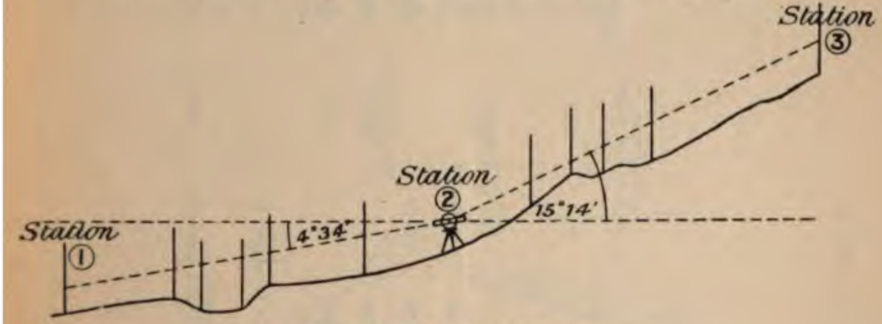

Fig. 129.—Levelling by angles.

From	To	Inclined length.	Vertical angle. R = rising. F = falling.	Staff-readings.
Station 1	Station 2	Feet. 550	4° 34′ R.	Feet. 000—4·40 270—4·40 120—4·90 420—6·00 190—7·00 550—4·40 230—8·00
" 2	" 3	490	15° 14′ R.	100—4·00 190—4·40 150—2·00 490—4·40 175—3·00
" 3	" 4	270	6° 7′ F.	Uniform slope
" 4	" 5	950	3° 17′ F.	" "

Fig. 130.—Mode of booking when levelling by angles.

From	To	Inclined length.	Vertical angles. R = rise. F = fall.	Sine of angle.	Cosine of angle.	Vertical height = sine × inclined length.		Height above datum.	Horizontal length = cos. × inclnd. length.
						Rise	Fall.		
								Feet. 50·00	Feet. 000·00
Station 1	Station 2	Feet. 550	4° 34′ R.	0·0796190	0·9968254	Feet. 43·79	Feet. —	93·79	548·25
„ 2	„ 3	490	15° 14′ R.	0·2627506	0·9648638	128·74	—	222·53	472·78
„ 3	„ 4	270	6° 7′ F.	0·1065533	0·9943070	—	28·76	193·77	268·46
„ 4	„ 5	950	3° 17′ F.	0·0572736	0·9983585	—	54·41	139·36	948·44[1]

FIG. 131.—Calculations necessary in levelling by angles.

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[1] In this case the length of the sight is over one-sixth of a mile, and the correction for curvature would be about 0·02 foot.

are measured by reading the depth below the line of sight. The instrument is now moved forward further up the hill, and fixed within sight of the last staff, and the operation repeated.

In levelling by means of angles the sight may be very long, and, therefore, corrections for curvature may be required.

The mode of booking is shown in Fig. 130, and the calculations in Fig. 131, and the plotted section in Fig. 132.

The intermediate staff-readings have not been plotted on Fig. 132, as it is drawn to too small a scale.

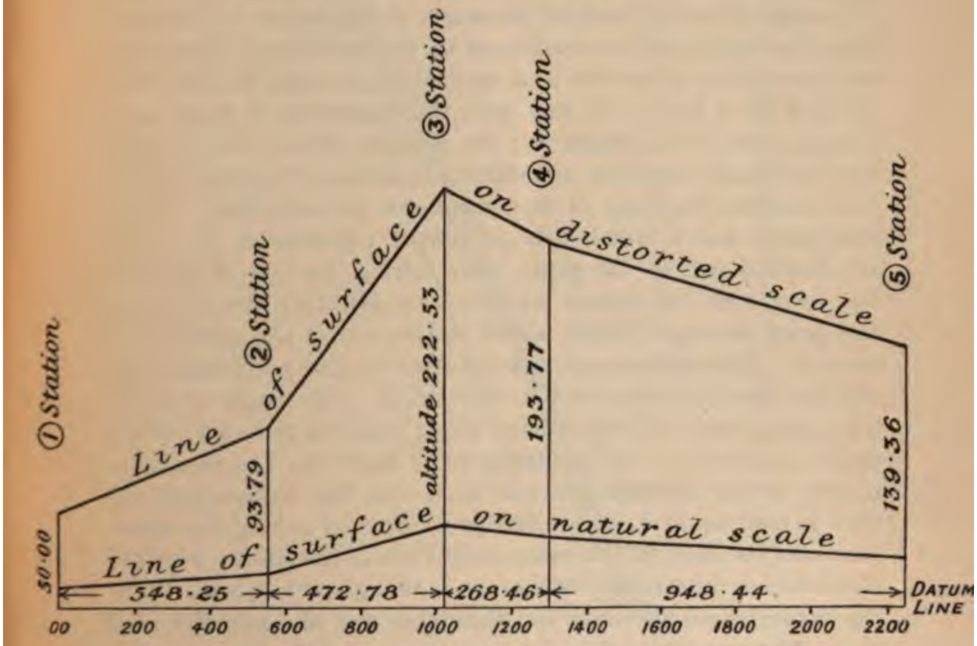

FIG. 132.—Section plotted from Figs. 130 and 131. The lower line represents the surface as plotted with both horizontal and vertical scales of 500 feet to 1 inch; the upper line as plotted with a horizontal scale of 500 feet to 1 inch, and a vertical scale of 100 feet to 1 inch.

The figure shows two sections, each plotted from the same datum. The smaller section is plotted to a natural scale; that is to say, the altitudes and the horizontal lengths are both plotted on the same scale, viz. 500 feet to 1 inch. The large section is plotted to a distorted scale, the horizontal scale being 500 feet to 1 inch, and the vertical scale 100 feet to 1 inch.

It is usual to plot sections of the surface, taken for the purpose of a proposed road or railway, on a distorted scale, in order that irregularities of the surface may be clearly shown. For such a section a horizontal scale of 2 chains to 1 inch, and a vertical scale of 20 feet to 1 inch is convenient, and a similar ratio for larger or smaller scales.

For sections made for geological or mining purposes, a natural scale is to be preferred, as a distorted scale presents a confusing and therefore a misleading picture.

Measuring Altitudes by Traversing in a Vertical Plane.—The use of the bubble-tube on the theodolite may be dispensed with if, instead of merely reading the angle of depression or elevation from the horizontal as ascertained by the spirit-level, the angles are observed as a traverse in a vertical plane using the first line of sight as a base. In this case the theodolite is fixed at *b*, looking back to the station *a*; the relative levels *a* and *b* have been previously carefully ascertained by ordinary spirit-levelling, and therefore the angle of depression can be calculated. If the instrument has a bubble-tube in proper adjustment, the angle as observed will be the same, after setting the line of collimation level with the vernier at zero, and directing the telescope to a point the same height above station *a* that the telescope is above *b*. The telescope is now reversed on the horizontal axis, and the fore sight observed to the station *c*, the angle of elevation being read, the cross-hairs being fixed on the staff at the same height above *c* as the telescope is above *b*. The theodolite is now moved forward to *c* and fixed over the station, and the staff is held up at *b*; the telescope is directed so that the cross-hair cuts the staff at the same height above the station *b* as the theodolite is above the station at *c*, though not necessarily at the same height as when the theodolite was at *b* and the staff at *c*. The vernier is now fixed at the angle of elevation read when the theodolite was at *b* looking towards *c*; if the instrument has been levelled, the cross-hairs in the telescope should now coincide with the point *b*, but if not, the telescope may be brought down on to the object *b* by means of the two screws that move the arc to which the vernier circle can be clamped. The vernier circle is now unclamped, and the telescope reversed and fixed on the upper station *d*, and the angle read. In this way, the use of the spirit-level on the telescope is merely a check, and is not essential. This plan, however, is not to be recommended in preference to the use of the spirit-level at each station, because any mistake made in the reading of any one angle is multiplied by the entire length of the survey,

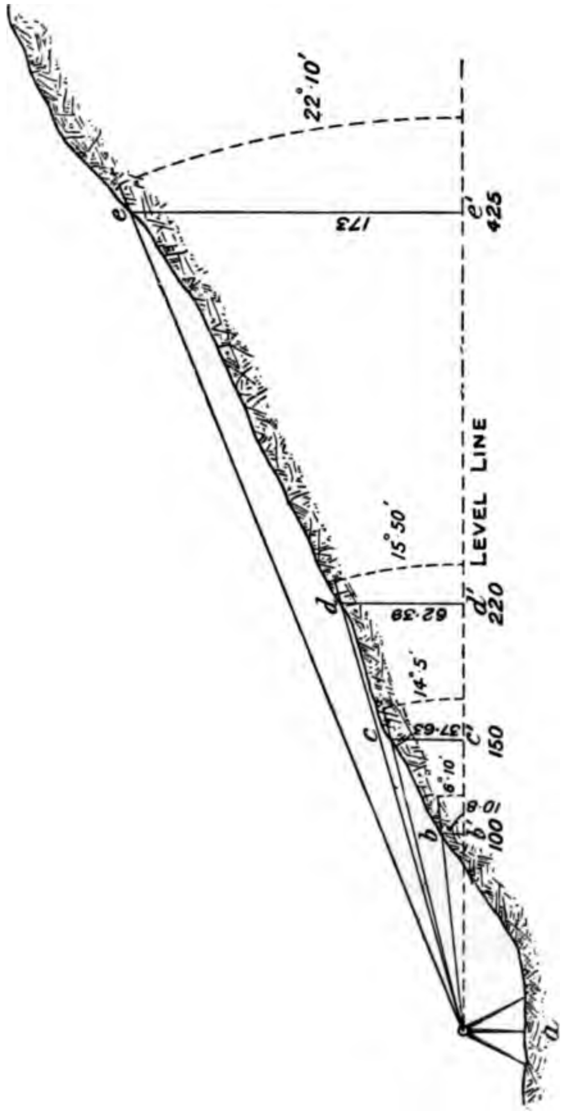

Fig. 133.—Method of obtaining heights when horizontal distances are known.

whereas if the spirit-level is used every time the instrument is set up, and all the angles measured from the horizontal at each station, the errors in reading from that station are confined to the distances measured from that station to the next. In a traverse of this kind the correction for curvature and refraction must be made as if the whole length of the traverse was one sight.

Calculation of Heights from Observed Angles, the Horizontal Distance being known.—The altitudes of various stations, the distances of which from the instrument can be determined by reference to some plan, can also be conveniently obtained approximately by the theodolite, as in Fig. 133. Here the theodolite is fixed at *a*, a known spot on the plan; *b*, *c*, *d*, and *e* are also marked on the plan; the angles of elevation may thus be read, say, *b*, 6° 10′; *c*, 14° 5′; *d*, 15° 50′; *e*, 22° 10′. The lengths *ab*′, *ac*′, *ad*′, and *ae*′, which are known from the plan, may be considered as radii of the corresponding arcs, and the vertical altitudes *bb*′, *cc*′, *dd*′, *ee*′ are tangents, the value of which is found by multiplying the natural tangent of the angle by the corresponding radius, thus—

Angle of elevation of $b = 6° 10'$ nat. tangent $= 0{\cdot}1080462 \times 100 = 10{\cdot}8$ feet
" " $c = 14° 5'$ " " $= 0{\cdot}2508734 \times 150 = 37{\cdot}63$ "
" " $d = 15° 50'$ " " $= 0{\cdot}2835999 \times 220 = 62{\cdot}39$ "
" " $e = 22° 10'$ " " $= 0{\cdot}4074139 \times 425 = 173{\cdot}15$ "

For long sights the correction for curvature must be made (see p. 214).

The method of levelling just described with the theodolite can be done with other instruments of less precision, as, for instance, by means of the dial with a vertical semicircle or circle, by means of the box sextant, by the clinometer, or by an Abney's level.

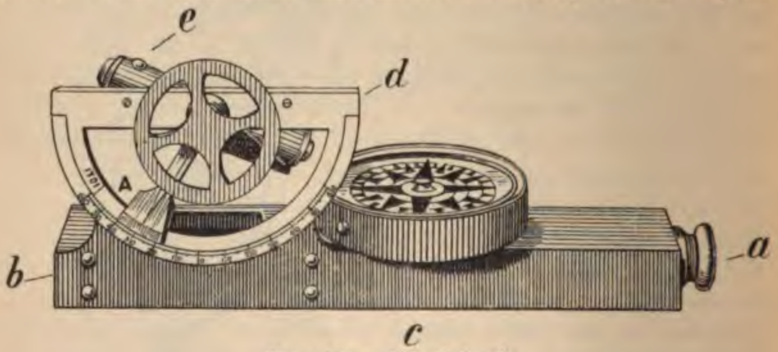

FIG. 133A.—Abney's level.

Abney's Level.—An illustration of Abney's level is given in

Fig. 133A. It consists of a tube, *c*, provided with eye-piece, *a*, and cross-hairs, *b*. Attached to the tube is a graduated semicircle, *d*, and at the centre of this is an axis carrying a small bubble-tube, *e*, which can be revolved; an index with vernier shows the inclination. Across half of the sight-tube is a reflector, adjusted at an angle of 45°, so that when the bubble is in the centre of its run, its reflection is seen by the eye of the observer. It will thus be seen that if a sight is taken to some object, and the bubble-tube is moved till the bubble appears in the centre, the vernier will record the angle of elevation or depression of the object sighted.

Advantage of Levelling by Angles.—The advantage of levelling by angles is only where the inclination is considerable; if the inclination is such that sights of 5 to 10 chains can be taken with the ordinary level, no time is gained by taking the angles; but where the inclination is such that the length of sights is reduced with a 16-feet staff to a couple of chains, the levelling process demands a good deal of time, and where—as is not infrequently the case, especially in mines—the length of sights is reduced to less than half a chain, the levelling process is a very slow one indeed. The speed of levelling by angles is, except for very steep roads, independent of the inclination, but is limited by the uniformity of the incline; the altitude of a uniform slope of any length within the clear vision of the telescope can be measured with the theodolite. This renders this mode of levelling particularly useful for geological purposes, and for preliminary surveys where minute accuracy is not required. It must be borne in mind that the longer the sights the larger the errors likely to be made.

Using Theodolite for Ordinary Levelling.—The theodolite can be used in the same way as an ordinary level by clamping the vertical circle at zero and bringing the bubble level in the usual way with the screws on the parallel plates or tripod.

Contouring.—Contour lines are marked upon some of the maps of the British Ordnance Survey. A contour line is so called because it is a level line which, like a canal, follows the contour of the surface. A contour line may be marked out on the surface of the ground in the following manner: Let the level be fixed at any point, say *a* (Fig. 134), and let the staff be held at the point *b* upon a peg, the top of which is nearly level with the surface of the ground, and which is, say, 225 feet above the sea. The cross-hairs read the figure 10·2 on the staff. The staff is now moved in the direction of the point *c*, which is distant, say 1 chain. The assistant holds the lower end of the staff close to the surface of the ground, and walks up and down hill as directed by the surveyor until the cross-hair of the telescope is in line with the figure 10·2 on the staff; the ground is here 10·2 feet below the level of the telescope, and therefore it is at the same level as the top of the peg *b*. A peg may be driven down here. The staff is then moved to another point, *d*, and the place is found where the cross-hair of the telescope reads 10·2, as before, when another peg is put down. In this way as many pegs are put down as are within sight of the telescope on the same level. A survey may subsequently be made of these pegs, and their positions marked on the plan; a line drawn on the plan from peg to peg will be a contour line. The surveyor, having carried this line as far as it is required, will then level up the hill, say 25 feet, and fix a peg at *m*, 25 feet above the peg *b*; he will then proceed to range a line of pegs, *m*, *n*, *o*, etc., which are on the contour line 250 feet above the sea.

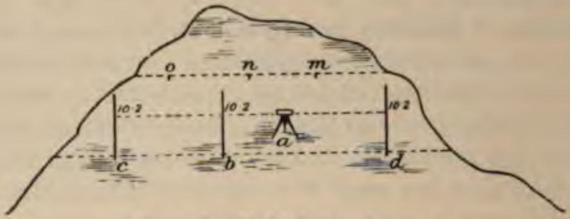

FIG. 134.—Method of contouring.

In the same way, contour lines may be shown on a mining plan, but since the view of the surveyor in a mine is confined to the narrow road in which he stands, the only method of contouring is to take levels of each road and mark them in writing in the way shown in Fig. 135. Here every change of level of 10 feet is marked with a dot, and the altitude shown in figures, the figures giving either the depth below some station, such as the shaft-top, or else the Ordnance datum is used, the correct distance of the shaft-bottom above or below Ordnance datum having first been carefully obtained.

All the marks of equal altitude may now be connected by lines. It is obvious that where the seam is steep these lines

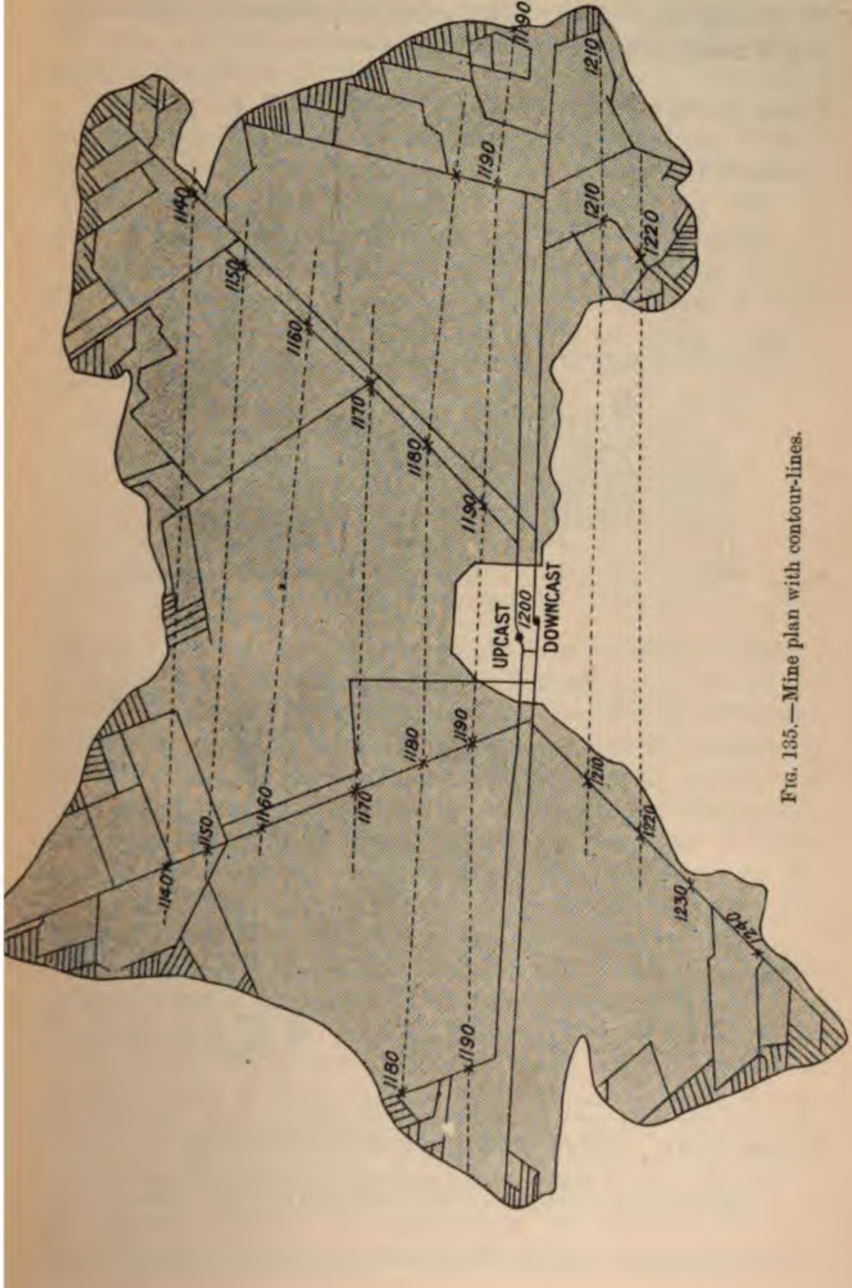

FIG. 135.—Mine plan with contour-lines.

will come close together, and where the seam is flat they will be a long way apart. The universal practice of contouring would result in many economies.

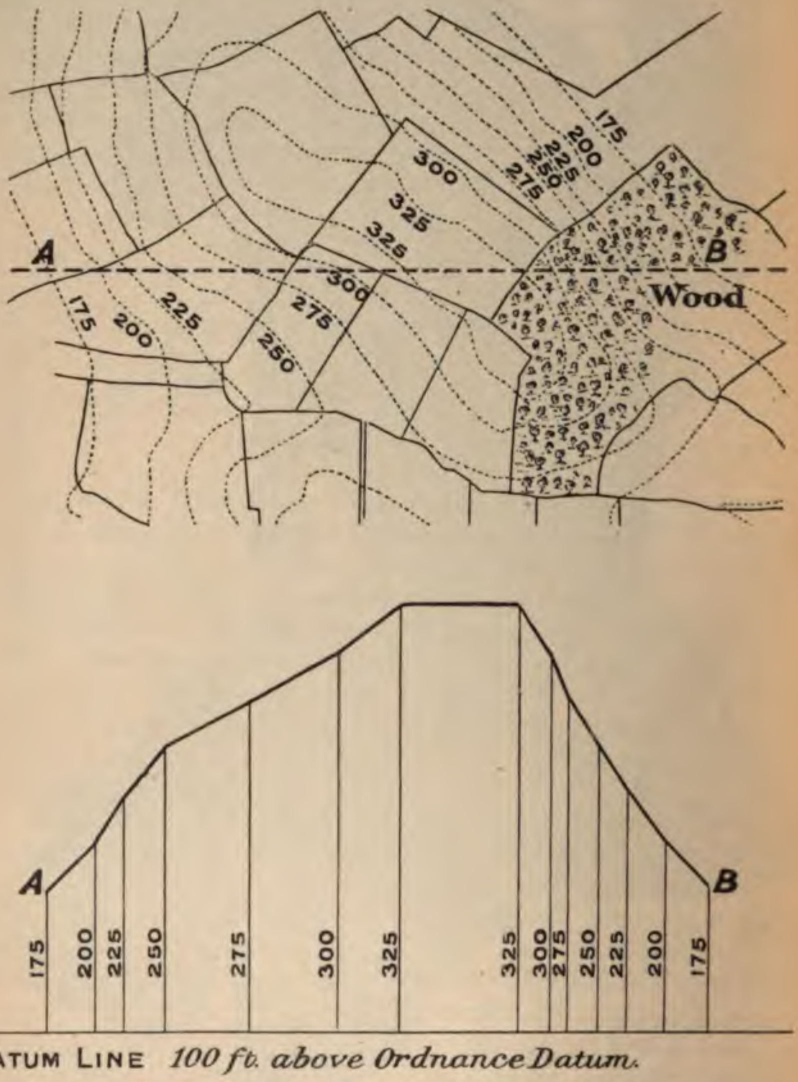

FIG. 136.—Plan of surface, showing contours and section plotted from same. Horizontal scale, 6 inches to 1 mile; vertical scale, 1 inch = 100 feet.

The surveyor should know how to draw a section from a contoured plan. Fig. 136 shows a plan of the surface with the surface contours in dotted lines; a line is marked across the plan, and the corresponding section showing the changes in level is also drawn.

Levelling by Barometer.—The barometer is of great use to explorers, enabling them to ascertain the approximate altitude of any place above the sea-level. The theory of barometric levelling may be best understood by reference to Fig. 137. In this case a section is shown of part of the earth's surface with the atmospheric covering. The thickness of the air, though liable to occasional variation, is usually fairly constant at all seasons of the year, and the weight of the air at the sea-level is on the average about 14·7 lbs. to the square inch, or nearly 15 lbs. This is equal to the weight of a column of mercury 1 inch square and 30 inches high. (Mercury weighs (at 32° F.) 0·491 lb. per cubic inch.) Owing to atmospheric disturbances, the thickness of air over any particular place is occasionally reduced about 10 per cent., or say down to 28 inches, and occasionally increased to nearly 31 inches. A very low reading, however, seldom lasts long, the ordinary variation in Great Britain at sea-level not exceeding 3 per cent., or say the barometer is between 29 and 30 inches, the average height of the barometer at sea-level being, say, 29·95 inches.

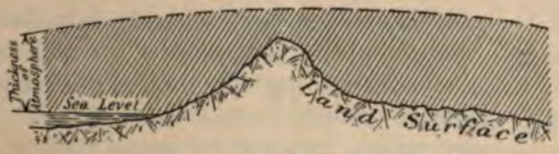

Fig. 137.—Theory of barometric levelling. The shading represents the atmosphere.

On ascending, Atmospheric Pressure falls; descending, Atmospheric Pressure increases.—It is evident that if the observer ascends a hill, he will have a certain weight of air below him varying with the elevation he has attained, and since the total weight of air is constant, the weight of air above him must be correspondingly reduced. If, therefore, he can ascertain the weight of air above, he can, by subtraction from the total weight, obtain the amount of air below him; this is the method of barometric levelling. At each station, the altitude of which is required, the observer measures the weight of air above, then subtracts that quantity from the total weight of the atmosphere at sea-level, the difference being the weight of air between the station of the observer and sea-level.

Explanation of Barometer.—The barometer is simply a weighing-machine applied to weighing the atmosphere. The weight of a column of air is equal to the pressure of a column of equal size over the area of the base of the column; thus if a column of air the height of the atmosphere and 1 inch square has a pressure of 15 lbs., then 15 lbs. is the weight of a column of air of which the cross-section is 1 square inch and the height is equal to the height of the atmosphere; thus in ascertaining the pressure of the air we ascertain its weight.

The mercurial barometer (see Fig. 138) consists of a long tube, say 36 inches long, one end of which is sealed, and the other bent round and enlarged to form a cup. The tube is placed vertically with the top of the cup upwards; the cup is filled with mercury, or quicksilver, as it is sometimes called. By a process which it is not necessary here to describe, all the air has been removed from the tube, but the air is present on the surface of the mercury in the cup, and presses it down. As the tube is curved, the mercury, as it goes down from the cup, must rise up the long vertical leg, and it continues to rise until the weight of mercury in the tube above the level of the mercury in the cup has a pressure per square inch equal to the pressure of the atmosphere per square inch.

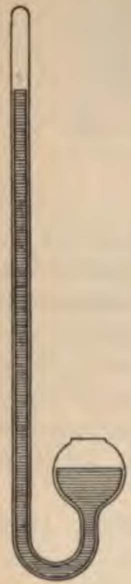

FIG. 138.—Simple barometer.

It will be understood by the student that up to the level of the top of the mercury in the cup, the mercury in the tube and the mercury in the cup balance; above that level on the cup side there is no mercury, and on the tube side there is no atmosphere; therefore the mercury in the tube has to balance the atmosphere. The cup is made very large as compared with the tube, in order that a great variation in the height of the column of mercury in the tube may take place with a very small variation in the height of the mercury in the cup. As already stated, at the sea-level 30 inches of mercury (29·95 inches in England) balance the atmosphere on the average. At a higher level, say 1000 feet, the column of air above the cup is less, consequently the pressure of the air on the top of the cup is less, and a less column of mercury is required to balance this pressure; the mercury therefore falls to the extent of the weight of the 1000 feet of air which are below the cup. This weight, if the barometer reading at sea-level is 30 inches, and the temperature 52°, will be equivalent to a pressure of about 0·541 lb. per square inch, equal to a column of mercury about 1·1 inch high, and the barometer therefore will read 30 inches − 1·1 inch, or about 28·9 inches. Therefore, if the barometer reads 28·9, the observer knows that there is a weight of air below him equal to 1·1 inch of mercury, which, if the temperature is 52°, and the barometer at the sea-level 30, is equal to a column of air about 1000 feet in height.

It is, however, necessary to correct the above calculation by the consideration that the column of air 1000 feet high will not have the same density throughout; thus, whilst 1 cubic foot at the sea-level would weigh 0·077 lb., 1 cubic foot at the 1000-feet level would weigh 0·0753 lb., and the average weight of a cubic foot of air in the whole distance would be the mean of these two figures, or 0·07615 lb., and the weight of a column of air 1000 feet high and 1 foot square at the base would be 76·15 lbs., dividing this by 144, we get the weight of a column of air 1 inch square at the base, which is about 0·529 lb.

Assuming the mercurial column at the sea-level to be 30 inches, then the fall of the column at the 1000-feet altitude will be found by the following rule of three sum: $14{\cdot}7 : 0{\cdot}529 :: 30 : x$. Here $x = 1{\cdot}08$ inch.

In a similar manner, we can calculate the amount the barometer will fall for any other elevation more or less than 1000 feet, and also for any depression. The student will readily see that here is a means of calculating the height that a barometer is raised or the depth that it is depressed by the corresponding fall or rise of the mercurial column; thus, should he walk up a mountain and observe that the barometer has fallen from 30 inches when he was at the base to 28·92 inches at the top, he knows that the height of the mountain is 1000 feet, that is to say, assuming that the temperature of the air is the same as in the previous observations, namely, 52°. Should, however, the temperature be different, then the height will not be 1000 feet, but it will be more or less than 1000 feet, because 1000 feet of air at 42° weigh more than the same volume at 52°, and, of course, would have a greater effect upon the mercurial column; thus, in calculating the height of a hill or the depth of a pit, it is absolutely necessary always to take the temperature of the air, not only at the upper and lower stations, but at intermediate places, so as to arrive at the mean temperature of the air.

The correction for temperature must be made in accordance with the following ascertained rules. If air having a temperature of 0° F. is heated to a temperature of 1° F., it will expand $\frac{1}{459}$ of its volume, and if it is heated to any other temperature, say 100°, the expansion will be in the same ratio, or $\frac{100}{459}$ of its volume. If a volume of 459 cubic feet of air at 0° is raised to a temperature of 1° (pressure constant), it will occupy a volume of 460 cubic feet; if it is raised to a temperature of 100°, it will occupy a volume of $459 + \frac{100}{459} \times 459 = 559$. In this way the relative volumes of the same weight of air for any difference in temperature can be at once ascertained by adding the observed temperature to 459. Thus taking four temperatures—0°, 32°, 41°, 71°—the volumes would be 459, 491, 500, and 530, and the relative densities will be in the inverse ratio; thus the weight of 1000 feet of air at 32° : weight of 1000 feet of air at 41° : : 500 : 491; and again, the weight of 1000 feet of air at 32° : weight of 1000 feet of air at 71° : : 530 : 491.

As freezing-point is a temperature that can be easily verified, the expansion of air for an increase in temperature of 1° F. is often spoken of as the $\frac{1}{491}$ part of its volume at freezing; it would be quite as convenient to say that the expansion of air was $\frac{1}{500}$ part of its volume at 41° for every increment of 1°, 500 being a much more convenient figure for division than 491 or 459.

Supposing that, in observing the barometer on the hill referred to, the temperature was found to be 60° at the bottom and 58° at the top, or an average temperature of 59°, or 7° higher than the previously assumed temperature of 52°, then the air will, of course, have expanded: at 52° temperature the volume of the air will have expanded from zero $\frac{52}{459}$, making a volume of 511; if the temperature rises to 59°, the expansion will be $\frac{7}{511}$ of its volume at 52°, increasing the volume to 518. The density of the air is inversely as the temperature, thus the density of the air at 59° : the density of the air at 52° : : 511 : 518;

then the height of the column will be increased in the same proportion: 511 : 518 : : 1000 : 1013·70 feet. If, however, the temperature, instead of being increased from 52° to 59°, had been decreased to 41°, the density of the air would have been increased in the ratio of 500 to 511, and the height of the column would have been decreased in the same ratio, that is to say, 511 : 500 : : 1000 : 978·47 feet.

Compensated Barometers.—The mercury itself is affected by temperature; thus a column of mercury 30 inches high and 70° temperature weighs much less than a column of mercury 30 inches high and 32° temperature, therefore the barometric readings must be corrected for temperature. Mercury expands with great regularity, the expansion between freezing-point and boiling-point, that is, between 32° and 212° F., or a rise of 180°, is 0·018153, about $\frac{1}{55}$, or, taking a column 30 inches high, $\frac{30}{55}$ inch, and the expansion for 18° would be $\frac{3}{55}$ inch; thus between freezing-point and 50° temperature, the barometric column would rise $\frac{3}{55}$ inch, whilst the atmospheric pressure remained constant. For a rise of 1° the expansion would be 0·0001, or $\frac{1}{10000}$; for a column 30 inches high the expansion for 1° would be $\frac{30}{10000}$ or 0·003; thus, the correction for every rise of temperature of 1° above the standard is approximately a reduction of $\frac{1}{333}$ for every degree, that is to say, assuming the standard temperature to be 52°, and the actual temperature 53°, and the barometric reading 30·003 inches, this reading must be corrected to 30 inches. If, however, the actual temperature were 51°, and the barometric reading was 29·997, this reading must be increased by the addition of 0·003, which would make the correct reading 30 inches.

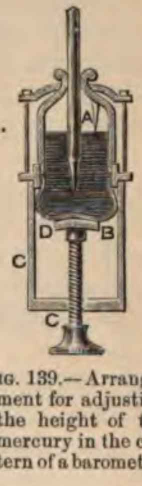

FIG. 139.—Arrangement for adjusting the height of the mercury in the cistern of a barometer.

Some mercurial barometers have a means of correction for temperature by adjusting the height of the mercurial cistern for various temperatures; this, however, is not usual. A carefully made barometer generally contains the mercury in a glass cup, which allows of the level of the mercury within being seen. The bottom of the cylinder is made of flexible leather, **DB**, and can be raised or lowered by the screw **C** (Fig. 139). At the top of the cup is an ivory pointer, A, and before taking an observation, the level of the mercury is carefully adjusted to this mark, which corresponds with zero on the scale, and the correction for temperature is made by calculation.

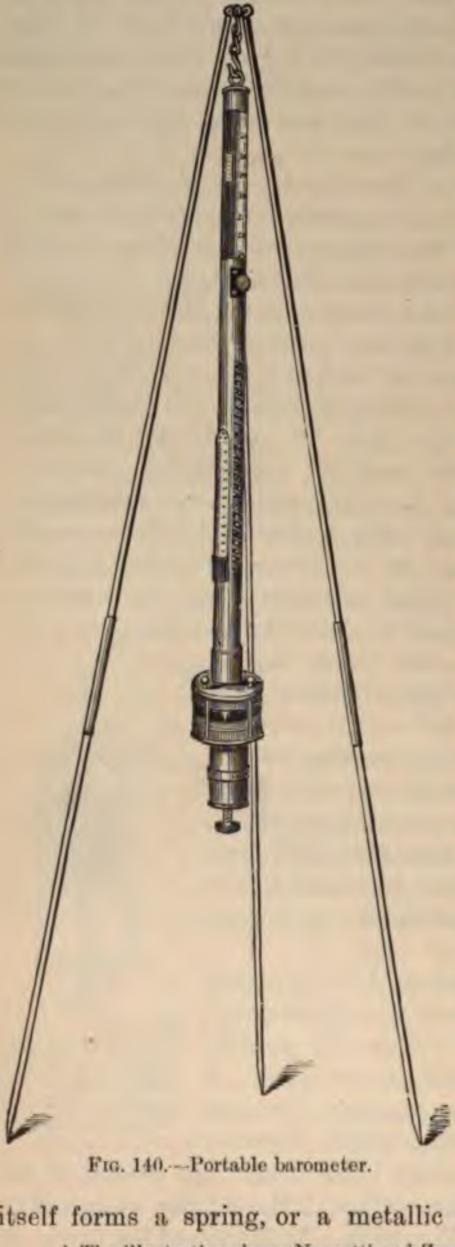

FIG. 140.—Portable barometer.

Portable Barometer.—In order that the barometer may be portable, the flexible diaphragm is raised by the screw until the mercury is pressed to the top of the cup, the opening to the atmosphere being closed; by the same operation the long glass column is also filled with mercury, so that there is nothing to shake about. The barometer is fixed in a strong wooden or metal tube, and a light tripod stand is carried by which it can be suspended in a vertical position (see Fig. 140).[1]

Aneroid Barometer.—The most portable form of barometer is called an aneroid. In this case, instead of the mercurial tube, there is a metallic box from which the atmosphere has been partially exhausted; the cover of the box either itself forms a spring, or a metallic spring is attached to the

[1] The illustration shows Negretti and Zambra's mountain barometer.

cover and prevents it from collapsing. When the pressure of the atmosphere increases, the box-cover is pressed in; when the pressure of the atmosphere decreases, the spring causes the box-cover to come out. By means of multiplying-gear, the movement of the box-cover is shown by a pointer on a dial-plate. These instruments may be made to work with great nicety and regularity; but they must be carefully tested from time to time by means of a mercurial barometer. The instrument is made in sizes from 2 inches diameter up to about 5 inches diameter; if the case is made of aluminium, the latter size is quite portable, and is suitable for levelling operations, either on the surface or in the mine, in cases where an error in level of say 10 or 20 feet is not material.

With a well-tried 5-inch aneroid, levellings may be taken to within 10 feet of the correct level, and if the levellings are repeated four or five times, the error may be reduced from 10 feet to 2 or 3 feet, or even less; but it must be remembered that whilst in some cases the levelling with the aneroid is correct to a foot, in other cases, even with the best aneroid, there may be an error of 10 feet, and therefore this instrument must not be used where great accuracy is required.

Fixed-scale Barometer.—It is convenient for the surveyor to have an approximate rule always ready, either in his head or in his note-book, and for this purpose a scale may be calculated to suit the average temperature. English aneroids are usually fitted with a scale showing the altitude in feet above the sea-level for any given barometric pressure with an average atmospheric temperature of 50°.

50° is rather more than the average temperature of the air on the surface of the earth in the latitude of Yorkshire, taking the average of winter and summer, the actual average at Bradford being 49·3°.[1] At Greenwich the mean average temperature, according to Glaisher, is 49·2°; Dresden, 47·3°; Moscow, 38·5°; Rome, 59·7°; Jamaica, 79°.[2]

The mechanism of the best aneroids is compensated for variations of temperature in the instrument itself, so that the reading will be the same, whether it is inside a warm room or out of doors in the frost, provided that the atmospheric pressure is the same in each case. When the atmospheric temperature happens to be 50°, no calculation at all is necessary in levelling with this instrument (that is, with the altitude scale attached as named above), except the subtraction of the lesser height from the greater, to show the difference in level; thus, if station A reads 324 feet on the barometric scale, and station B 560 feet on the same scale, the difference in level is 560 − 324 = 236 feet.

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[1] Table published by Messrs. J. McLandsborough, M.I.C.E., etc., and A. E. Preston, M.I.C.E.

[2] Box on *Heat*.

The difference of atmospheric pressure, however, between two different stations is less at high temperatures than it is at low ones, consequently the scale needs correcting. The variations of altitude shown by the fixed scale at a less temperature than 50° are too great, and at higher temperatures are too small. Fig. 140A shows the correction.

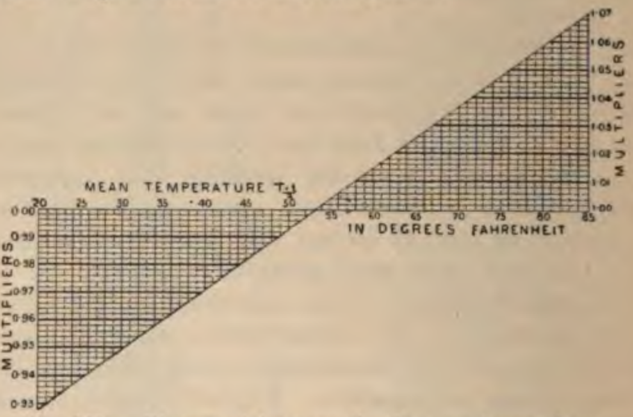

FIG. 140A.—Diagram giving correction for mean temperature.
(From Gribble's *Preliminary Survey.*)

It will be seen that with the mean temperature at 53° no correction is necessary. With the mean or average temperature as 85°, the reading on the scale of the aneroid must be multiplied by 1·07 in order to get a correct altitude.

Levelling by Boiling-point Thermometer.—The temperature at which water boils in an open vessel is dependent on the pressure of the atmosphere, so that when the atmospheric pressure is less, the temperature of boiling water, or the temperature of steam at the atmospheric pressure, is also less; and inversely, when the pressure increases, the temperature at which water boils, or the temperature of the steam at atmospheric pressure, is also greater. Thus, whilst under the ordinary atmospheric pressure water boils when it is heated to 212°, if this water is put into a receiver in which the atmospheric pressure is reduced by means of an air-pump to say 5 inches of mercury, then it will boil at a temperature of 134°. On the other hand, if water is put into a well-made steel boiler, and subjected to a pressure of ten atmospheres, it will not boil until a temperature of 357° F. is reached. This quality of water (and other liquids) has been utilized for the purpose of measuring the atmospheric pressure, numerous experiments having determined the exact temperatures at which water vaporizes for a great number of pressures.

Table XII.[1] shows the temperature at which water boils, that is, the temperature of the steam given off by the boiling water, for pressures varying between 17 inches of mercury and 31 inches of mercury.

TABLE XII.

TEMPERATURE AT WHICH WATER BOILS FOR PRESSURES VARYING BETWEEN 17 INCHES OF MERCURY AND 31 INCHES OF MERCURY.

Pressure in inches of mercury.	Boiling-point. Fahr.	Pressure in inches of mercury.	Boiling-point. Fahr.	Pressure in inches of mercury.	Boiling-point. Fahr.	Pressure in inches of mercury.	Boiling-point. Fahr.
17·048	185°	21·038	194·8°	24·492	202·1°	28·011	208·7°
18·000	187·5°	21·530	195·9°	25·000	203·1°	28·521	209·6°
18·512	188·8°	22·033	197·0°	25·517	204·1°	29·040	210·5°
19·036	190·1°	22·498	198·0°	26·043	205·1°	29·508	211·3°
19·490	191·2°	23·019	199·1°	26·523	206·0°	30·041	212·2°
20·037	192·5°	23·502	200·1°	27·012	206·9°	30·522	213·0°
20·511	193·6°	24·042	201·2°	27·507	207·8°	31·010	213·8°

Calculation of Altitude by Boiling-point Thermometer.—In order to ascertain the difference of altitudes corresponding with any difference of pressure or with any difference in the temperature of the boiling-point, the following rule, given by Theodore G. Gribble, is useful:—[2]

Rule.—Let B = temperature of boiling-point in degrees F. deducted from 212°; H = height of station above sea-level; K = 540 for a mean temperature of intermediate air of 53°, and varying as explained below. $H = KB + B^2$.

EXAMPLE.—Boiling-points, 211·37°, 210·14°; the mean temperature of the atmosphere, 82° F.: required the difference of elevation.

$$H = 540 \times 1{\cdot}064 \times 0{\cdot}63 + 0{\cdot}63^2 = 362{\cdot}37$$
$$H' = 540 \times 1{\cdot}064 \times 1{\cdot}86 + 1{\cdot}86^2 = 1072{\cdot}14$$

Ans. Difference in feet 709·77

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[1] From *Hints to Travellers.*

[2] *Preliminary Survey* (Longmans, Green, and Co.).

In the above example, K is 540. If the mean temperature had been 53°, no correction would be necessary; the mean temperature, however, is 82°, consequently K must be increased, and the multiplier is found from Fig. 140A to be 1·064, which is the correction made on account of the temperature of the air; the figure 0·63 is the difference between 212° and 211·37°; and $0·63^2$ is the square of this difference; the figure 1·86 is the difference between 210·14° and 212°.

The boiling-point thermometer is often constructed for use with a spirit-lamp and small portable boiler and telescopic tube, the whole of the apparatus fitting into a circular tin case 6 inches long and 2 inches diameter. The mode of using is shown in Fig. 141.

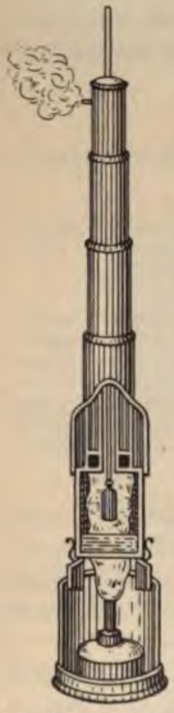

FIG. 141.—Boiling-point thermometer.

Method of Levelling by means of Barometer or Boiling-point Thermometer.—A single observation of the barometer or boiling-point thermometer does not give the altitude of any station that may be observed; it only gives the pressure of the atmosphere at that particular time, and this, as is well known, may vary from hour to hour and day to day. All that can be known from the observation of these instruments is the comparative pressure of the atmosphere at different places; thus if the surveyor starts from the sea-level at 6 a.m., observing the barometer (or boiling-point thermometer), and also recording the temperature of the atmosphere, he may proceed up or down hill, observing the barometer at every change of inclination, noting the station and atmospheric temperature; returning in the afternoon by the same route, he may again observe the instruments at the same stations as in the morning.

If it is apparent that the atmospheric pressure has been constant all day, the relative levels of all the various stations can be calculated from the observations made. It might, however, not improbably happen that on returning at night to the starting-point of the morning, the barometer reading is, say ½ inch lower than in the morning; it is obvious that all the readings made will have to be corrected for this variation in the total atmospheric pressure, and the surveyor, if working single-handed, may have means for facilitating this correction. For instance, if, at noonday, having finished his outward journey, he observes the barometer, then, remaining at the same place for one hour, he observes the barometer again, he will see if it is stationary. If it has fallen, say $\frac{1}{12}$ inch, he will note the circumstance; again, on the return journey, he will note that the barometer shows continuous signs of falling as compared with the observations made in his outward march. In order to judge of the rate at which the barometer is falling, the hour of each observation should be noted. In this way the surveyor will ascertain whether the fall in the atmospheric pressure of $\frac{1}{2}$ inch which has occurred during the day is in consequence of a regular decline or a sudden drop. If it is a regular decline, the corrections in the readings can be easily made; suppose the decline to be at the rate of $\frac{1}{12}$ inch per hour, then the readings as observed must be increased by $\frac{1}{12}$ inch for every hour that has elapsed since the first reading; if, however, the fall has occurred suddenly, say during the last hour, then all the readings taken up till then require no correction.

Levelling with Two Observers with One Fixed and One Movable Barometer.—If in the case above described a barometer had been fixed at the starting-point, and an assistant left there, he would have observed at every hour, or at more frequent intervals, the pressure and temperature of the atmosphere; and the surveyor would have been able to correct all his observations by the rise and fall of the barometer as read by his assistant. Thus, if at 6 a.m. the stationary barometer reads 30 inches, and if at 8 it reads 29·95; and at 10, 29·9; at 12, 29·85; at 2 p.m., 29·9; at 4, 29·95; and at 6, 30; the surveyor will correct his barometric readings as follows: Suppose his reading at 8 a.m. was 29, he will correct it to 29·05; if at 10 a.m. his reading was 28·50, he will correct it to 28·60; if at noon his reading was 28, he will correct it to 28·15; if at 2 p.m. his reading was 27·50, he will correct it to 27·60; if at 4 his reading was 28·50, he will correct it to 28·55, and at 6 p.m. his reading will need no correction. For any intermediate observations he will make a correction on the assumption that the variation of pressure has been going on at the same rate between the hours observed.

Levelling with Two Observers and Two Portable Barometers.—A still better method of levelling is for the assistant to follow the surveyor on his route. Before starting, two barometers and thermometers are compared, and the watches of the surveyor and his assistant set to read the same time. The surveyor now starts, and on reaching the station whose altitude he desires to measure, he plants a staff or makes a mark that can be easily recognized by his assistant; the assistant, who remains at the starting-point, observes his barometer, thermometer, and watch at the same time that the surveyor makes his observations; if they are within sight, the time for reading can be fixed by the waving of a flag; if they are not within sight, the time for reading must be made simultaneous in some other way: if the distance is not too great and the other conditions suitable, communication may be made by the discharge of a gun, otherwise the readings must be taken at times agreed upon, the assistant always reading his barometer at the stations left by his leader. In this way the observations of the pressure and temperature at the upper and lower of each pair of stations are recorded simultaneously, and the difference in level can therefore be calculated without regard to those changes in the atmospheric pressure or atmospheric temperature which might occur in the interval if the upper and lower readings were not simultaneous.

It may be difficult to effect the readings of the two barometers in all cases simultaneously; therefore, to prevent errors that might otherwise arise, the leader should fix main stations, say every quarter of an hour, so that the assistant will be at the last main station at the moment that the leader is recording his barometer at the advanced main station, the readings at the intermediate stations being taken at approximately the same time. In this way, as much accuracy is obtainable as can be expected from the instruments used, the care of the observers, and the accuracy of their calculations.

Levelling with Three Barometers.—Where the difference in level of two stations is known, a barometer may be fixed at each of these stations, and, the height being known, the density of the air can be calculated. With a third barometer, readings are taken at stations the altitudes of which are unknown, but which can be calculated from the known density of the air as recorded by the two barometers at the fixed stations; thus, two barometers being observed, say every hour or oftener, any changes in the density of the air will be noticed, and the altitude of the other stations calculated from the density ascertained at the hour of reading.

This method of levelling dispenses with the observation for the temperature of the atmosphere or for the moisture of the atmosphere, and also with corrections for gradient, if the two base stations are in the vicinity of the new stations. This method of hypsometry is fully described in a very valuable paper by Mr. G. K. Gilbert.[1]

The rules adopted for the calculations are as follows: There are three stations, lower, upper, and new, denoted by L, U, and N. The height of U above L is found exactly by spirit-levelling, and constitutes the base B; the height of the new station which is required is the height above the lower station; this height is called A. Barometric readings are now taken at all three stations, and the height of the base B may be calculated approximately on the assumption that the air is dry and has a uniform temperature of 32°; this approximate height is called *B*. The height of the new station A may also be calculated from the barometric readings on the same assumption, and this approximate height is called *A*; then the actual height of the new station A may be found from the following rule of three sum: Approximate height (*B*) of the base-line : true height (B) of the base-line : : the approximate height (*A*) of the new station : true height (A) of the new station; whence $\frac{B}{\mathrm{B}} = \frac{A}{\mathrm{A}}$. In this way we find the true height (A) of the new station.

Let A represent the true height of the new station N above L.
,, a ,, uncorrected height of the new station N above L.
,, l ,, barometric reading at the station L.
,, u ,, ,, ,, ,, ,, U.
,, n ,, ,, ,, ,, ,, N.

B represents the actual height of the base; then—

$$a = \mathrm{B}\,\frac{\log l - \log n}{\log l - \log u}$$

This is what Mr. Gilbert calls the logarithmic term of the formula, and he gives the following example: Barometric reading, station L, 29·879; station U, 23·336; station N, 27·475; altitude, B, 6989 feet.

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[1] Published in the second Annual Report (1880-81) of the United States Geological Survey.

$$
\begin{aligned}
\log l = \log 29{\cdot}879 &= 1{\cdot}47537 \\
\log n = \log 27{\cdot}475 &= 1{\cdot}43894 \\
\log u = \log 23{\cdot}336 &= 1{\cdot}36803 \\
\log l - \log n &= 0{\cdot}03643 \\
\log l - \log u &= 0{\cdot}10734 \\
\log 0{\cdot}03643 &= \bar{2}{\cdot}56146 \\
\log 0{\cdot}10734 &= \bar{1}{\cdot}03076 \\
\text{Difference} &= 1{\cdot}53070 \\
\log B = \log 6989 &= 3{\cdot}84441 \\
\text{Sum } (\log a) &= 3{\cdot}37511 \\
a = 2372{\cdot}0 \text{ feet}
\end{aligned}
$$

This result, however, has to be corrected by what Mr. Gilbert calls the thermic term; and the full formula, as given by Mr. Gilbert, is as follows:—

$$A \text{ (in English feet)} = B \frac{\log l - \log n}{\log l - \log u} + \frac{A(B - A)}{490000}$$

or—

$$A \text{ (in metres)} = B \frac{\log l - \log n}{\log l - \log u} + \frac{A(B - A)}{149349}$$

in which last formula A is the correct height.

In calculating the thermic term $\frac{A(B - A)}{490000}$, A may be taken as equal to a, the uncorrected height, to facilitate calculations, and it will be sufficiently near for most purposes.

Applying this to the figures above given—

$$\frac{A(B - A)}{490000} = \frac{2372(6989 - 2372)}{490000} = 22{\cdot}4$$

we get a correction of 22·4 feet to be added, making the total altitude A 2372 + 22·4 = 2394·4.

In order to save calculating this thermic term, Mr. Gilbert gives a table of its value for altitudes of A of 10,000 feet above and 5000 feet below the lower station of the base, and for a vertical base varying from 1000 to 10,000 feet.[1]

It must be noted that if A is a vertical distance below U, it becomes a minus quantity in the formula, and $\frac{A \times (B - A)}{490000}$ is equivalent to $\frac{-A \times (B + A)}{490000}$; but the value of A as ascertained by logarithmic term is also a minus quantity, so that the thermic correction has to be added.

When the new station is higher than the upper station U, B − A becomes negative, and renders the thermic term negative, so that the correction due to the thermic term has to be subtracted from the altitude calculated from the logarithmic term.

If N is below U, the height is minus, and the correction, being also minus, is added.

Where minute accuracy is not required, the thermic term may be disregarded, and the altitude calculated from the formula $a = B\frac{\log l - \log n}{\log l - \log u}$. The correction for the thermic term varies from 0 up to about 2 per cent. When A and B are equal, and on the same level, there is no correction, and the required correction increases as the difference between A and B increases. Thus if B is 1000 feet and A is 100 feet above L, the correction is +0·2 feet, or $\frac{1}{5}$ per cent.; when A is 500 feet, the correction is 0·5 feet, or $\frac{1}{10}$ per cent.

Temperature of the Atmosphere.—This is difficult to ascertain, owing to the difficulty of placing the thermometer in a place free from the effects of radiation from hot or cold objects. Thus a thermometer placed in the shade at 8 a.m. near a north wall might give the reading less than that due to the temperature of the air owing to the coldness of the wall which had been cooled down during the night; again, a thermometer placed in the shade near to a south wall might give a reading higher than that of the temperature of the air due to radiation from the wall which had been heated by the sun's rays. In the same way, a thermometer placed in the shade near to the ground may be cooled by radiation to the earth, which is cold owing to the coolness of the night, or the thermometer may be raised above the temperature of the air by the radiation from the earth, which has been heated by the sun's rays.

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[1] The reader is referred to Mr. Gilbert's paper for this table, as it is too large to insert here.

But the difficulty of obtaining the temperature of the air within 4 or 5 feet of the ground is by no means the only difficulty or the chief difficulty. What is really required is the temperature of the air above the ground for a height of several hundred or several thousand feet, and a surveyor walking along the surface of the ground has no chance of measuring this. Walking up a hillside (see Fig. 142), the surveyor measures the temperature of the air within say 4 feet of the ground, the ground, having been greatly heated by the sun's rays, has warmed the air; the average temperature from **A** to **B** is say 65°, while the average temperature **A** to **C** is unknown; but this is the temperature which is really required. On a cloudy day and on a windy day the temperature **AB** is likely to approximate to the temperature **AC**; on a calm, bright day the temperature **AC** will be much less than the temperature **AB**. In clear weather the temperature of the ground during the sunshine is much greater than that of the air, and during the night is much colder than the air; but it is probable that the average temperature day and night **AB** approximates to the average temperature day and night **AC**.

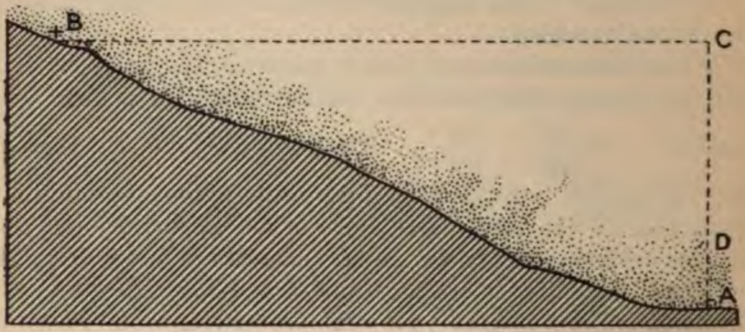

Fig. 142.—Variation in temperature of air.

According to observations made in Switzerland, and calculations made by Plantamour, Rühlmann, and others, quoted by Mr. Gilbert, the average range of temperature in the body of the air in Switzerland is, in summer, 4° F. between the early morning and noon, and in winter less than 2° F., whilst near to the ground the range of temperature of the air varies from 10° to 20° at the seashore, and from 20° to 35° in the interior of continents between the hottest and coolest periods of the daytime.

It follows, therefore, that where there is a great variation in the atmospheric temperature between the night and day, it would be better to take the mean temperature of the 24 hours than to take the temperatures observed in the daytime, though a more correct result would be obtained by making a correction for noon or sunrise, according to the figures above quoted for Switzerland.

These corrections must be applied to the mean temperature of the air as ascertained by readings day and night; thus, if the observations are made in January, and the mean temperature of the air near the ground day and night is say 37·5°, this might be taken as the temperature of the air at sunrise, and the temperature at noon as 39·5°. If the observations were taken in August, and the mean temperature of the air day and night were 62·5°, this temperature should be corrected by the addition of 4° for observations made in the warmer part of the day.

The difference of altitude between stations **A** and **B** may, however, be taken by a series of readings, A, d, d^1, d^2, d^3, d^4, and so on; the height from **A** to d is say 100 feet, and it is evident that the temperature of the air in this stratum will more nearly approximate to the temperature of the air near the ground

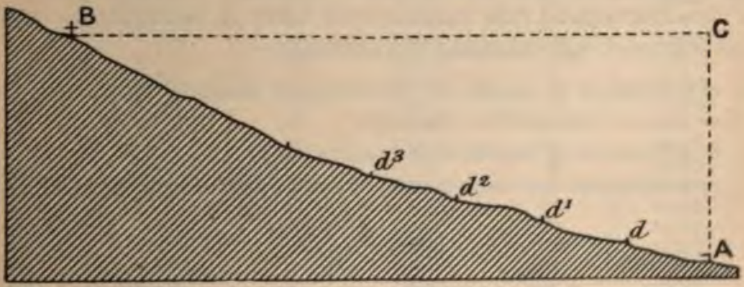

FIG. 143.—Method of taking barometrical observations to avoid error due to temperature.

than will the temperature of the air in stratum **AC**, which is 1000 feet high.

One method of obtaining the temperature of the air is given by Nansen (*First Crossing of Greenland*), who tied the thermometer to a string, and then whirled it in a circle in the air, thus forcing it into such contact with numerous particles of air as to minimize the effect of radiation either from the sun or from snow, and obtained with great accuracy the temperature of the air, within say 8 feet of the ground.

It is comparatively easy to obtain the temperature of the air in a mine. If the surveyor proceeds say 20 feet down the downcast shaft, where there is a rapid current of air, the effect of the sun's rays or of frosty skies will be but trifling; a thermometer held in the air-current will probably give the temperature of the air, and the temperature of the air here given will be approximately the temperature of the air outside in the sunshine. Assuming, of course, that the velocity of the air is considerable, say 500 feet a minute or more, a thermometer held in the air will give approximately the temperature of the air, unless it is held in view of a fire.

The surveyor must be cautioned that, on the surface, the observation of the temperature of the air is the most difficult observation he has to make; as regards the temperature of the barometer there is no difficulty.

Rule for calculating the Difference in Height of Two Stations from Barometric and Thermometric Readings.—Mr. Gribble gives a very convenient rule and a useful table of constants by which the surveyor can calculate the altitudes—

H = difference of height in feet between stations.
S = sum of barometric readings.
D = difference of barometric readings.
K = a constant for each degree of temperature from zero to 102°.

$$H = \frac{K \times D}{S}$$

As above said, K varies with the temperature of the air, that is to say, the average temperature of the column of air between the two stations. This average or mean temperature[1] is found by adding together the readings of the thermometer taken at the two stations and at equidistant intermediate places, and dividing their sum by the number of readings. Thus, if the reading at the lower station is 50°, and at the upper station 60°, their sum is 110; this, divided by 2, gives 55°, the average temperature. Or again, if the lower reading is 50°; reading one quarter of the way, 48°; reading half of the way, 45°; reading three quarters of the way, 43°; reading at the upper

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[1] In case the variation of level is rapid and great, the readings at each station are averaged to give the day and night temperatures, so as to get the real temperature of the air column.

TABLE XIII.

VALUE OF K IN FORMULA $H = \frac{K \times D}{S}$

Degrees of mean temperature, Fahr.	K.	Degrees of mean temperature, Fahr.	K.	Degrees of mean temperature, Fahr.	K.
0	48753	35	52813	69	56757
1	48869	36	52929	70	56873
2	48985	37	53045	71	56989
3	49101	38	53161	72	57105
4	49217	39	53277	73	57221
5	49333	40	53393	74	57337
6	49449	41	53509	75	57453
7	49565	42	53625	76	57569
8	49681	43	53741	77	57685
9	49797	44	53857	78	57801
10	49913	45	53973	79	57917
11	50029	46	54089	80	58033
12	50145	47	54205	81	58149
13	50261	48	54321	82	58265
14	50377	49	54437	83	58381
15	50493	50	54553	84	58497
16	50609	51	54669	85	58613
17	50725	52	54785	86	58729
18	50841	53	54901	87	58845
19	50957	54	55017	88	58961
20	51073	55	55133	89	59077
21	51189	56	55249	90	59193
22	51305	57	55365	91	59309
23	51421	58	55481	92	59425
24	51537	59	55597	93	59541
25	51653	60	55713	94	59657
26	51769	61	55829	95	59773
27	51885	62	55945	96	59889
28	52001	63	56061	97	60005
29	52117	64	56177	98	60121
30	52233	65	56293	99	60237
31	52349	66	56409	100	60353
32	52465	67	56525	101	60469
33	52581	68	56641	102	60585
34	52697				

station, 40°; then the sum of the readings = 50° + 48° + 45° + 43° + 40° = 226; this, divided by 5 = 45·2°, the average temperature. In Table XIII. the column of mean temperature is the average temperature so found.

It will be seen that the value of K varies from 48753 at 0° F. to 60585 at 102° F. This shows the importance of the temperature-readings; this, of course, is an extreme range. At 50° temperature the value of K is 54553; at 60°, 55713, showing a variation of 2 per cent. in the value of K for a change of 10°.

EXAMPLE.—If the barometric reading at the upper station was 25·5 inches
" " " lower " 30·0 "
S = sum of readings 55·5 "
D = difference 4·5 "

Temperature at upper station 50°
" " lower " 65°
115°
Mean temperature 57·5°

K for 57° = 55365
K for 58° = 55481
$2\,\overline{)\,110846}$
K for 57·5° 55423

$$H = \frac{55423 \times 4·5}{55·5}$$
H = 4493·7

If we assume a mean temperature of 50°, then $H = \frac{54553 \times D}{S}$; if D = 1 and S = 60, the upper reading being 29·5 and the lower reading 30·5, then $H = \frac{54553}{60} = 909·21$. Let us assume that D = 1 and S = 59, that is to say, that the upper station reads 29, and the lower station 30, which is a very usual set of readings, then $H = \frac{54553}{59} = 924·62$.

Or again—

D = 1 S = 68 $H = \frac{54553}{68} = 802·25$
67 $\frac{54553}{67} = 814·22$
66 $\frac{54553}{66} = 826·56$
65 $\frac{54553}{65} = 839·27$
64 $\frac{54553}{64} = 852·39$
63 $\frac{54553}{63} = 865·92$
62 $\frac{54553}{62} = 879·88$
61 $\frac{54553}{61} = 894·31$
60 $\frac{54553}{60} = 909·21$
59 $\frac{54553}{59} = 924·62$
58 $\frac{54553}{58} = 940·56$

$D = 1$	$S = 57$	$\frac{54553}{57} = 957{\cdot}07$
	56	$\frac{54553}{56} = 974{\cdot}16$
	55	$\frac{54553}{55} = 991{\cdot}87$
	54	$\frac{54553}{54} = 1010{\cdot}24$

This table gives the difference in height corresponding to a difference of 1 inch in the barometric readings for a mean temperature of 50° F. at different altitudes. It will be seen that the less the pressure—that is to say, at great altitudes, 1 inch of pressure represents a much greater altitude than at great depths. The height due to a difference of pressure of less than 1 inch can be easily calculated; thus, if the difference in pressure is 1·4 inch, and the upper station is 28·6 inches, we take the altitude due to a difference of 1 inch of pressure between 29 and 30 inches; then $\frac{4}{10}$ of the altitude due to a difference of 1 inch between 28 and 29 inches. To correct for temperature, if the temperature exceeds 50°, we increase the altitude for every 1° F. above 50°, 2 per 1000, or 1 in 500. Thus, if the altitude as calculated without the correction for temperature was 500, and the temperature was found to be 51°, the real altitude would be 501; if the temperature were found to be 60°, the real altitude would be $500 + \frac{2 \times 10}{2} = 510$. If, on the other hand, the temperature should be found to be 49°, the column must be reduced by 2 per 1000, so that a 500-feet column, as calculated without correction for temperature, would be really 499; if the temperature were 40°, the 500-feet column should be corrected to 490, and so on.

Measurement of Vertical Shafts.—The determination of the depth of a vertical shaft may be done in one of several ways. A rough way is to let a cord down the shaft; holding the lower end at the bottom, pull it tight, mark the top of the shaft, then, drawing the cord to the surface, measure it; this is inaccurate, owing to the stretching of the cord and the contraction that may follow from wetting.

A more accurate mode is to let a wire down the shaft, with a weight at the end. The wire should be unrolled from a barrel, and, as it is lowered, it should be measured on the surface in convenient lengths of say 50 feet, the wire being stretched by the weight all the time. The wire should also be remeasured as it is rolled up.

Another method is to measure the winding-rope in convenient lengths by means of a steel tape or other accurate measure as it is being wound up and lowered down.

A fourth method is to measure the shaft-guides or other smooth continuous surface. If the guides are of wood, a nail may be driven in at the surface and a chain or steel tape suspended; at the bottom of the chain a second nail is driven in, and the chain lowered down and suspended from this second nail. If the second nail is driven in just below the last ring, so that the end of the chain just touches the top of the nail, it is evident that from the top of the first nail to the top of the second nail will be the extreme length of the chain minus the thickness of the ring at the top end of the chain by which it is suspended. When the chain is suspended by the second nail, and a third nail driven in just below the chain, but so that the last link can just touch it, it is evident that the length from the top of the second nail to the top of the third nail will be the length of the chain minus the thickness of the top ring of the chain by which it is suspended; therefore, the length as recorded will be greater than the actual length by the thickness of this ring multiplied by the number of times the chain is suspended, therefore the recorded length must be reduced by that amount. If the chain or measuring-tape used is accurate, the measurement obtained in this way will be accurate.

The measurement is facilitated by a contrivance described by Mr. B. H. Brough. At the length of a chain or other measure above the cage, a seat is fastened to the winding-rope, on which a miner can sit and hold the upper end of the chain or steel band to marks made on the guide. The cage having been lowered down the shaft the length of the measure, the surveyor applies the lower end of the chain to the guide, and marks the place carefully; the cage is now lowered down the chain-length; the miner holds the top end of the chain to the first mark, whilst the surveyor makes the second mark below; this operation is repeated throughout the whole depth of the shaft. Instead of a chain or tape, rods may be used.

There should be no serious error in the measurement of a shaft, and with care a shaft 1000 feet in depth may be measured with an error of less than ¼ inch.

Measurements of Inclined Shafts.—Whilst the measurement of vertical shafts is thus simple and easy, the measurement of the depth of inclined shafts is often very tedious, and resolves itself into a process of levelling with straight-edge and spirit-level (see Fig. 144). In this case a straight-edge is fixed level by means of a prop of some kind. For accurate work the end of the straight-edge which is raised above the ground should be clamped to a vertical rod when it has been carefully adjusted by the level; from the end of the straight-edge a plumb-line is dropped to the ground, and, on some bar or mark firmly fixed, the exact position of the centre of the plumb-line is marked with great care, and then the length from the straight-edge to the ground is measured, the measurement being taken to hundredths of an inch. The straight-edge is now lowered and fixed on the mark made by the plumb-bob, and the operation repeated. A set square may be attached to one end of the straight-edge, and the vertical rod fixed against this, dispensing with a plumb-line. The vertical rod should have a scale marked on it, and it should be erected on a smooth stone, brick, or bar, on which the straight-edge can be placed afterwards, or has been placed previously.

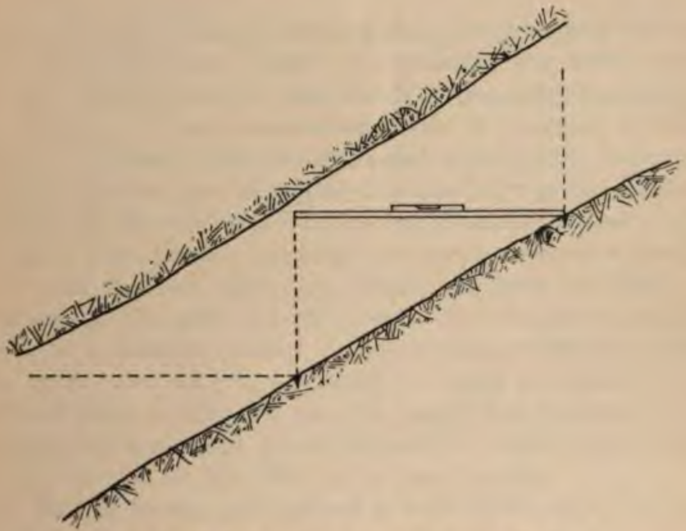

FIG. 144.—Method of measuring inclined shafts.

If the shaft twists, it must be surveyed, although the direction is of no importance for ascertaining the depth; the straight-edge used being of a convenient length, the horizontal measurement of each set is known. A dial may be attached with clips to the straight-edge, and the bearing of each position noted. This, of course, can only be done in case there is no attraction, so that the loose needle can be used.

CHAPTER XIII.

CONSTRUCTION OF PLANS.

As already stated, English mining plans are generally drawn on large sheets of paper capable of containing the whole survey. The kind of paper used is that which is generally described by the makers as "best antiquarian," and is mounted on brown holland. A sheet of the required size has frequently to be made specially to order, and is prepared by pasting together a number of sheets of the size made by the paper-manufacturer; where one sheet joins another the edges are pared down to a bevel, so that when the two edges are placed one over the other, the thickness is the same as one sheet; the two pieces are then united by a suitable paste. At the corners where four sheets join, great care has to be taken to make a sound junction without having a lump. A sheet of paper thus mounted should be made months, if not years, before it is wanted, the surveyor keeping a stock in his office in a chest in the centre of the room, that is to say, not against a wall which might be damp. Mounted plan-paper can also be obtained in rolls up to 81 inches in width, and the length required for a plan cut off. The plan, when made, is rolled up and put into a drawer when not in use. As dust generally finds its way into the drawers, it is necessary, to prevent dust from getting inside the roll, to cover the ends with paper. The plan is often kept in a case of tinned iron painted on the outside, with a hinged lid and fastened with a padlock; in this case the plan may be safely carried without fear of injury; without such a case the plan would soon get damaged in transport. If carefully used, the plan may serve for a generation without being much the worse. When the plan is confided to the charge of assistants who do not feel the responsibility of the cost of replacing it, it is frequently damaged by being bent over the edges of tables in such a way as to break the paper or seam it with cracks; it is soon made black by being exposed to the dust, and by being rubbed with dirty articles. A plan when laid out on the table should be kept down by leaden weights covered with leather (usually weighing about 2 lbs.), the edges of which are rounded. The weights should be always dusted before using, and the table dusted before the plan is laid down. When working on the plan, it should be covered up with clean paper, except that part which has to be exposed for work, and if the draughtsman finds it necessary to rest his arms upon the paper, he should lay down a sheet of clean paper underneath his arms or other portion of his body that may be pressing on the paper.

In order to bring that part of the paper on which he is working within his reach, it is frequently necessary to draw the plan near the edge of the table. For this purpose the table is fitted with a beading reaching to a depth of say $3\frac{1}{2}$ inches from the top, and the corner of the edge planed off until the section of the edge with the beading below is a semicircle. If any segment of a circle less than a semicircle is used, there will be a sharp edge, and in bending the plan over that edge it may get injured. If the draughtsman leans against the plan drawn over the table, he will soon dirty and injure it; he must therefore cover up the plan at this part with paper or calico.

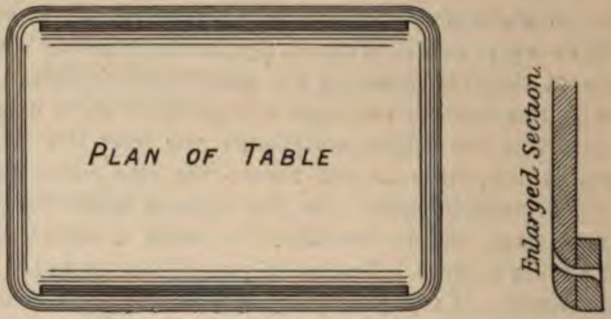

FIG. 145.—Improved drawing-table.

Drawing-tables may be made with an outer bar, against which the draughtsman rests, as shown in Fig. 145.

Miscellaneous Notes on the Preparation of Plans.—To make plain those parts of the plan from which the minerals have been extracted, it is usual to colour it with water-colours. Colouring is apt to lead to shrinkage of the plan, and should therefore be done as sparingly as possible, although the surveyor must remember that colouring may be essential to the utility of the record. It may be in some cases advisable to keep a skeleton plan of the workings with no colouring, by which to preserve the accuracy of the main stations, and to correct from time to time the working plan, which has been distorted by colouring and comparatively rough usage.

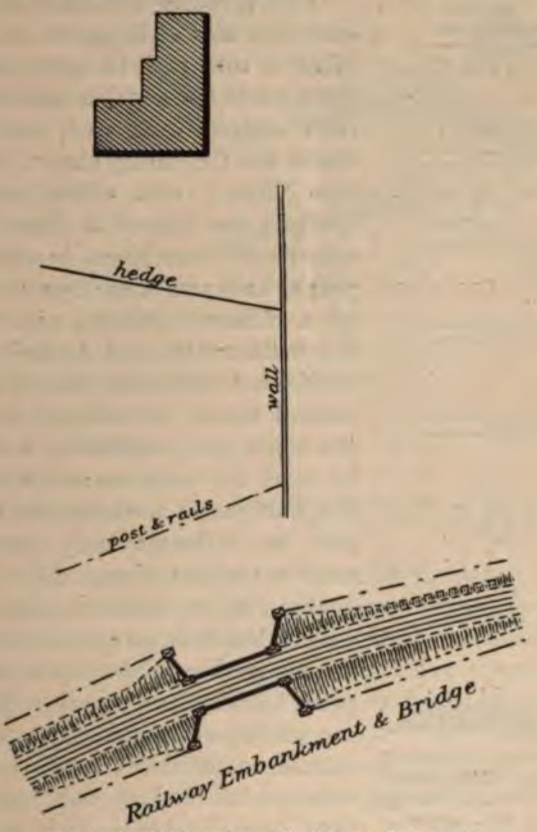

FIG. 146.—Delineation of buildings, fences, etc.

The advantage of a large plan, showing the relative position of all the different workings connected with one concern, is obvious, conveying forcibly and at once the whole situation to the mind of the engineer; on the other hand, the exact distances and bearings can generally be better ascertained by calculations contained in the office survey-book.

The survey is laid down in fine pencil-lines, and afterwards inked over; the surface is invariably drawn in Indian ink, walls, buildings, hedges, etc., being indicated in the way shown in Fig. 146.

Underground workings in the Midland Counties of England are generally inked in with pink lines (crimson lake); those parts from which the mineral has been entirely excavated, washed lake; faults are frequently shown with dotted blue lines; and other conventional symbols are shown in Fig. 147. It is sometimes convenient to indicate each year's, half-year's, and quarter's survey by a different colour; at other times the written date and a shaded line are considered sufficient (see Fig. 148). If several seams of coal are shown upon the same plan, a different colour should be used for each seam, in which case the half-year's workings in each seam can be indicated only by the dates written on the plan. There must, of course, be a separate plan for each seam; but it is very convenient to show all the workings also upon one general plan, to facilitate the true understanding of the situation. The north point should be indicated by an arrow of an ornamental kind at one corner of the plan (see Fig. 148). This arrow is not to be used in plotting, but merely to indicate approximately the direction; the real meridian line is represented by a long thin line drawn with the aid of a steel straight-edge across the plan. If it is the magnetic meridian, the date is written against it.

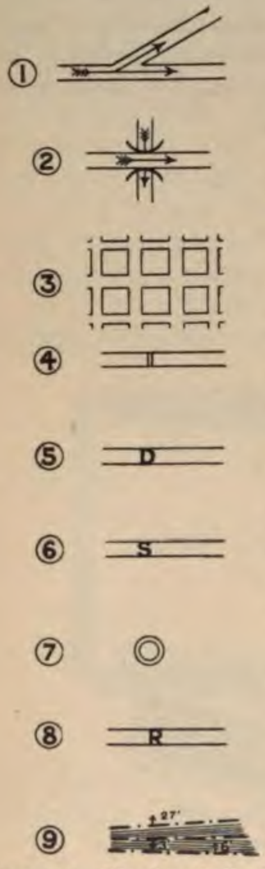

FIG. 147.—Usual methods of delineating underground workings, etc. (1) Arrows indicating air-currents; (2) air-crossing, overcast and undercast; (3) pillars and stalls; (4) stopping; (5) door; (6) sheet; (7) pit-shaft; (8) regulator; (9) faults and faulty ground.

Copying Plans.—Plans can be most easily and quickly copied on tracing-paper or tracing-cloth. Tracing-paper is the more pleasant to work on, but is easily torn; tracing-cloth makes a permanent copy, but is liable to be much distorted by colouring. A cloth tracing, however, often makes a good working plan for rough usage, and is serviceable, and when folded into a leather case may be carried about the mine. When the smooth or greasy nature of the surface makes it difficult to draw upon, a little prepared ox-gall mixed with the ink or colour obviates the difficulty; powdered chalk also is sometimes useful when rubbed over the surface.

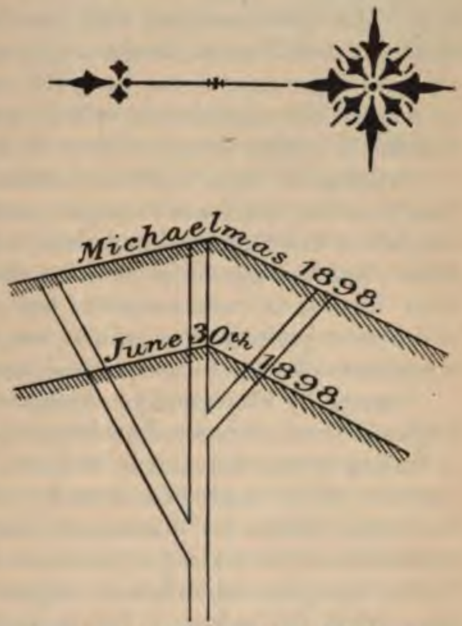

FIG. 148.—North point. Method of showing coal worked during the quarter.

Glass Table for tracing through Thick Paper.—Plans drawn on unmounted paper may be traced on to drawing-paper, by placing the plan and the sheet on which it has to be traced upon a glass. In order to get the light through this glass, it should be placed in a frame near a window, and light from below thrown upwards through the glass by a reflector; the reflector may be made either of looking-glass or of white paper. The surface of the paper on which the draughtsman is working should be shaded by a blind. If the work is done at night, a brilliant illumination of reflected gaslight can be used, or better still, electric lamps may be placed immediately under the glass.

Transferring.—British mining plans being generally made on mounted paper, sufficient light will not pass through to enable them to be traced on to thick paper, in which case they may be transferred. The usual practice is to make a tracing on thin paper, then to place the tracing over the new mounted paper, and, between, to place a transfer paper specially made of very thin paper, one side of which is blackened with black lead. By means of a steel point (style) and a flat ruler, a fine line may be traced on to the paper below. Great care is required in doing this. If a blunt point is used, the line transferred will be too thick; if a fine point is used, it is apt to cut the tracing. If accidental pressure is put on to the tracing-paper, a black mark is left on the plan below, which may make the survey-lines indistinct, though it may afterwards be cleaned off.

Pricking through.—Another method is to place the original plan over the new sheet of paper, carefully fasten it down with weights or drawing-pins, and then to prick through to the plan below, and subsequently join the prick-marks by pencil-lines. This method is very accurate, but requires great care in the subsequent pencilling in, which has to be done by the aid of continual reference to the original plan.

Copying by Photography.—Architects and engineers reproduce copies of their plans by the action of light on sensitized paper. A tracing of the drawing is made on very transparent tracing-paper or cloth (dead-black lines for the drawing, and vermilion or burnt sienna for dimension lines). The sensitized paper covered with the tracing is placed in a frame and exposed to the light; the point of sufficient exposure is indicated by various changes in the colour of the sensitized paper; the sensitized paper is then immersed in a bath (either of water or acid, depending on the process used) and washed till the lines on the tracing appear upon the paper, owing to the circumstance that these lines have shielded the paper from the action of the light. According to one process, the lines appear white on a blue ground; by another process they appear black on a white ground.

The sensitizing solution for the ferrotype or blue process, in which white lines are given on a blue ground, may be easily made as follows:—

Solution A: Citrate of iron and ammonia, 100 grains; water, 1 ounce.

Solution B: Red prussiate of potash, 70 grains; water, 1 ounce.

These solutions will keep indefinitely before mixing, but after mixing they should be used at once or left in the dark.

To prepare the paper, mix equal quantities of A and B, and apply to one side of the paper with a sponge. The sponge should be as full as it will hold of the solution, which should be liberally applied to the paper for about two minutes. Then squeeze out the sponge and wipe off all the solution from the surface of the paper, care being taken to use the sponge lightly without abrading the surface. The paper, which is now of a bright yellow colour on the prepared side, should be hung up to dry in the dark.

Reduction and Enlargement of Plans.—It is frequently necessary to enlarge or reduce a plan. It is not, as a rule, advisable to make a plan on a large scale from an original plan drawn on a small scale, because any error in the original plan will be multiplied as much as the plan is enlarged, and an error imperceptible on the small-scale plan may become important on the large-scale plan, therefore a large-scale plan should, as a general rule, be made from the original survey notes by replotting the survey on the required scale. In reducing a plan any errors in the original will be also reduced.

A common mode of reducing or enlarging a plan is to treat the original plan as if it were the works, mine, or estate, that had to be surveyed, and to make a survey of it by drawing triangles, measuring offsets, etc., and then reproducing these triangles, offsets, etc., with the aid of a smaller scale on another piece of paper. The surveying of the original plan, and the reduction, may be accomplished with the aid of two scales—say the original plan is on a scale of 2 chains to an inch, and the reduced plan is to be on the scale of 6 chains to an inch, then the lines are measured with the 2-chain scale, and plotted with the 6-chain scale. If the original survey notes are available, however, it would no doubt be more accurate and expeditious to plot them afresh.

Enlarging or Reducing by Photography.—Another method of reducing and enlarging plans is by means of a lens and camera obscura. This may be done by the ordinary process of photography. Thus, supposing the plan to be 6 feet square, it might be photographed on to a plate 12″ × 12″, or of any other dimensions to suit the camera of the observer.

If the size of the negative is too small for the required plan, an enlargement may be produced by placing the negative in an enlarging camera, inside which is a lens which enlarges the view and prints it on a larger piece of paper at the other end of the camera.

This process of reduction by photography may be done with great accuracy if sufficient care is taken. It is essential that the plate on which the negative is formed should be parallel to the drawing which is being photographed, and it is desirable that the camera should be opposite to the centre of the plan. The lines on the plan to be photographed must not be too fine, otherwise the lines on the reduced plan become too thin to be clearly visible. A line $\frac{1}{300}$ inch in width is perfectly clear, but if that line were reduced by photography to one quarter of that width, it would be too fine for ordinary distinctness; if, therefore, it is proposed to reduce the plan to one quarter its original size, and if it is decided that the minimum thickness of lines on the reduced drawing should be $\frac{1}{400}$ inch, the lines on the original plan must not be less than $\frac{1}{100}$ inch in thickness.

The reduction or enlargement of plans by photography is not usually practised by the mining engineer, because the cost and trouble of procuring and arranging the apparatus is more than the saving in labour to be gained by the process. It is also necessary that the plan to be photographed should be all in black and white. The system, however, is suitable for the illustration of a report of which say a dozen or more copies are required.

Pantagraph.—This instrument, Stanley's improved form of which is illustrated in Fig. 149, is used for the mechanical copying of drawings, either upon the same scale or upon a reduced or enlarged scale. It consists of four arms jointed together in pairs. On one of these arms is a tracer, and on another a pencil-holder, and by means of scales engraved on

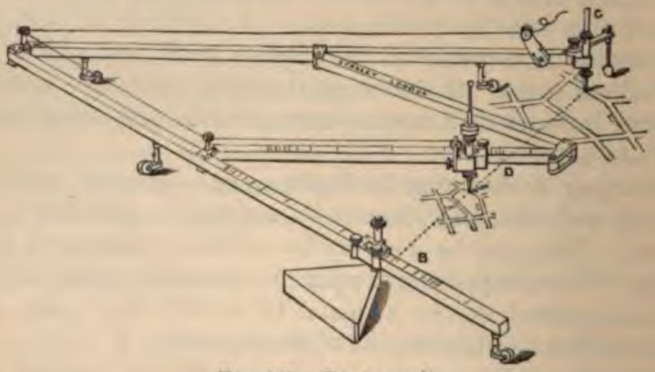

Fig. 149.—Pantagraph.

the instrument the relative positions of these can be so arranged that the figure drawn by the pencil bears a definite proportion to that which is followed round by the tracer.

A similar instrument, called the eidograph, illustrated in Fig. 150, is said to be superior to the pantagraph; it is, however, much more expensive, and for that reason some firms [1] send the instrument out on hire for temporary purposes.

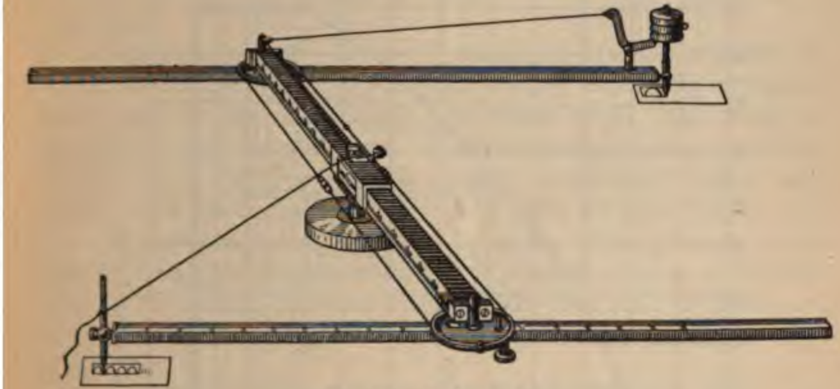

FIG. 150.—Eidograph.

Proportional Compasses.—Proportional compasses may be used instead of or in addition to the scales. These compasses, as shown in Fig. 151, consist of two straight bars pointed at each end. Each of these bars is slotted to about two-thirds of its length, a slide fits into each slot, and a pin with a milled head passes through both slides; each bar is graduated with cross-lines marked from 1 up to 10. If the slide is fixed at 1, and the bars twisted round the centre pin, the points at each end will remain equidistant; if the slide is fixed at 2, the points at one end will open twice as far as the points at the other end; if the slide is fixed at 3, the points at one end will open three times as far as the points at the other, and so on. Another side of the bar is graduated $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{2}{5}$; thus if the slide is fixed at $\frac{3}{4}$, the

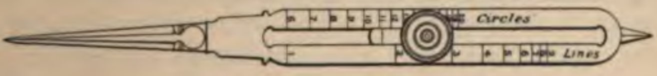

FIG. 151.—Proportional compasses.

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[1] Amongst others Messrs. Halden and Co., 8, Albert Square, Manchester.

points at one end will move 4 inches, while the points at the other move only 3 inches. This instrument is very convenient for marking off lengths on reduced plans.

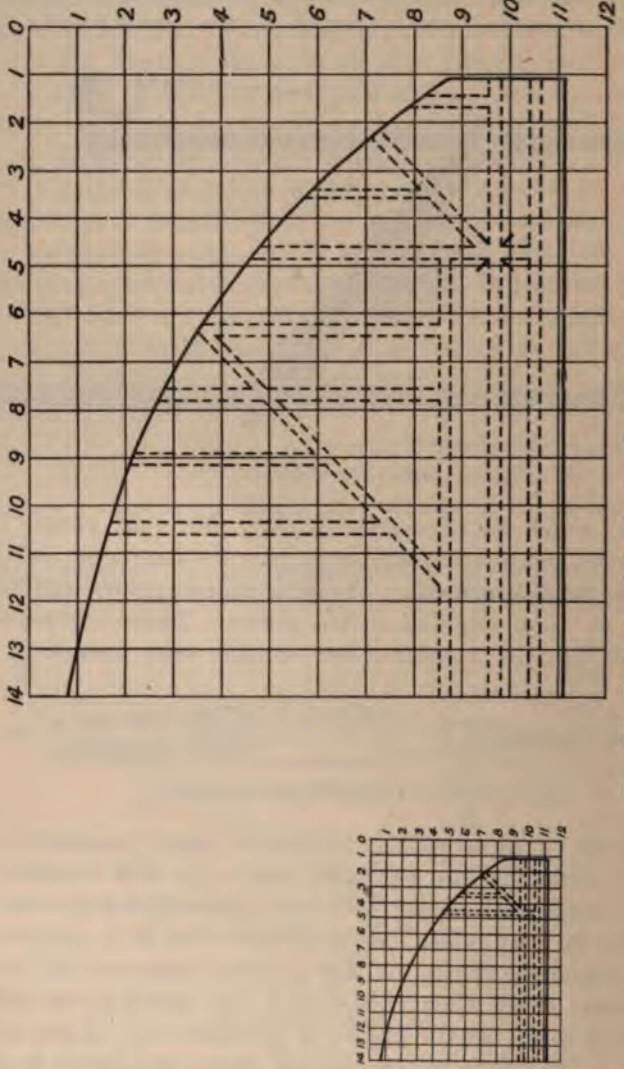

Fig. 152.—Reduction of plan by use of sectional paper.

Enlarging or Reducing by Sectional Paper.—To facilitate the surveying of one plan and the plotting of another, it is a common practice to divide each plan into squares; thus the 2-chain plan will be ruled into squares, measuring ½ inch on each side, and the paper for the 6-chain plan into squares ⅙ inch on each side; or, if this latter size is found inconveniently small, the 2-chain plan may be ruled into squares 1 inch on each side, and the 6-chain plan into squares ⅓ inch on each side. The plan may now be reduced by sketching with a fine-pointed pencil (see Fig. 152).

In order to save the labour of ruling the squares, and also to avoid the disfigurement of the plan, tracing paper with sectional lines ruled in fine blue ink is sometimes used. The mode of reduction would be as follows: Over the 2-chain plan a tracing divided into 1-inch squares is placed; a tracing divided into ⅓-inch squares is now placed over a piece of white paper, and the reduced plan sketched on it by hand. The plan so made may be subsequently transferred to a piece of paper; or, instead of that, the section lines for the reduced plan may be ruled on the paper, and the sectional tracing-paper merely used for the large plan.

For many purposes drawing-paper ruled with fine blue sectional lines is exceedingly useful. This is not generally used for mining plans, partly owing to the disfigurement of the paper by the sectional lines, and partly owing to the difficulty of procuring extreme accuracy in the ruling of these lines; but in many other cases this sectional paper is extremely convenient.

The Opisometer (Fig. 153) is an instrument for roughly measuring distances on plans which, owing to their sinuosities would require the expenditure of a great deal of time with an ordinary scale. It consists of a small wheel with a milled edge, which revolves upon a screw for an axis. The screw moves through the arms which carry it, being propelled by the movement of the wheel, and a scale is attached, showing the distances corresponding to any movement of the screw.

Relief Plans and Mine Models.—Models of mines, showing the configuration of the surface and the veins of minerals, or seams of coal, faults, etc., are very useful and instructive, but are exceedingly expensive. They are chiefly used for educational purposes, and for displaying in exhibitions and museums. In so far, however, as they represent actual mines, they must be prepared from data which can all be put on to plans and sections; and the effect of a model can be given to a great extent by a skilfully prepared drawing, the cost of which is insignificant in comparison with the cost of a model.

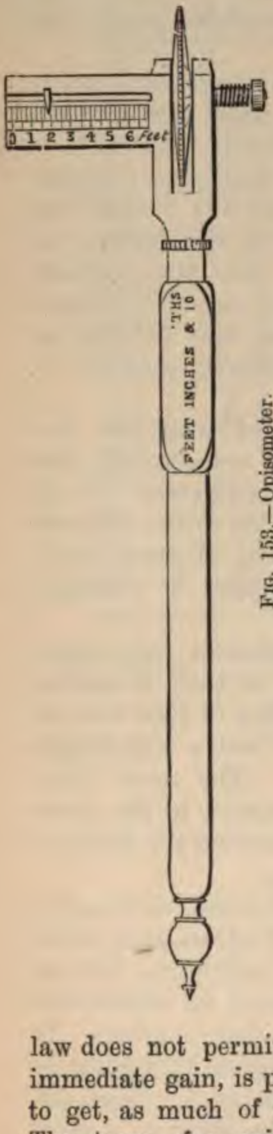

Fig. 153.—Opisometer.

In the construction of a model, rocks are generally represented by wood, painted, to represent the different strata, seams of coal, or veins of mineral. If machinery is shown, this is also generally of wood, painted where necessary to represent iron; real brass may be used to represent portions of the machinery made of that metal. Fig. 154 is prepared from a photograph of a model in the museum at South Kensington.

Calculation of Area and Quantity of Coal worked.—One of the most important uses of a mineral plan, particularly in connection with collieries, is the calculation of the extent of mineral got in cubic feet, cubic yards, cubic metres, or in areas of square feet, square yards, square metres, or acres and decimals, or acres, roods, and perches.

Royalty.—In Durham, Northumberland, Wales, and other parts, the lessee of a colliery pays to the lessor a royalty, as it is called, of so much per ton. The derivation of the word "royalty" is probably derived from the fact that the minerals, as also the land, in former days belonged to the king. In the United Kingdom, the land, and with the land the minerals, is now generally the property of private owners, such ownership being either absolute, as when the land is held in fee simple, or limited, as when the owner has only a life interest, having to transmit the estate to heirs. The owner of a life interest of an estate, while the law does not permit him to destroy the surface for his own immediate gain, is permitted to get, or grant to others a licence to get, as much of the mineral as he can during his lifetime. The term of a mineral lease granted on an entailed estate

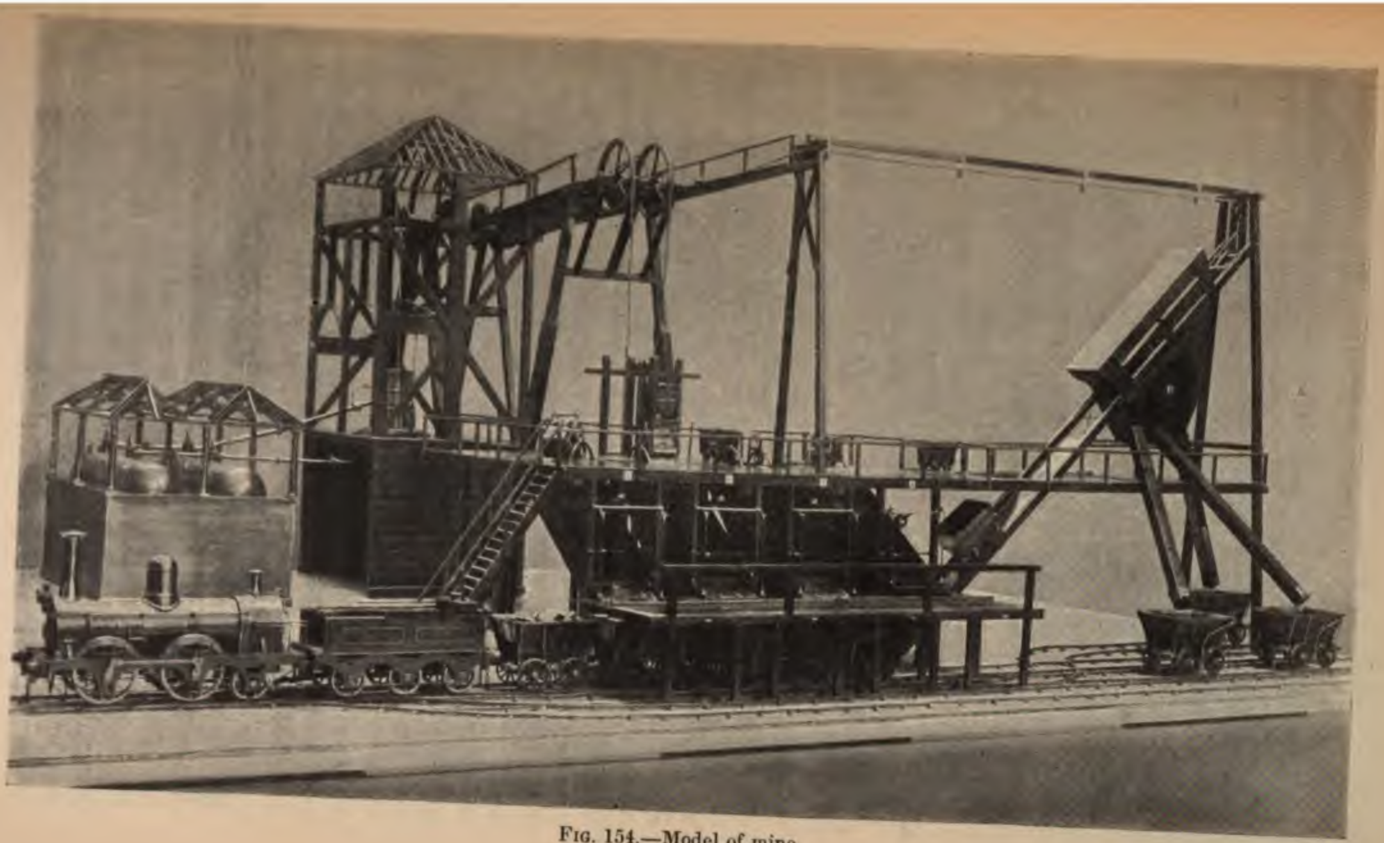

FIG. 154.—Model of mine.

cannot exceed 60 years, except by special leave of the Court of Chancery.

The minerals reserved to the British Crown at the present time are only gold and silver, and those lying underneath Crown lands, as, for instance, the coal and ironstone in the Forest of Dean; the tin, lead, and copper in the Duchy of Cornwall; the minerals in the Duchy of Lancaster; and the minerals underlying the foreshore on our sea-coasts, that is to say, the minerals underlying that portion of the coast which is covered by the tide, and extends to a distance of three miles from the shore, except in cases—as, for instance, the estuary of the River Dee—where the king has made a grant of these minerals to a subject.

This royalty is often a fixed sum, as, say 6*d*. a ton, or 1*s*. 4*d*. per chaldron (a chaldron being 53 cwt.); or it may be one price on large coal and another price on small coal, say 6*d*. per ton on lumps that pass over a screen, the bars of which are 1 inch apart, and 3*d*. a ton on the slack which falls through these bars. In some districts, as, for instance, in North Staffordshire and North Wales, the royalty is a fixed proportion of the total value of the mineral sold, as, for instance, one-eighth, one-tenth, one-twelfth, etc. In other parts of the country, as, for instance, in Yorkshire, Derbyshire, Nottinghamshire, Leicestershire, Warwickshire, etc., an acreage royalty is paid, the royalty being, say £100 an acre on each seam. If several minerals are worked together in one working, as, for instance, ironstone, coal, and fire-clay, there will be a separate royalty on each, say £50 an acre on the ironstone, £50 an acre on the coal, and £50 an acre on the fire-clay. Sometimes the royalty is so much per acre for a given thickness of coal, say £30 per acre per foot thick, so that if the seam were 5 feet thick, the royalty would be £150 per acre, the thickness of the seam being ascertained each time the mine is surveyed.

Measuring Acreage by Division into Triangles.—In calculating the area got, the ordinary process is as follows: The area of coal to be measured (which may represent the whole extent that has been worked, or only the area got in one half-year) is divided into trapeziums and triangles, the edges of the area being straightened by give-and-take lines; the trapeziums are divided into triangles by diagonals, and from the apex of each triangle of the base a perpendicular is let fall (see Fig. 155). Each of the triangles and trapeziums is numbered 1, 2, 3, etc., the base of each triangle is measured, the diagonal of a trapezium forming the base of two triangles; the perpendicular of each triangle is also measured. The area is equal to the base multiplied by the perpendicular divided by 2. The area of a trapezium is equal to the diagonal multiplied by the sum of the two perpendiculars divided by 2. Thus, referring to the figure, the area of triangle No. 1 is equal to 1000 (the base) × 200 (the perpendicular) ÷ 2 = 100,000; and the area of No. 2, which is a trapezium, is equal to 1000 (the diagonal) × 156 + 200 (the perpendiculars) ÷ 2 = 178,000; therefore the sum of the areas 1 and 2 = 100,000 + 178,000 = 278,000. If the measurements are in links, we thus have an area of 278,000 sq. links.

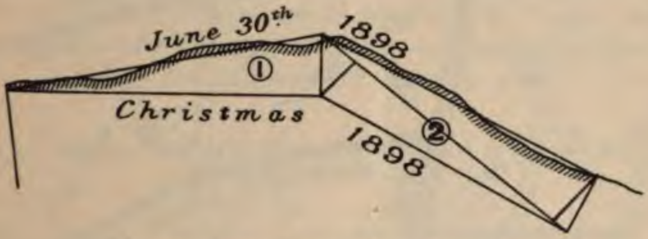

FIG. 155.—Scaling by triangles and trapeziums.

The area of an acre is 100,000 sq. links, so that to turn the square links into acres we must divide by 100,000. The process of division by 100,000 is exceedingly easy, consisting in simply putting the decimal point before the fifth figure from the right-hand end of the number; thus, in dividing 278,000 by 100,000, we put the decimal point before the 7, which is the fifth figure from the right-hand end of the number, and we have the answer 2·78000 acres. To turn 2·78 acres into acres, roods, and perches, we multiply the decimal part by the number of roods in an acre; there are 4 roods in an acre, so we multiply 0·78 by 4, and the result is 3·12 roods. To turn the decimal of a rood into perches we must multiply by the number of perches there are in a rood, which is 40; 0·12 × 40 = 4·8; therefore the number of perches is 4·8; the total acreage is therefore 2 acres 3 roods 4·8 perches. It is, perhaps, mainly on account of the ease with which square links can be reduced to acres that the use of the 100-link chain is so popular with mining surveyors.

If the foot chain were used, then the area measurements might have to be calculated in square feet. To reduce square feet to acres, they must be divided by 43,560, the number of square feet in an acre.

It must, however, be borne in mind that, in the scaling of the plan, it is immaterial whether it has been made by the measurement of links or of feet, because the measurements can be taken off the plan by means of a scale of links, even though it was plotted from a scale of feet, just as the engineer can take from a plan plotted from measurements in links any distance he desires in feet by a scale prepared with that object. Scales are often made to read feet on one side and links on the other.

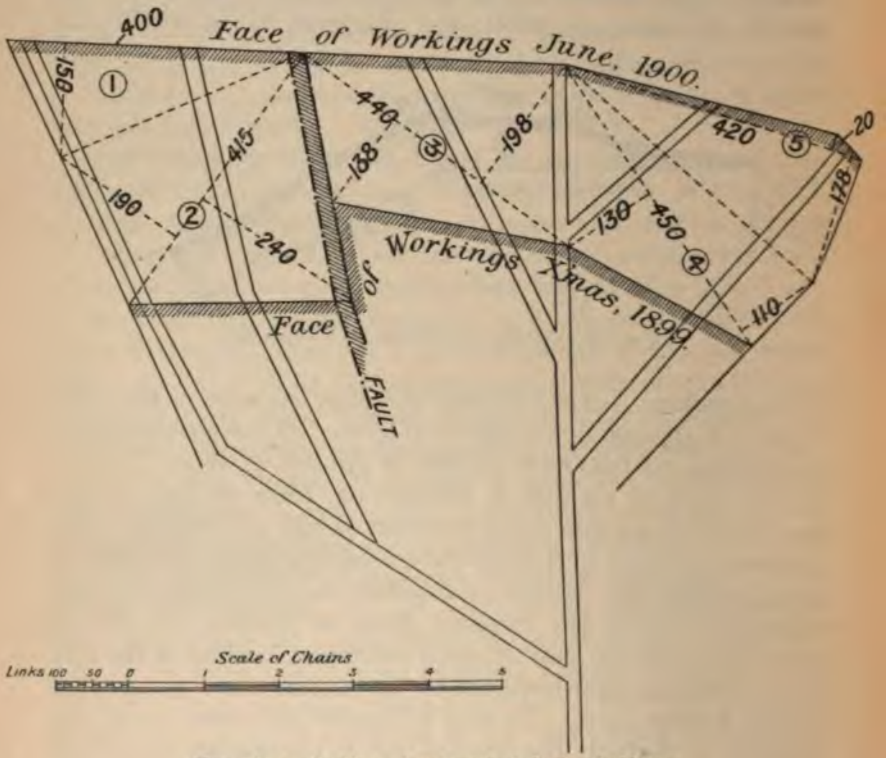

FIG. 155A.—Scaling of coal worked during half-year.

In order to avoid scoring a plan with scaling-lines, it is usual to place a piece of tracing-paper over the plan, and to make a very careful tracing with fine lines of the areas to be measured, and upon this to draw, with a fine-pointed pencil, give-and-take and dividing lines, and then to ink these in, writing upon them the measurements. These scaling tracings are copied in a book, and form a useful and permanent record of all the measurements made. The advantages of this system of measurement for the purposes of reference, and the ease with which the scalings and calculations can be checked by assistants, commend the system so strongly to the practical surveyor that it is likely to hold its own as long as the system of acreage royalties prevails.

The method of scaling up an area of coal worked is shown more fully in Fig. 155A. The triangles and trapeziums into which the area is divided are numbered 1, 2, 3, etc.; and the lengths of the base and perpendiculars are marked on the lines.

The entry in the scaling-book for this area would be as follows:—

——— *Colliery. Scaling of Freehold Coal worked during the Half-year ending June* 30, 1900.

$$
\begin{array}{rr}
(1)\ 400 \times 150 = & 60000 \\
(2)\ 415 \times (190 + 240) = & 178450 \\
(3)\ 440 \times (138 + 198) = & 147840 \\
(4)\ 450 \times (130 + 110) = & 108000 \\
(5)\ 420 \times (20 + 178) = & 83160 \\
\hline
& 2)\overline{577450} \\
& 288725 \\
\text{Deduct faulty coal } (340 \times 20) = & 6800 \\
\hline
& 2{\cdot}81925 \\
& 4 \\
\hline
& 3{\cdot}27700 \\
& 40 \\
\hline
& 11{\cdot}08000
\end{array}
$$

2 acres 3 roods 11 perches at £100 per acre.

	£	s.	d.
2 acres at £100 per acre	200	0	0
3 roods at £25 per rood	75	0	0
11 perches at 12*s*. 6*d*. per perch	6	17	6
0·08 " " "	0	1	0
	£281	18	6

The above is the usual way of getting out acreages and royalties, but it is evident that a better and quicker way is to take the acres and decimals thus—

2·81925 acres at £100 per acre
100
281·92500
20
18·510
12
6·120 £281 18*s.* 6*d.*

Statute Acre and other Acres.—The term "acre," as applied to the measurement of land, is generally understood to refer to the statute acre of 160 perches, which was established by law about the thirteenth century. There are, however, different acres in various parts of this country (see Table XIV.), but the use of these varying measures is rapidly giving way to the statute acre, and they will soon be quite obsolete.

TABLE XIV.[1]

LIST OF VARIOUS ACRES.

1 Statute acre contains	4840	sq. yards.	
1 Scotch acre contains	6150·4	„	(48 Scotch acres equal nearly 61 statute acres)
1 Irish „ „	7840	„	(100 Irish acres are nearly equal to 162 statute acres)
1 Welsh „ „	4320	„	(sometimes called "erw")
1 Cornish „ „	5760	„	(equivalent to about 1·19 statute acre)
1 Leicestershire acre contains	2308·75	„	(equivalent to about 0·477 statute acre)
1 Westmoreland „ „	6760	„	(equivalent to about 1·397 statute acre)
1 Cheshire „ „	10,240	„	(equivalent to about 2·115 statute acres)
1 Lancashire „ „	7810	„	(equivalent to about 1·613 statute acre)

Planimeter.—For the rapid measurement of numerous areas with sinuous boundary-lines, Amsler's planimeter is of great use to the surveyor. By means of this instrument, shown in Fig. 156, the area of any given figure is measured in square inches; the area so obtained can be converted into square chains by simple multiplication. Thus if the area measured is 6 sq. inches, and the scale of the plan is 1 chain to an inch, the area is 6 sq. chains; if the plan is on a 2-chain scale, the area is four times as great, because each square inch contains 4 sq. chains, therefore the 6 sq. inches represent 24 sq. chains; if the scale of the plan is 3 chains, then each square inch contains 9 sq. chains, and the 6 sq. inches contain 54 sq. chains: 10 sq. chains equal an acre; therefore the area, as measured in square chains, if divided by 10, gives the area in acres and decimals of an acre.

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[1] The authority for most of these figures is the Century Dictionary, recently published by the *Times*, London.

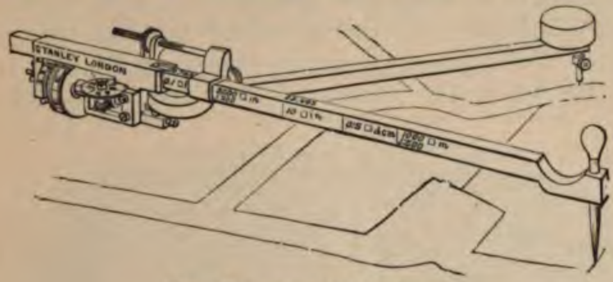

FIG. 156.—Amsler's planimeter.
(Kindly lent by Messrs. W. F. Stanley and Co., Ltd.)

Rule.—Let X = area in square inches, Y = number of chains per inch of scale, Z = area in square chains. Then $Z = X \times Y^2$.

EXAMPLE.—Let the area measured in square inches (X) be 5·64; let the scale of the plan be 3 chains to an inch, or $Y = 3$; then the area $(Z) = 5{\cdot}64 \times 3^2 = 50{\cdot}76$; the acreage $= 50{\cdot}76 \div 10 = 5{\cdot}076$. To write the decimal portion down in roods and perches, we multiply by 4 for roods, giving 0·304 of a rood; to turn this decimal into perches, we multiply by 40, giving 12·16; the area is therefore 5 acres 0 roods 12·16 perches.

The method of working the planimeter is to fix one arm with a weight, and to move the pointer at the end of the other arm round the boundary of the plot; the number of square inches is then shown on a scale marked on a revolving drum. The accuracy of the work done with this planimeter is more than equal to that of ordinary scaling, and the method is much quicker.

Stang Planimeter.—An exceedingly simple instrument for measuring areas is the Stang planimeter made by Knudsen, of Copenhagen. This instrument is shown in Fig. 157. It will be seen that it consists of a light metal rod supported on two legs; one leg ends in a fine point, the other ends in a narrow edge about ½ inch wide like the edge of a small axe.

Goodman's Planimeter.—The above instrument has been modified by Professor John Goodman, by engraving on the bar a scale by which the area measured can be read without calculation. This scale constitutes the difference between the Goodman and the Stang planimeters.

The method of using the instrument is thus described by Professor Goodman: "Choose a point **A** (see Fig. 158), as near the centre of the figure as can be judged by eye, and from it draw a line **AB** to the boundary. Hold the tracing-leg of the instrument in the right hand, placing the point at **A** and the hatchet at **X**—*i.e.* with the instrument roughly square with **AB**—and press the hatchet in order to make a slight dent in the paper at **X**; then, the finger having been removed from the hatchet, the tracing-point of the instrument is caused to traverse the line **AB** and the boundary in the direction indicated by the arrows, returning to **A** *viâ* **AB**, when it will be found that the hatchet has taken up a new position, and it must be again lightly pressed in order to make a fresh dent in the paper at **Y** (Fig. 158). The instrument being held in this position, revolve the paper on which the figure is drawn through about 180° (by eye), using the point of the instrument as a centre, and taking care that neither the point nor the hatchet shifts while the paper is being turned. The line **AB** will again be roughly at right angles to the axis of the instrument, but in a reversed position (see dotted figure, Fig. 158). Now cause the tracing-point to traverse the boundary as before, but in the opposite direction, as indicated by the

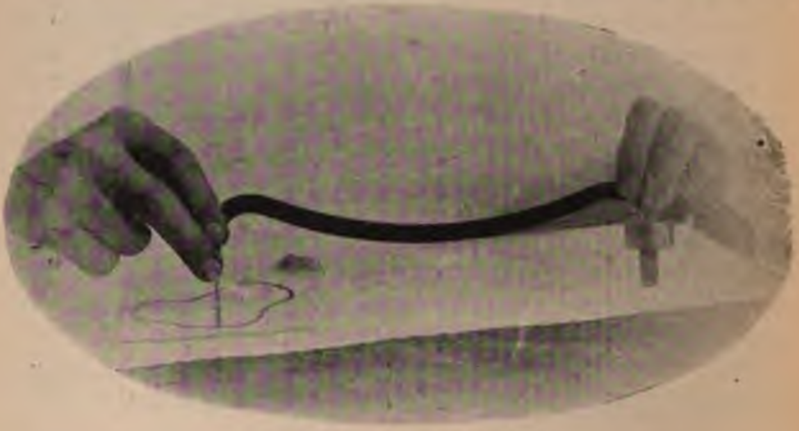
FIG. 157.—Stang planimeter.

dotted arrows. The hatchet will take up the new position X_1, which may or may not coincide with X; then, *the mean of* XY *and* X_1Y *measured on the scale engraved on the instrument is the area of the figure;* this can be readily read off by pricking a central point, as shown between X and X_1 by eye. When it is inconvenient to turn the paper round, the instrument itself may

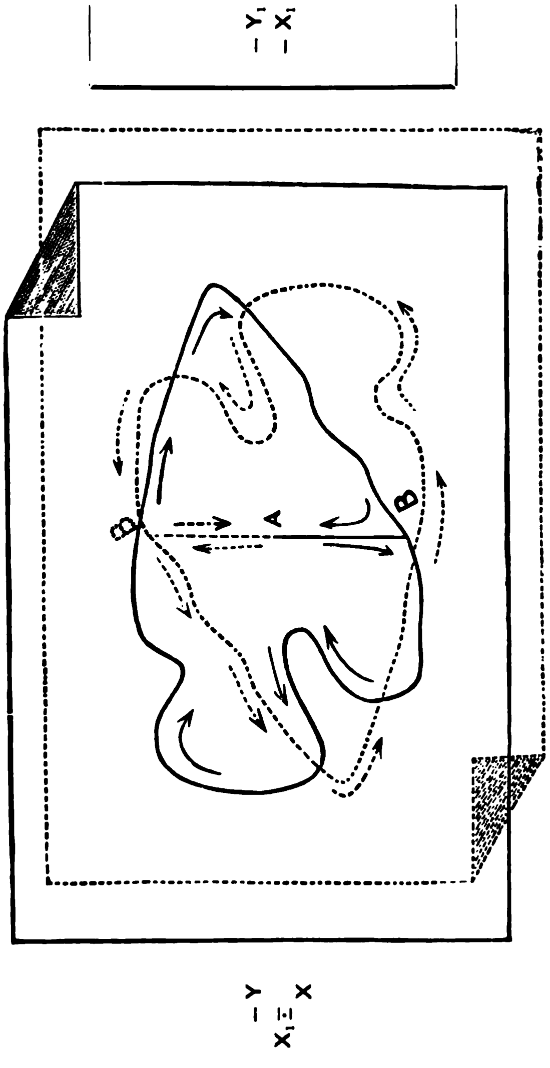

FIG. 158.—Method of using Goodman's planimeter.

be turned round to form a dent, **X_1**, on the *opposite* side of the figure, as shown on the right-hand side of Fig. 158. Then, by following the boundary in the direction of the arrows, **Y_1** is obtained. The area is the mean of the lengths **XY** and **X_1Y_1** measured off on the scale as before, or the area $= \frac{XY + X_1Y_1}{2}$.

"When the area is large, the instrument will move through a large angle, and consequently, if approximately square with **AB** at starting, it will be a long way out at the finish. In such a case all that is necessary is to see that the *mean* position of the instrument is square with **AB**."

Professor Goodman considers that his planimeter is quite accurate; in comparing it, however, with Amsler's planimeter, he would liken his to an ordinary foot rule, and Amsler's to a carefully made vernier reading-gauge.

Area-computing Scale.—Mr. W. F. Stanley makes a useful scale for computing areas. The scale and method of using it can be explained on reference to Fig. 158A. The area to be measured is covered with a sheet of transparent paper on which parallel lines are ruled, the distance between these lines being equal to 1 chain on the scale of the plan; thus, if the scale is 2 chains, the lines will be half an inch apart.

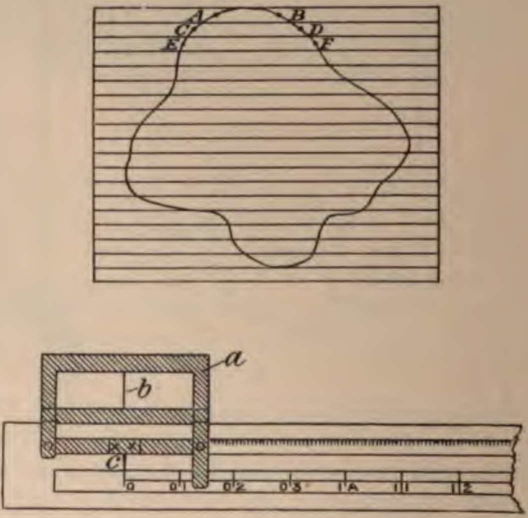

FIG. 158A.—Stanley's scale for computing areas.

The scale has a sliding frame, *a*, attached to it, across which is stretched a fine wire, *b*, and under the wire is a pointer, *c*.

The pointer is put at 0 on the scale, and the wire is then placed over the point **A** on the figure to be scaled, and the slide moved to **B**. The scale is then moved so that the wire is over the point **C** and the slide moved to **D**, and so on to the bottom of the figure. The reading of the pointer on the scale gives the acreage.

Slide Rule.—In the calculation of areas and of many other figures required by the surveyor, the slide rule (shown in Fig. 159) is of great use. By means of this instrument, calculations can be rapidly accomplished without any strain on the head, the detection and elimination of errors being achieved by repetition of the calculations by several persons. For the method of using the slide rule, the reader is referred to one of the numerous treatises on the subject.[1]

Professor Fuller's Calculating Slide Rule.—This form of calculating machine (shown in Fig. 160), which is said to be the simplest yet made, is found to facilitate very greatly the numerous arithmetical calculations required in the office of the engineer, architect, and actuary.[2]

Its range is greater than most arithmetical machines, as, besides the operations of multiplication and division which many instruments can only perform, results requiring the reciprocals, powers, roots, or logarithms of numbers can be quickly and easily obtained by its use.

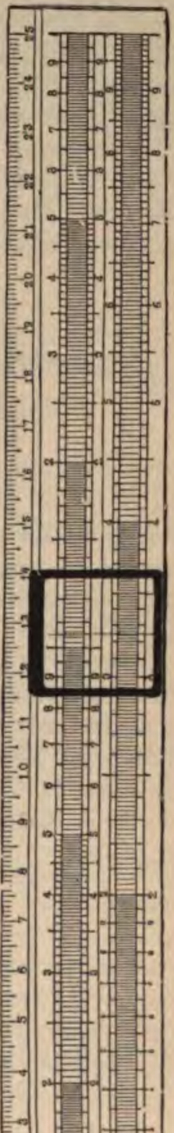

Fig. 159.—Slide rule.

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[1] "The Slide Rule," by Charles N. Pickworth, Whit. Sch., price 2*s.*; "The Slide Rule, its Principles and Applications," by John W. Nasmith, price 3*s.* 6*d.*

[2] See pamphlet by Professor George Fuller, on the Spiral Slide Rule, published by E. and F. Spon, London.

The rule consists of a cylinder, *d*, that can be moved up and down upon, and turned round, an axis, *f*, which is held by a handle, *e*. Upon this cylinder is wound in a spiral a single logarithmic scale. Fixed to the handle is an index, *b*. Two other indices, *c* and *a*, whose distance apart is the axial length of the complete spiral, are fixed to the cylinder *g*. This cylinder slides in *f* like a telescope tube, and thus enables the operator to place these indices in any required position relative to *d*.

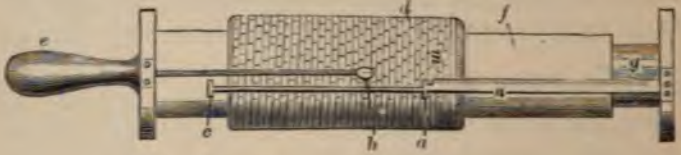

FIG. 160.—Fuller's slide rule.
(*Kindly lent by Messrs. W. F. Stanley and Co., Ltd.*)

Two stops are so fixed that when they are brought in contact, the index *b* points to the commencement of the scale, *n* and *m* are two scales, the one on the movable index *n*, the other on the cylinder *d*.

The use of slide rules has been confined to roughly approximate calculations, as the length of scale hitherto made was sufficient only for about 160 divisions. In the new rule above shown the length of scale is 500 inches, and the number of divisions 7250; consequently, the approximation obtained by its use is sufficient for most of the calculations required by engineers.

CHAPTER XIV.

MEASUREMENT OF MINERAL TONNAGES—CALCULATION OF CONTENTS OF PIT-HILLS—CALCULATION OF EARTHWORK, ETC.

Calculation of Tonnages.—To calculate the tonnage of coal contained in any given acreage, it is necessary to know the specific gravity of the coal and the average thickness of the seam.

Specific Gravity.—By "specific gravity" is meant the ratio of the weight of any substance to that of a standard substance (usually pure distilled water).

The specific gravity of any solid which is not soluble in water may be found as follows :—

Weigh the body in air, then in pure distilled water. Then—

$$\text{specific gravity} = \frac{\text{weight in air}}{\text{weight in air} - \text{weight in water}}$$

If the substance is lighter than water, a known weight is attached to it to cause it to sink, which is afterwards deducted.

Knowing the weight of the standard (water), we can find the weight of the substance if we know its specific gravity. Thus, if the specific gravity of a certain coal is 1·3, the weight of a cubic foot may be calculated. The specific gravity of water is 1, and the weight of a cubic foot is 62·5 lbs.; the calculation will therefore be as follows :—

$$1 : 1{\cdot}3 :: 62{\cdot}5 : 81{\cdot}25$$

Weight of a cubic foot of coal whose specific gravity is 1·3, 81·25 lbs.

Table XV. shows the specific gravity of various coals and other substances, and the authority :—

TABLE XV.

SPECIFIC GRAVITIES OF VARIOUS SUBSTANCES.

Substance.	Specific gravity.	Authority.	Substance.	Specific gravity.	Authority.
Coal—			Sandstone—		
Anthracite	1·53	Molesworth	Caithness	2·638	Molesworth
	1·3 to 1·84	Trautwine	Derby Grit	2·4	Molesworth
	usually 1·5		Cheshire		
Cannel ...	1·272	Molesworth	Red ...	2·15	Molesworth
Fire-clay ...	1·8	Molesworth	Slate—		
Granite—			Anglesea	2·87	Molesworth
Aberdeen	2·62	Molesworth	Welsh ...	2·88	Molesworth
	2·56 to 2·88	Trautwine	Basalt—		
Limestone ...	2·58	Molesworth	Scotch ...	2·95	Molesworth
	2·4 to 2·86	Trautwine	Sand-pit—		
Limestone—			Coarse ..	1·61	Molesworth
Blue Lias	2·467	Molesworth	Fine ...	1·52	Molesworth
Portland	2·423	Molesworth	Shingle ...	1·42	Molesworth
Bath ...	1·978	Molesworth	Earth from	1·52	Molesworth
	2·1	Trautwine	to	2·00	
Sandstone—			Gypsum ...	2·286	Molesworth
Bramley			Shales ...	2·4 to 2·8	Trautwine
Fall ...	2·5	Molesworth			

The ordinary coal of this country weighs from 78 lbs. to 82 lbs. a cubic foot: 80 lbs. may be taken as an approximate average (or specific gravity = 1·28).

It is usual to calculate the weight of coal per foot thick per acre; thus an acre contains 4840 sq. yards, or 43,560 sq. feet. At a foot thick, 1 acre contains 43,560 cub. feet, which, at 80 lbs. to the cubic foot, weigh 3,484,800 lbs., or about 1555 tons. Probably no coal in this country weighs less than 1500 tons per foot thick per acre, and very few seams, except anthracite, reach 1600 tons per foot thick per acre; 1550 tons may be taken as an approximate average weight. Having fixed on the weight per foot thick per acre, it is a simple matter to multiply this by the average thickness of the seam in feet and decimals; thus, if the coal averages 4 feet 8 inches in thickness, the weight per acre is say 1550 tons × 4·66, or 7233 tons.

Produce of Coal Seams.—Owing to loss in working, the tonnage of coal obtained per acre is, of course, less than the actual tonnage existent; a very usual figure taken to represent the actual produce of coal is 110 tons per inch per acre. Thus, if the thickness of the seam is 5 feet, the produce will be 110 × 60 = 6600 tons per acre. The actual weight of coal existing per acre, supposing it to be of the average specific gravity, will be 1550 × 5 = 7750; thus the allowance for waste in working in this case is just under 15 per cent.

Increase of Area or Thickness due to Inclination.—When the acreage of coal is measured off a plan, it is necessary, in order that the correct tonnage may be found, that the thickness of the coal should be measured on a line perpendicular to the plan, that is to say, on a vertical line. If the seam is inclined, the thickness measured on a vertical line will be greater than the thickness as measured at right angles from roof to floor. If, therefore, the thickness of the seam as given is a measurement at right angles to the dip, it will be necessary to increase the thickness to that given by the measurement of a vertical line through the coal. The proper increase in thickness can be ascertained from a drawing on a large scale (see Fig. 161). A horizontal line is shown, and the seam is drawn according to the angle of inclination, say 25°, and is plotted to the thickness measured, 4 feet; a vertical line is now drawn, and the thickness of the coal on this line can be scaled.

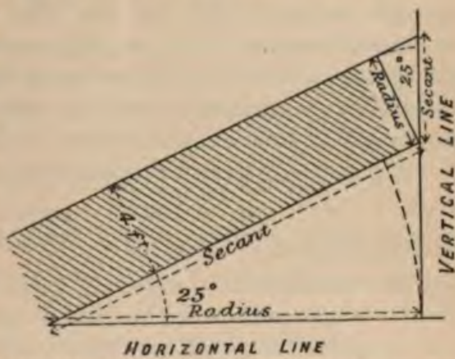

FIG. 161.—Increased thickness of coal measured on a vertical line when seam is inclined.

Instead of making this drawing, the thickness on the vertical line can be more quickly and accurately calculated. It will be seen from the figure that if the horizontal line is treated as the length of the radius of an arc, the inclined line (of which the horizontal line is the plan) is equal to the secant. In the same way, if the thickness of the seam measured at right angles to the dip is treated as the radius of an arc, the secant of that arc is equal to the thickness of the seam measured on a vertical line.

Assuming the inclination of the seam to be 25°, and radius 1, the secant is 1·1033779, or say 1·10338; this, multiplied by 4, gives the thickness of the coal in a vertical line. The tonnage of coal under any horizontally measured area is then found by multiplying it by this increased thickness in inches, and the tons weight per inch. The same result is arrived at if, instead of increasing the thickness, that is taken as measured at right angles to the dip, and the acreage is increased from that measured on the plan to that which might be measured on the slope; the increase of measurement will be in the ratio of the radius to the secant. Thus, if the acreage on the plan was 100, the acreage of a seam of coal lying at an inclination of 25° will be found as follows:—

1 : 1·10338 :: 100 : 110·338

Increased acreage 110·338.

The shape of the figure from which the acreage is scaled is immaterial as long as the whole acreage is on the same inclination.

Increase of Tonnage due to Inclination.—In case the tonnage has been calculated from the thickness of the seam as measured at right angles to the dip and from the area on the plan, the increase of tonnage due to the dip can be calculated by increasing the tonnage in the same ratio as the secant of the angle of inclination is greater than the radius; thus, if the tonnage calculated is 1000, then, to allow for the increased tonnage due to a dip of 25°, we have the following sum:—

1 : 1·10338 :: 1000 : 1103·38

Increased tonnage, 1103·38.

The calculation of tonnages extracted from veins or pockets of ore or quarries and sandpits is much less simple, owing to the irregular shape of the excavation, and a number of longitudinal and transverse sections are often necessary for accurate calculations.

Calculation of Contents of Cuttings and Embankments.—The contents of a cutting may be calculated from the average width, depth, and length. The quantity is generally given in cubic yards; if the calculation is made in feet, the sum must be divided by 27 to give the result in cubic yards.

Fig. 162, *a*, shows a cutting in section, from which it will be seen that the average depth of the cutting is ascertained from the measurement of the depth in six equidistant lengths of cross-section, measuring respectively 3, 9, 12, 12, 9, 3, the sum of which, divided by 6, gives the average depth as 8 feet. The total width of the cutting is 60 feet, and the area of the cross-section is therefore 60 × 8 = 480 sq. feet, and if the length is 100 feet, the cubical contents are 480 × 100 = 48,000 cub. feet.

In case the cutting varies in section, measurements must be taken to ascertain the average area of the cross-section. Thus, referring to Fig. 162, *b*, a cross-section is shown of which the average width is 40 feet and the average depth 4·5 feet; the area of the cross-section is therefore 40 × 4·5 = 180 sq. feet. If the distance between *a* and *b* is say 100 feet, and *the change of section is regular and gradual*, the average width of the top of the cutting will be 50 feet, and the average depth at the centre will be 9 feet. The area of a cross-section midway between *a* and *b* will thus be (20 × 9) + (15 × 9) = 315 sq. feet. The cubic contents may then be found by the prismoidal formulæ.[1]

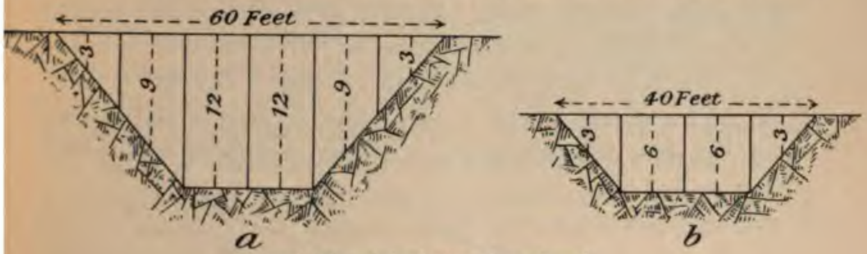

FIG. 162.—Calculation of earthwork.

Prismoidal Formulæ.—Let A_1, A_2, A_3 be the areas of three sections at equal distances apart, then the volume of the portion between A_1 and A_3 will be $V = \frac{A_1 + 4A_2 + A_3}{6} d$, where d is the distance between A_1 and A_3. In the case above taken—

$$V = \frac{480 + (4 \times 315) + 180}{6} \times 100 = 32{,}000 \text{ cub. feet}$$

Where the cross-slope of the ground is considerable, and the depths of the cutting differ widely, cross-sections must be taken at each place, and the areas calculated independently; and the cubic contents should then be calculated by the prismoidal formulæ.

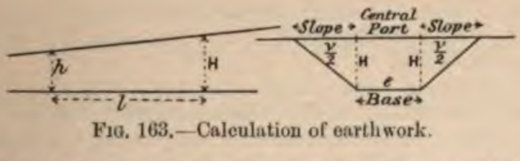

FIG. 163.—Calculation of earthwork.

The following formulæ may also be used in the measurement of earthworks.[1] Referring to Fig. 163, **H** and h are heights of section in feet at each end of a length l in feet; **V** is the sum of the areas of the two slopes at one end; v is the sum of the areas of the two slopes at the other end; and e is width of base of cutting. The slopes are calculated as the frustum of a pyramid, the centre as that of a wedge.

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[1] If the central cross-section had been multiplied by the length, it would give the contents as 31,500 cub. feet, or within 2 per cent. of the real quantity.

B = cubic contents of both slopes $= \frac{1}{3}(\mathrm{V} + \sqrt{\mathrm{V}v} + v) \times l$
D = cubic contents of central part $= \frac{1}{2}(\mathrm{H} + h)e \times l$
C = total cubic contents of length l in feet $= \mathrm{B} + \mathrm{D}$

If the transverse section is on a slope instead of horizontal, as in Fig. 164, the two slopes must be calculated separately.

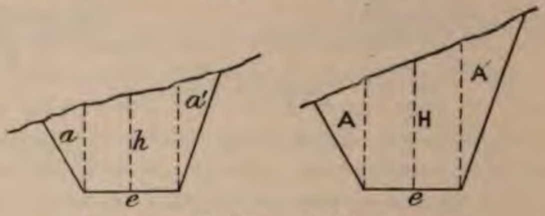

FIG. 164.—Calculation of sidelong ground.

A and A_1 are the areas of slopes at one end, and a, a_1 at the other.

$$\text{Then } \mathrm{B} = \tfrac{1}{3}(\mathrm{A} + \sqrt{\mathrm{A}a} + a)l + \tfrac{1}{3}(\mathrm{A}_1 + \sqrt{\mathrm{A}_1a_1} + a_1)l$$
$$\mathrm{D} = \tfrac{1}{2}(\mathrm{H} + h)e \times l$$
$$\mathrm{C} = \text{as before } (\mathrm{B} + \mathrm{D})$$

Earthwork tables are published which are designed to make the calculations shorter.[2] They are all based on similar formulæ to the above.

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[1] *Royal Engineers' Aide Memoire Pocket-book.*

[2] "Handy General Earthwork Tables," by I. H. Watson Buck (Crosby Lockwood and Son, London). Price 3*s.* 6*d.*

"Earthwork Tables," by Joseph Broadbent (Crosby Lockwood and Son, London). Price 5*s.*

"Earthwork: A Method of calculating the Cubic Contents of Excavations and Embankments by the aid of Diagrams," by J. C. Trautwine, C.E. (E. and F. Spon, London). Price 8*s.* 6*d.* net.

"Earthwork: Diagrams for the Graphic Calculation of Earthwork Quantities," by Alfred Henry Roberts, C.E. (E. and F. Spon, London). Price 10*s.* 6*d.* net.

"Earthwork Tables," by W. Macgregor (E. and F. Spon, London). Price 6*s.*

"Earthwork Tables," by David Cunningham, C.E. (E. and F. Spon, London). Price 10*s.* 6*d.*

"Tables showing the Contents of Excavations, Areas of Slopes, etc.," by George P. Bidder (published by Vacher and Sons, Westminster). Price 2*s.*

The calculation of the contents of embankments is similar to that of cuttings.

Restoration of Damaged Land.—When a mine is abandoned, it is often necessary to restore the land to an agricultural condition, and, if there are heaps of shale or other waste, they must be levelled and covered with soil. The soil from under this heap will probably have been removed before the heap was tipped, and will now be available; if not, it might be necessary to incur the increased expense of removing the shale-heap in order to excavate from beneath it the soil so buried. This, of course, would be a very expensive process, in many cases more than the value of the land. In levelling the heap, however, it will cover a greatly increased area: if the soil from this increased area is removed and respread over the levelled shale, it may be made to cover the whole area.

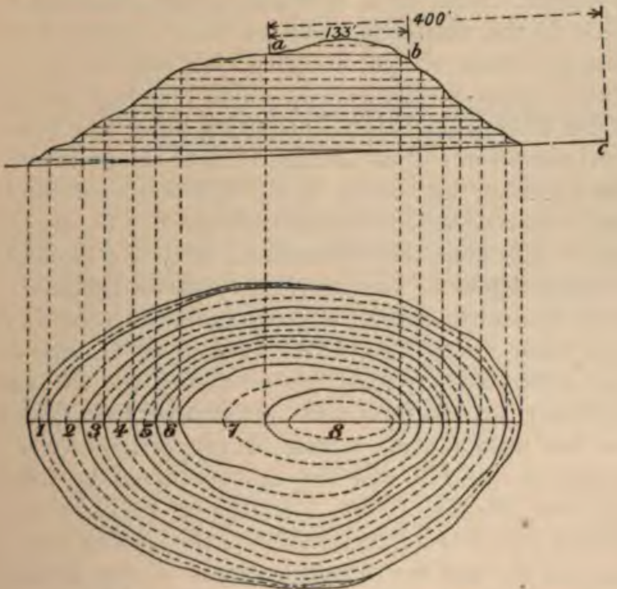

Fig. 165.—Calculation of contents of shale-heap.

Calculation of Contents of Shale-heap.—It is sometimes desired to calculate the area that will be occupied by the shale-heap when levelled, and to do this the cubical contents of the shale-heap are measured. This may involve a great deal of calculation and measurement. Fig. 165 shows a plan and elevation of a shale-heap. In order to obtain the exact cubical contents of this heap, it would be necessary to mark out, with the level, contour lines, as shown on the plan, and then to survey the position of the pegs. If these contour lines are at equidistant altitudes of say 6 feet, the heap will be divided into a series of horizontal sections, shown on the plan 1, 2, 3, 4, 5, 6, 7, 8. The average area of each section will be the area enclosed by the dotted lines half-way between the contour lines; and the cubical contents of each section can be calculated by multiplying this area in square feet by the depth in feet, which, in this case, is 6 feet.

Where there is no particular reason for desiring to know the number of cubic yards in the heap, the area it will occupy when levelled can be roughly ascertained with much less labour. Thus, if it is decided that the slopes of the waste-heap, when it is "levelled," shall be 1 in 10, it is only necessary to know the profile of the shale-heap, and to draw a give-and-take line of section at a slope of 1 in 10, equalizing cutting and bank, to find approximately the distance to which the slope will extend.

It must be borne in mind, in drawing the give-and-take line, that it represents one of an infinite number of equidistant radial lines drawn from the centre of the hill to the circumference, and that the width of the cutting between any two radial lines is therefore less near the centre than near the circumference, and for equal depths of cutting the amount of material on any given line of section between any two radial lines varies directly as the distance from the centre. If the hill is levelled down from the centre in every direction all round, the average depth of the cutting on the hill must be greater than the average depth of the embankment formed, in proportion to the area of the cutting on the hill and the area occupied by the ground removed from the hill. Thus, referring to Fig. 165, the centre of the hill is at *a*, the average radius of the cutting may roughly be taken as *ab*, and the average radius of the ground to be filled up may be roughly taken as *ac*. Suppose the length *ab* to be 133 feet, and the length *ac* to be 400 feet, then the relative areas of the circle and ring as described by those radii is as $133^2 : 400^2 - 133^2 ::$ average depth of bank : average depth of cutting; therefore the depth of the cutting at *b* will be 8 feet for 1 foot in depth of embankment at *c*. It must, however, be borne in mind that the cutting at *b* will be more solid than the embankment at *c*, and, making some allowance for this, the depth of the cutting at *b* may be say six times the depth of the embankment at *c*. In order to arrive at this result, it may be necessary to draw a few trial sections, and modify the radii *b* and *c*.

In cases which very frequently happen in practice, no calculation is necessary (see Fig. 166). In this case the slope agreed upon as suitable for restored land, say 1 in 10, is first ruled off as shown on the longitudinal section, giving approximately equal bank and cutting. This reduces the height of the hill, and this reduced height is shown on the transverse section by a dotted line. A give-and-take line at a slope of 1 in 10 is then drawn on the transverse section, and the height of the hill still further reduced. The area occupied by the heap spread out can be scaled off the plan made by projecting the give-and-take section lines.

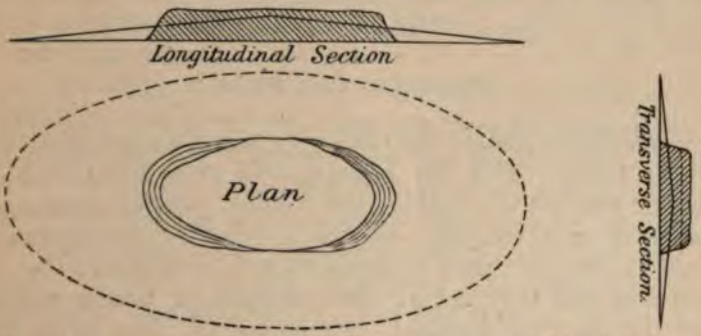

FIG. 166.—Approximate method of finding the area occupied by shale-heap when levelled.

CHAPTER XV.

SURVEYING BORE-HOLES.

It frequently happens that bore-holes are deflected from their vertical course. This has been noticed particularly with the diamond boring tool, of which the grinding action is more easily carried on in a crooked hole than boring by percussion. It is not surprising that bore-holes should be crooked; the only wonder is that they are so often straight, or nearly straight. Take the case, for instance, of rope boring. The bore-hole is made by a falling chisel at the bottom of a straight, stiff bar of iron not more than 30 feet in length. Such a bar cannot, of course, go round an angle, but it can go round a curve, just as a railway waggon with a long wheel-base can go round a curve. Suppose that there are enlargements on the rod intended to nearly fit the hole, these enlargements being 20 feet apart and ½ inch less in diameter than the hole, then the rod may lie at an inclination of 1 inch in 20 feet to the direction of the hole, or 1 in 240. Considering, however, the rod as the chord of an arc, and the 1-inch play as the versed sine, this corresponds to an angle of about 58′, and the radius of the circle will be found as follows: The natural sine of 58′ is 0·0169 feet; the actual sine taken in the bore-hole is 10 feet; then 0·0169 : 10 :: 1 : radius of the circle. The bore-bole is thus started on a curve with a radius of about 593 feet, and if it should continue on this curve for a distance equal to the radius—that is, 593 feet—the direction of the bore-hole may be changed to the extent of 60°. There is, however, a continual tendency on the part of the falling weight to straighten the hole. If the length of the straight, stiff bar between guides is less than 20 feet, the angle of possible deflection will be greater than 58′.

In boring with rods there is the same liability to deflection, and the angle of deflection does not depend on the length of the boring-rods, but on the length of absolutely stiff rod below the sliding-joint or free-fall arrangement. Although the boring-rods, when taken individually, may seem very stiff, yet when several hundred feet are screwed together, they make a very flexible rod, that will easily go round a curve. As in the case of rope-boring, the tendency of the cutting tool is to go straight, and a crooked hole is the result of some deflecting cause, such as a hard pebble or boulder, or the hard surface of some highly inclined stratum.

In considering the action of a revolving or grinding borer like the diamond rock-drill, somewhat similar considerations prevail, but there is a greater tendency to deflection from a straight line; part of the weight of the rods necessarily rests upon the boring head in order to give the requisite pressure to grind away the ground. This weight would naturally tend to bend the rods in case of any jar, tending temporarily to deflect them from a perfectly straight line; the stiffness of the rods, and the length of the stiff part, and the tightness with which they fit the bore-hole, have to be relied on to keep the hole straight.

In the diamond borer the crown fits the hole, but the core-tube, to permit the free passage of water and sand up the bore-hole, is frequently a good deal smaller than the bore-hole, thus permitting of a considerable deflection, the amount of which can be calculated in the same manner as that given in the example for rope boring.

If the bore-hole should be deflected by coming in contact with a highly inclined smooth surface of rock, there is no reason why the deflection should not continue until some other surface is met with, tending to cause a deviation in the other direction. This accounts for the circumstance that bore-holes frequently deviate greatly from the vertical, sometimes, it is said, to the extent of 40° or 50°, or even more; and, indeed, there is no absolute security that a bore-hole will continue to descend; it might gradually turn into a horizontal direction, or even into an upward direction.

In order to ascertain the course that has been taken by a bore-hole, and to prevent great and wasteful deviations from the intended line, it is just as necessary to survey a bore-hole as it is in tunnelling to survey the tunnel. There are, however, great difficulties in surveying a hole which naturally suggest themselves. These difficulties have been overcome by several ingenious instruments. The first of which the writer has any knowledge was designed by Mr. G. Nolten, Dortmund, Germany.[1]

Nolten's Instrument.—The object of this instrument is to ascertain the inclination of the bore-hole and the direction of the inclination—that is to say, whether the inclination is towards the north, south, east, west, or some intermediate point of the compass. One of the most important parts of the instrument is shown in Fig. 167. This is a glass cup, in which is a liquid, the level of which is shown by the line *ab*; this liquid may be hydrofluoric acid, which acid has the quality of dissolving glass. If, therefore, it is allowed to stand in a glass cup, the glass below the surface of the liquid will be gradually dissolved. Suppose, then, that the glass is inclined so that the level surface of the liquid is on the line *ab*; if the liquid is allowed to rest with the cup in this inclined position, it will eat away the glass up to the line *ab*.

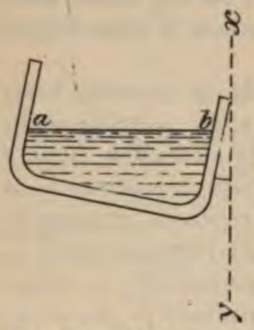

FIG. 167.—Nolten's instrument, showing glass cup inclined.

If, instead of pure hydrofluoric acid, a mixture of 1 part acid and 4 parts water is used, half an hour will be sufficient time to make a clear and permanent mark at the surface of the liquid. If some of the liquid is now poured out of the glass, and the vessel is allowed to stand in a perfectly vertical position, the surface of the liquid *dc* (see Fig. 168) will be level, and if left stationary for half an hour, this lower surface-line will be etched upon the glass. It will be readily seen that the angle the line *ab* makes with the line *dc* is equal to the angle made by the sides of the glass with the vertical line *xy* (Fig. 167) when it is held in an inclined position, or in other words, the angle *aec* (Fig. 168) is equal to the angle *bxy* (Fig. 167).

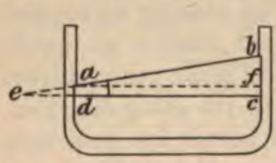

FIG. 168.—Nolten's instrument, showing glass cup, level and angle of inclination marked on glass.

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[1] *N. Eng. Inst. M.E.*, vol. xxix. pt. 2: Paper by C. Ziethen Bunning and J. Kenneth Guthrie.

Then, in order to ascertain the inclination of a bore-hole, it is only necessary to lower such a glass, partly filled with this acid liquid, rigidly fixed in a straight line with the sides of a long tightly closed tube.

Let this tube, then, be lowered down the bore-hole to the bottom, or to any other required depth. Whilst it is being lowered, the acid will shake about and will leave no clear mark upon the glass; when it has reached the required depth, if it is allowed to remain unmoved for half an hour, the level of the liquid in the cup will be etched upon the glass, and will form a permanent record of the angle of inclination of the glass, and also of the tube in which it was fixed. The glass may now be withdrawn and placed upon a perfectly horizontal table, and the difference between the surface of the liquid and the mark upon the glass will show the angle of inclination of the bore-hole. If a little of the liquid is poured out of the glass, and it is then placed on the horizontal table, and left stationary for half an hour, a line corresponding with the horizontal line will be permanently etched on the glass, and the angle of inclination can afterwards be measured at leisure.

In order to record the direction in which the hole is proceeding, in case it is not perfectly vertical, another instrument is combined with this one, and fixed in the same tube. This instrument consists of a compass-needle free to revolve in a horizontal plane on a vertical pivot, and of a watch which can be set to operate a lever, so that the needle can be clamped at the exact time to which the watch is set. Then, suppose the instrument to be lowered down the bore-hole, the watch having been set to operate in three quarters of an hour, one half-hour is occupied in lowering the instrument to the required depth in the bore-hole; one quarter of an hour remains for the needle to steady. At the expiration of that time, by the action of the watch, the needle is clamped in the magnetic meridian. Since the compass-box is fixed in the same case as the glass containing the acid, the direction of the inclination of this glass can be ascertained before they are disconnected one from the other.

The construction of the apparatus can be gathered by reference to Figs. 169 and 170. The points to be noted are that the recording apparatus must be placed in a water-tight case made of brass or some other metal which has no attraction for the needle; this case must be not only water-tight at ordinary pressures, but at very high pressures, as it may be lowered to the bottom of a hole (containing water) several thousand feet in depth. It must also be remembered that, if the instrument is put in a hole already cased with iron tubing, the compass-reading will not be exact, although the average of a number of readings may be approximately correct; where, however, there is no iron the reading of the needle will be correct, unless there are magnetic minerals or rocks, the existence of which will be discovered in boring.

As the sides of the hole are probably not perfectly straight, the longer the tube in which the apparatus is put the more likely will it be to show the correct inclination. The apparatus has been constructed small enough to go into a 3-inch hole, and doubtless, if necessary, one could be made to suit the smallest size of bore-hole.

The following is a description of the figures, given by Messrs. Bunning and Guthrie:[1]—

"The cylindrical casing of the instrument is shown in section in Fig. 1. The opening *aa* of the cylinder **A**, Fig. 1, is shown in Fig. 3 in section, and in Fig. 4 in perspective. Into the space *dp* (Fig. 1) is fixed, by means of a rod, the instrument shown in Figs. 2 and 5, which consists of three plates, *a*, *e*, and *o* (Fig. 2), shown by dotted lines in Fig. 1.

"These plates, or divisions, are placed at right angles to the longitudinal axis of the instrument, and are connected together by three vertical strips of brass, shown in Fig. 5.

"Fig. 4 shows the inner projecting flange of the cylindrical casing, also seen in *ad* (Fig. 1). This flange is divided into six equal parts, three of which are alternately cut out. Fig. 5 shows how the three plates are similarly cut out, so that they can slide through the projecting flange. After sliding the inner instrument through the flange in Fig. 4, it is turned one-sixth of its circumference towards the right, the catch *z* preventing it from going further; then the three outer projections of the upper plate in Fig. 5 will stand under the three inner projecting parts in Fig. 4.

"The cover *oa* (Fig. 1), is now placed over the rod, which, by means of the nut *m*, can be tightly screwed down. The rounded end-pieces in Fig. 1, held by nuts, are only used to round the

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[1] For convenience in reference, Figs. 169 and 170 are subdivided into separate Figs. (1 to 17), and these latter are the figures referred to in this description.

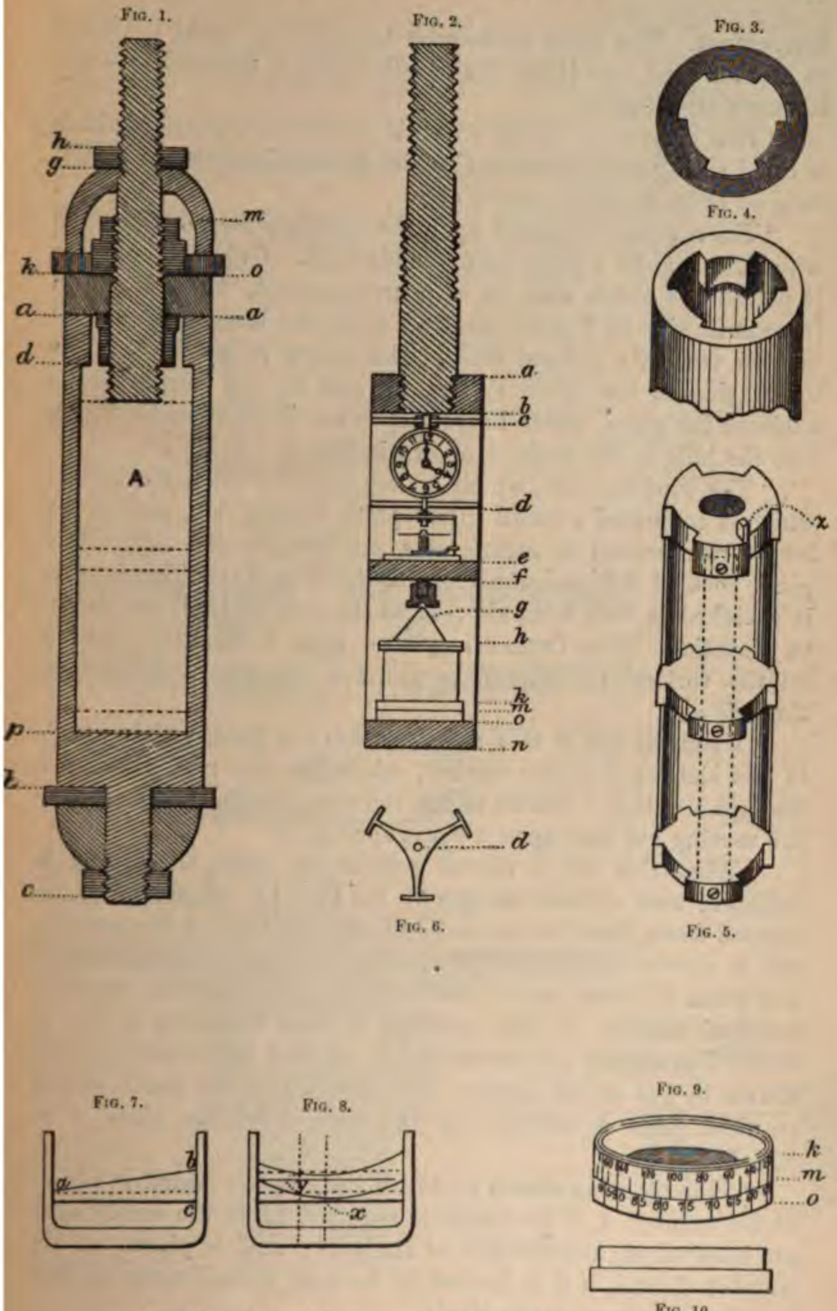

Fig. 1. Fig. 2. Fig. 3. Fig. 4. Fig. 5. Fig. 6. Fig. 7. Fig. 8. Fig. 9. Fig. 10.

Fig. 169.—Nolten's instrument. Details of working parts.

instrument. The glass containing the etching liquid is placed in a brass ring, *kmo* (Figs. 9 and 10), which is fastened into the lowest plate (Fig. 2).

"This glass is closed by a flat lid, the lower surface of which is lined with gutta-percha, and which is kept in its place by the cone *gh* (Fig. 2), and a screw above.

"The compass is placed upon the middle plate, the pin upon which the needle swings being made high. Over the compass is placed the watch with its stop arrangement, which is shown in natural size in Fig. 11, and in Fig. 13 the lever arrangement is seen twice its natural size. The watch is fastened on its upper side to the plate *c* (Fig. 2), and on the lower side is soldered the pin *d*, which keeps the watch in position by fitting into the hole in the guide *d*, shown in Fig. 6.

"The winding axle, which is lengthened outwardly, and to which is connected a small metal plate, *m* (Fig. 11), pushes the lever arrangement by means of a pin towards the right; this pin is seen at *d* in the small anchor *drf* (Fig. 13). This anchor is sketched as seen from above, and the rods under it are drawn in elevation. The former is placed in a horizontal position on the vertical rod *abr*, upon which a movable rod turns on the axle *g*.

"Upon the top of this rod the catch *f* is fixed, which moves in the slotting *f* of the anchor, while the pin *r* of the latter fits into the hole *r* shown in the rod *abr*. In the lower catch *p* the moving rod acts upon a brass spring, *x*.

"When this rod is moved towards the right, the spring is released, and strikes the pin *h* in Fig. 11, which, on being pressed down, fixes the magnetic needle. In Fig. 12 the movable rod is shown in two positions, before and after being stopped. The point *d* is kept out of reach of the plate *m* by the movable rod, and retained in that position so that the watch is free to work. The anchor and movable rod are held fast in the position shown by the dotted lines. The stopping of the watch at any required time is effected by the placing of the plate *m* in Fig. 11.

"This plate, as shown in dotted lines in the figure, is placed at the number 4, if the watch is required to fix the needle after an interval of four-fourths of an hour; and is placed at the number 3 or 5, if it is desired to fix it at three-fourths or five-fourths of an hour respectively.

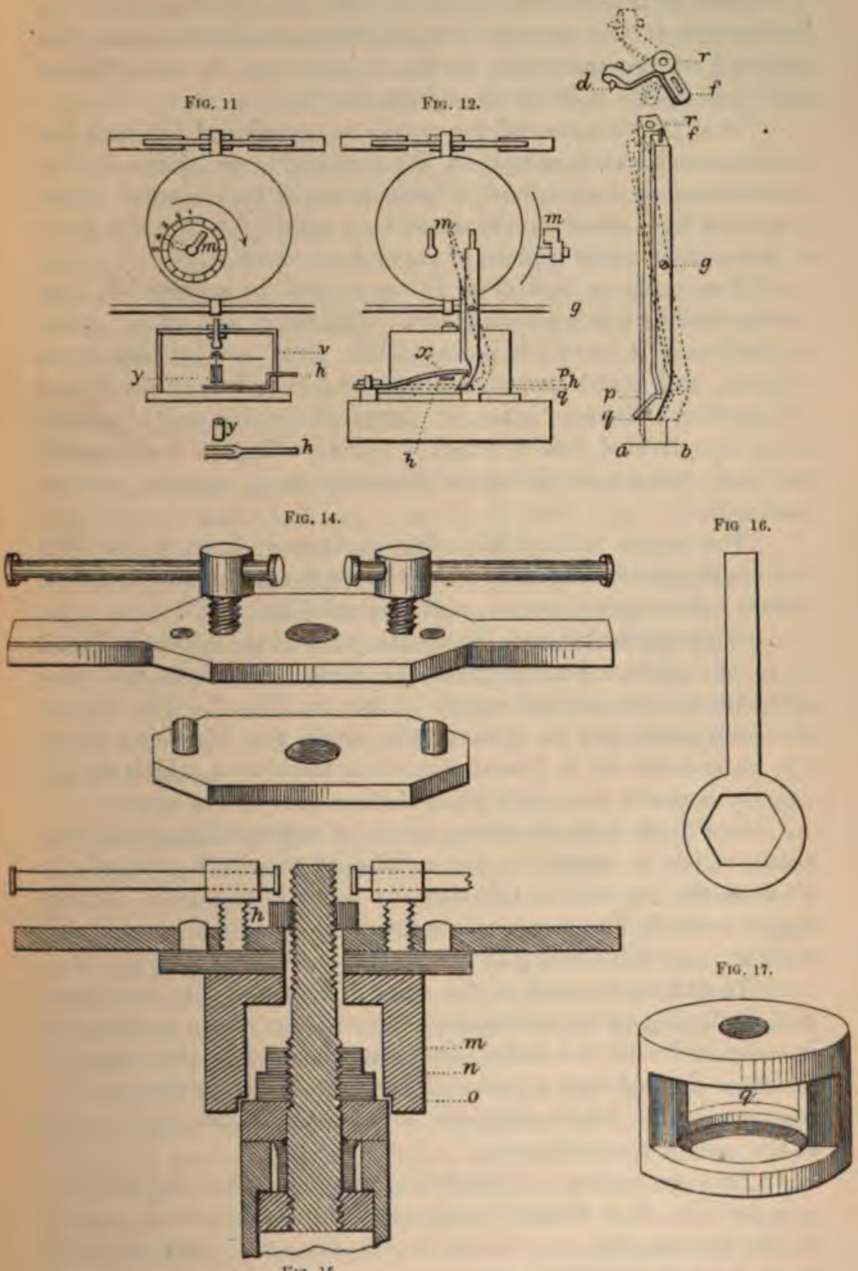

Fig. 170.—Nolten's instrument. Details of working parts.

"Should it take, for example, half an hour to lower the instrument to the measured depth where it is to remain, the stopping of the magnetic needle occurs when it is in perfect rest, if the plate *m* is set on the number 3.

"That the stopping of the needle is occasioned through the plate *m*, and not, perhaps, by the shaking loose of the spring in lowering, and also that it takes place at the required time, is proved by a small mark, made by a pencil fixed in the point of the anchor, upon a piece of paper fixed upon *m*.

"The compass case (Fig. 11) is covered by a glass lid, kept in its place by a pin placed above. The upper rim of this lid is divided into a hundred parts visible from within and from without, and stands concentric with the ring *kmo* (Figs. 9 and 10), in which the acid glass is placed, the under half of which, *mo* is also divided into a hundred parts, so that in both vessels the zero point and all other divisions stand vertically under each other.

"The upper half of this ring is turned down to half its thickness, seen in Fig. 10. Over this part a brass ring is fitted, which can be turned round, and is divided into 360°.

"Suppose, for example, the north point of the needle in Fig. 2 is to the right. Also suppose the glass fixed into the ring (Fig. 10) has the etched marks shown in Fig. 7. The dip of the instrument will be towards the south, the dip being where the water-level *ab* is lowest, therefore towards *a*, which stands exactly opposite the north point if the marks are as shown.

"In Fig. 8, with the lower curve as representing the lowest point, which is exactly in the middle of the back side of the glass *x*, the dip will in this case be towards the west. If the upper curve in Fig. 8 represents the lowest point of the etching fluid at *y*, so will the dip be south-west.

"To find easily, and at the same time accurately, the direction of the dip of the instrument, the position of the north point is compared with the before-described glass lid of the compass, which is divided into a hundred parts, and the number on the compass-holder which happens to be opposite the north point of the needle when stopped.

"Take, for example, this number to be 50. Then the movable ring *km* (Fig. 9) is turned round until the zero point is over 50 on the bottom ring *mo*, which is concentric with, and similarly marked to, the divisions on the compass case above mentioned.

Upon the movable ring, the zero point of which will be vertically under the north point of the needle, the bearing is read of the lowest point of the etched ring in the glass, which, in the example given in Fig. 7, is 180°, and in Fig. 8, 270° and 225° respectively.

"Finally, the Figs. 14, 15, and 17 represent the screw-press, which is placed over the lid of the instrument *ao* (Fig. 1). The key (Fig. 16) works in the space *q* of the part (Fig. 17). On this press being placed over the lid, pressure is brought to bear on the cover *ao* by means of the two screws shown in Fig. 14, the pressure being followed up by the nut *mo* turned by the key (Fig. 16); the whole of which is shown in section in Fig. 15. After sufficient pressure has been brought to bear, the nut *h* is loosened, and the parts 14, 15, and 17 taken off.

"This covering, together with the packing round the rod, has been proved water-tight at a depth of 3280 feet. The manner by which the middle rod is made water-tight is shown in Fig. 1, where *ad* represents a sort of metal gland, round the top inner edge of which packing is placed. . . . As regards the impermeability to water of the instrument, the following experience was gained: When hard or soft gutta-percha was used, it proved ineffective at a depth of 1312 feet; but when varnished paper was used at a depth of 3280 feet, no water was found in the instrument."

The writers of the paper found that when the instrument was put into a wrought-iron tube, the reading of the compass needle was sometimes 39° different from the true direction of the tube, but an average of three readings made within a length of 8 feet gave a bearing within 5° of the true direction. Therefore, in ascertaining the direction of a hole lined with iron tubes, it would be necessary to take at least three observations at a distance of 2 or 3 feet one from the other.

Messrs. Bunning and Guthrie refer to the use of this apparatus in ascertaining the inclination of some bore-holes. Referring to the bore-hole Sirius, near Crefeld, on being tested at the depths of 890, 1100, and 1230 feet, it was found that the inclination from the perpendicular was 3°, and the bearing W.S.W. Another bore-hole at Tellus, by Uerdingen, at depths of 750 and 796 feet, gave an inclination of 11°. In the bore-hole at Berggeist, at a depth of 600 feet, the inclination was 4½°. These three bore-holes were put down by percussion boring.

In 1874 the bore-hole Gustav Adolph, near Dienslaken, bored with a turning borer,[1] was stopped at a depth of 750 feet, and was tubed all the way with the intention of proceeding further at some future time. It was subsequently surveyed, with the following results:—

At a depth of	200 feet	... ...	2° inclination
,, ,,	300 ,,	... ...	3¾° ,,
,, ,,	430 ,,	... ...	8¼° ,,
,, ,,	750 ,,	... ...	47° ,,

After this it was decided not to proceed with the boring.

Other experiments were made in a bore-hole to a depth of 3280 feet. The diameter of the instrument used in the above experiments was 3 inches.

Macgeorge's Clinometer and Compass.—This ingenious instrument, designed by E. F. Macgeorge, of Victoria, Australia, is described in *Engineering* of March 13 and April 3, 1885. The principle upon which it is constructed is somewhat similar to the one last described; but gelatine is substituted both for the hydrofluoric acid and for the stop-watch. Ordinary gelatine is easily melted if the vessel containing it is immersed in hot water, whilst it solidifies at a temperature of about 70° F. When once the jelly has been melted, it takes several hours to stiffen, so that if a phial containing liquid gelatine is lowered into the bore-hole, it will not stiffen till some time after it has reached the bottom; if, therefore, a plumb-bob is suspended in the liquid gelatine in the phial, after the phial has been lowered to the bottom of the hole, or to some less distance, the plumb-line will hang in a vertical line; if the phial is now left for some hours, the jelly will stiffen round the plumb-bob. If the phial is vertical, its axis will be parallel to the plumb-bob; if, however, the phial is not vertical, the plumb-bob will not be parallel with its axis, and when it is withdrawn from the hole, the angle of inclination can be measured by putting the phial in a vertical position, and measuring the inclination from the vertical of the plumb-bob, which remains firmly embedded in the stiffened jelly.

If another phial is lowered down the hole in the same holder as that which contains the plumb-bob, and in this phial is a compass needle free to revolve in a horizontal plane upon a vertical pivot, and this phial is also filled with liquid gelatine, the needle will be free to swing into the magnetic meridian, and will take that direction as soon as the phial becomes stationary in the bore-hole. In the course of several hours the gelatine will stiffen, and the needle will be fixed in the meridian. If the hole is vertical, nothing is to be learned from the observation of this needle; but if the hole is inclined, the direction of the inclination is recorded.

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[1] Probably the diamond borer is meant.

Fig. 171 is a sketch, not of the instrument, but intended to show the principle upon which it acts. *aa* is part of a bore-hole; *bb* is a strong brass tube, watertight and capable of resisting the external pressure of water at the bottom of the bore-hole; *c* is a glass phial filled with gelatine; *d* is a small plumb-bob suspended in the phial; *e* is another glass phial filled with gelatine; and *f* is a compass needle on a vertical pivot; **N** is the north-seeking end, **S** the south-seeking end of this needle.

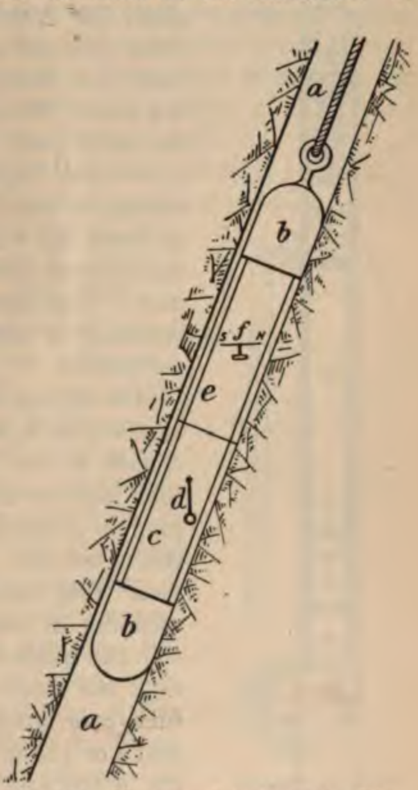

FIG. 171.—Macgeorge's instrument. Diagrammatic sketch, showing principle.

According to the above sketch, the bore-hole is inclined towards the south, and the angle of inclination is about 22½°. On looking at the figure, it is evident that the observer is on the east side of the needle, looking towards it, and the hole slopes towards the left hand, and is therefore sloping southwards. If it had happened that the hole was sloping towards the right hand, the slope would have been northwards; if the hole had been sloping towards the observer, it would have an easterly inclination, and if it had been sloping from the observer, a westerly inclination; and if the slope of the hole had been at some intermediate inclination, the exact bearing could be read off by careful observation.

Description of Macgeorge's Clinograph.—Fig. 172 shows the clinograph as made by Mr. Macgeorge. It is sketched by the writer from the descriptions given in *Engineering*. It shows a strong brass cylinder or guide-tube, *aa*, about 6 feet long, the diameter depending on the size of the hole. This tube is closed at the top with the solid plug *b*, the bottom of the tube is also closed tight; the tube and the plugs closing it must be strong and tight enough to resist the water-pressure at the bottom of the deepest bore-hole down which it may be sent. The kind of washer or other means required for making the plug water-tight are not shown.

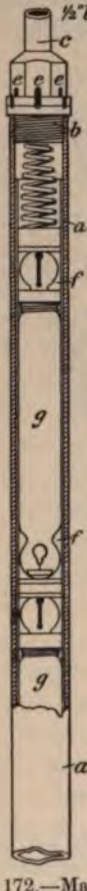

FIG. 172.—Macgeorge's instrument. Section, showing guide-tube in bore-hole, with the clinostats in position.

On the top of this guide-tube, and fastened to the plug *b*, is fastened a hollow brass tube *c*, with a bore in its upper part of not less than half an inch; where this tube joins the plug it is thickened and the bore enlarged; there are six holes, *ee*. It is intended to force cold water down this tube, which, escaping at the holes *ee*, will fall down outside the guide-tube *aa*, and so keep it cool in case the bore-hole should be warm. The tube *c* is made of brass, in order that it may not influence the magnetic needle; the top of the tube is jointed to a series of half-inch iron tubes reaching to the surface. These tubes serve not only for the passage of cooling water, but also for pushing the clinograph along the bore-hole to the required distance, the use of stiff tubes or rods being necessary if the hole should be horizontal or not very steeply inclined.

In the case of holes which are nearly vertical, or inclined at an angle of not less than 45° from the horizontal, and which are sufficiently cold for the jelly to congeal, these tubes are unnecessary, and a cord may be substituted as shown in Fig. 171. The lower part of the cord should be made of brass wire or vegetable fibre. Inside the strong guide-tube is a brass slide, *f*; in this brass slide are fixed a series of clinostats, one above the other, *g*, *g*. Mr. Macgeorge fixes five or six. The reason for having this number is in order that any accidental inaccuracy in the record given by one instrument may be checked by the record from another. On the top of the slide is a spiral spring, which keeps it tight and saves it from the shock of any concussion.[1] This instrument serves the purpose of taking the angle and direction of inclination of a bore-hole.

Inclination of Strata from Core.—It can, however, be adapted for the observation of the inclination of planes of stratification or other joints in the strata. For the purpose of this observation a core is left standing in the bottom of the bore-hole (Fig. 173); a core-holder is rigidly and securely attached to the bottom of the 6-feet brass guide-tube containing the clinostats. This core-holder is a brass tube set eccentrically to the guide-tube with a bell mouth, however, that guides the core-holder over the core. Inside the bell-mouthed tube is an inner split tube of brass, smaller than the core, and also bell-mouthed at the bottom. The apparatus is forced down by the tubes from the surface, and the bell mouth forced over the core; the inner tube is expanded as it is forced down over the core, and holds it firmly at the same time; the centre of this tube not being concentric with the core, great pressure is put upon the latter, and it is broken off by the descending movement of the holder.

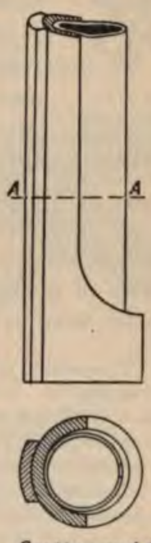

Fig. 173.—Macgeorge's instrument. Core-extractor.

The instrument is now left unmoved for several hours for the gelatine in the clinostats to stiffen, after which the whole apparatus is withdrawn, including the core, which will have on the surface the same position in regard to the compass needles and plummets of the clinostats as it had in the bore-hole.[1]

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[1] The writer presumes that there will be a similar spring at the bottom, and thus the slide will be saved from severe concussion both when being lowered down and when being drawn up.

The clinostat is shown on a larger scale in Fig. 174.[2] *a* is a straight cylinder of glass, fitting accurately within the guide-tube *f* (Fig. 172). The lower end of this glass cylinder terminates in a short neck and a bulb, *b*; the bulb is filled with liquid gelatine; a small glass float, *c*, carries a compass needle, *d*.[3] This lower bulb *b* is closed with a cork, *e*, thus preventing the escape of the needle and float. A small glass tube, *f*, passes through this cork to the top of the glass barrel, and out of the barrel through an air-tight cork and a screw capsule, *g*. The upper part of the tube *f* is enlarged into the bulb *h*; this bulb is also filled with liquid gelatine, in which is placed the plummet *i*. This plummet consists of a thin glass rod, *k*, terminating at the bottom in a plumb of solid glass; and at the top in a hollow glass bulb, *m*; this hollow glass bulb is a float. The size of this bulb is carefully adjusted to the specific gravity of the gelatine, so as just to carry the weight of the glass without being so light as to press with appreciable force against the top of the bulb *h*; the weight *l* naturally seeks the centre of gravity, and the glass rod *k* is in a vertical line, no matter what is the angle at which the barrel *a* is held.

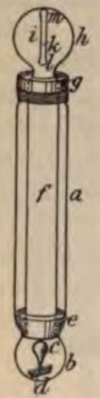

FIG. 174.—Macgeorge's instrument. Enlarged view of clinostat.

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[1] This method of taking the dip of the strata might be adapted to the hydrofluoric acid apparatus. It must, however, be borne in mind that any indications of the dip from a small bore-hole are apt to be misleading, as it is quite probable that the inclination of the piece of ground from which the core was extracted might be in the reverse direction of the general dip. If the inclination of a series of cores from top to bottom of a deep bore-hole is taken, there is a greater probability of being able to observe the general dip of the strata; but, in any case, the dip, as observed in the bore-hole, is only the dip that the rocks happen to have under the very small plot of ground through which the bore-hole passes.

[2] Sketched by the writer from the description given in *Engineering*.

[3] The description in *Engineering* refers to a pivot; it does not say what the pivot is. Possibly there may be a point underneath the needle, partly supporting it, but, if this is so, it is not plain, and it is not explained how the needle is kept from being jerked off the point. The writer is, therefore, driven to the conclusion above stated. If this is right, the only friction the magnetic force would have to overcome in drawing the needle into the meridian line would be that due to the liquid gelatine, and the friction of the upper part of the glass float pressing against the surface of the bulb; this pressure, however, would be very slight. As the float will have only just sufficient lifting power to carry the magnet clear of the bottom of the bulb, the needle itself can never come in contact with the sides of the bulb.

Before lowering into the hole, the gelatine is melted by warming; the apparatus is then lowered into the hole, and left stationary for say three hours, when it is withdrawn; the plummet and the compass needle are thus fixed in the transparent jelly in the relative positions they occupied at the bottom of the hole.

As the needle, when it is freely suspended, always turns to the magnetic north, it is only necessary to turn the phial till the needle points to the magnetic north in order to place the clinostat in the direction it had at the bottom of the hole; and since the plummet *i*, when freely suspended, always occupies a vertical position, it is only necessary to incline the barrel *a* till the plummet fixed in the jelly is in a vertical position, and then the instrument will be at the same inclination that it had at the bottom of the hole.

Macgeorge's Clinometer.—In order to facilitate the exact reading of the inclination and bearing of a bore-hole as recorded by the apparatus, before melting the jelly for future use, each clinostat—that is, the instrument shown in Fig. 174—is placed in a clinometer specially designed for this purpose. This clinometer is shown in Fig. 175. *aa* is a brass tube in which the clinostat is placed, and which it exactly fits; the bulbs appearing at either end, the plummet bulb at the top end, the compass bulb at the lower end. *aa* is fixed to a radial bar, *bb*, extending from and fastened to the horizontal axis *c*. Attached to this same radial bar are two microscopes, *d* and *d'*; the horizontal axis is attached by brackets to the horizontal tripod *ee*; the microscopes *d*, *d'* are so attached that they shall always be parallel to the horizontal tripod stand *ee*. If, therefore, this stand is

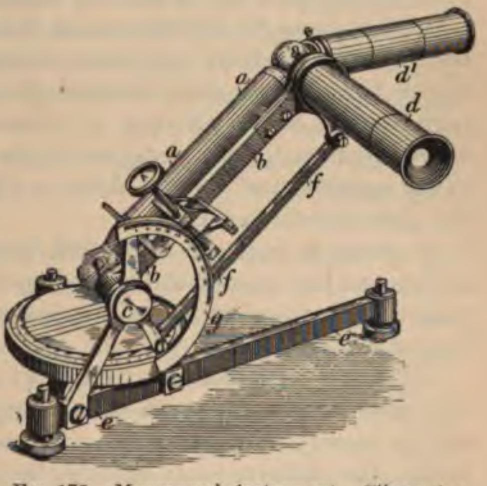

FIG. 175.—Macgeorge's instrument. Clinometer, with clinostat in position.

carefully levelled with a spirit-level, the plane of the microscopes *d*, *d'* will also be level; if the stand *ee* is inclined, the plane of the microscopes *d*, *d'* will be parallel to it.

It is not important that *ee* should be level. What is important is that the longitudinal axis of the microscopes should always be in a plane parallel to the stand, which is assumed to be a horizontal plane; but the phial-holder *aa*, attached to the radial arm *bb*, can be moved through an arc of 90°; therefore, as it is moved it is necessary that the microscopes should also be moved. The right-hand microscope *d* will always have its axis parallel to the base of the instrument; but the other microscope, *d'*, will require to be altered whenever the position of the radial arm *bb* is altered. For this reason it is fitted with a parallel motion, of which one rod is shown, marked *ff*; the action of this parallel motion keeps the longitudinal axis of *d'* always parallel to the base.

The object-glass of each microscope has one or more vertical lines drawn upon it. When the radial bar *bb* is moved, these vertical lines on the object-glass of the microscope *d'* will retain their vertical position, but those of the object-glass *d* will be moved through an angle similar to that through which the radial bar *bb* is moved; but, by means of the parallel motion above mentioned, the object-glass of the microscope *d* is adjusted to the movement of the radial bar, so that the vertical lines on the glass remain vertical.

It must be borne in mind that these lines are vertical only so long as the axis of the microscopes is horizontal; what is meant is that the lines are perpendicular to the plane of the base of the instrument, which is assumed to be horizontal.

These two microscopes are at right angles to each other, and with them the small plummet in the upper bulb of the clinostat can be clearly observed. The phial having been put into the holder, the observer looks through the microscope *d'*, and if the plummet is not parallel to the vertical lines, he turns the phial round in the tube until the plummet is parallel to the vertical lines. The observer then looks through the microscope *d*, and if the plummet is not parallel to the vertical lines on the object-glass, he moves the radial bar *bb* until the plummet becomes vertical. (The reader will bear in mind that all this time the jelly inside is congealed.) When the phial has been thus adjusted, so that the plummet appears vertical through both microscopes, the angle of inclination can be read by the pointer on the figures of the graduated arc *g*, which is attached to the base of the instrument.

The lower bulb is an inch or more above the centre of a horizontal revolving circular mirror with five parallel lines engraved across its face. Attached to the mirror is a graduated circle, which can be turned round in the ring that carries it. On the fixed ring is a pointer; this pointer is in a line drawn through the centre of the mirror at right angles to the horizontal axis *c*, and parallel to the direction of the radial arm *bb*, and opposite to the centre of the glass phial or clinostat. The centre line of the five engraved on the glass is coincident with the zero of the graduated circle; the mirror is turned so that the zero and centre line are coincident with the fixed pointer.

Reflected in the mirror will be seen the image of the needle embedded in gelatine, which, as we know, pointed north before it was fixed by congelation in the bore-hole. If, then, the reflected image of the needle is parallel to the lines engraved on the mirror, these engraved lines are in the magnetic meridian, and it follows that the clinostat, as placed in the tube-holder, is also in the magnetic meridian, and that the inclination of the bore-hole is northerly or southerly, according as the notched or north-seeking end of the needle, or the south-seeking end of the needle is pointing towards the index finger of the horizontal graduated circle.

If, however, the image of the needle is not parallel to the engraved lines, the mirror must be revolved until it is parallel, and until the zero of the graduated circle is opposite to the north-seeking end of the needle. The number of degrees through which the mirror has been moved will be the number of degrees from the magnetic north that the clinostat is now pointing. Thus, if the circle is moved through 20° to the right, it shows that the clinostat has been fixed 20° to the left of the magnetic meridian, that is, 20° north-west, and that is the direction of the bore-hole. If, however, the graduated circle were moved to the left 20°, it would show that the direction of the clinostat, as held in this clinometer, is 20° eastward of the magnetic meridian, and that is the direction of the bore-hole. If the graduated circle were moved 120° to the right, it would show that the clinostat is 120° to the left of the magnetic meridian. As 180° would be south, 120° is 60° east of south. If the graduated circle had been moved 130° to the left, it would show that the direction of the clinostat is 130° to the right of the magnetic meridian; and as 180° would be south, the direction of the hole is 50° south-west. Each of the six clinostats is observed in turn, and the mean of the observations is taken to represent the angle of inclination and direction of the bore-hole.

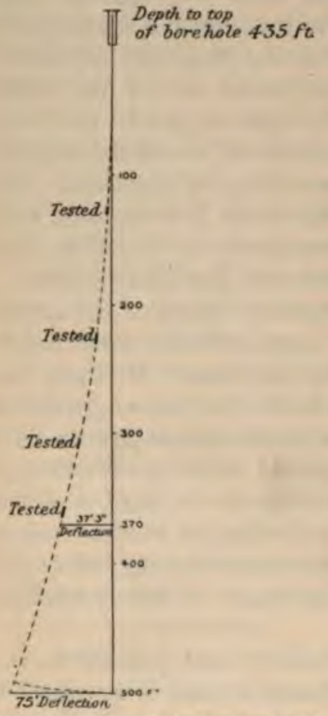

Fig. 176.—Bore-hole surveyed with Macgeorge's instrument.

The upper part of the clinostat which Mr. Macgeorge recommends for use with his core-extractor, differs from the preceding ones in having a minute compass floating in the hollow glass head of the plummet.[1]

This instrument has been used in surveying bore-holes in Australia. In one case, at a depth of 370 feet it was found that the bore-hole had deviated 37½ feet in a horizontal direction from the position of the bore-hole at the top.

Fig. 176 shows the survey of the bore-hole. The positions in which the clinostat was placed are there shown in the vertical section, the test being applied at 130 feet, 230 feet, 320 feet, and 367 feet. It will be seen that the deflection of the bore-hole is on a curve, which, if continued to a depth of 500 feet, would give a horizontal deflection of 75 feet.

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[1] It is not stated in the description from which the writer takes his account how the compass needle is fixed in the magnetic meridian before it is withdrawn from the bore-hole. If the hollow head of the plummet were filled with gelatine, the plummet would not float; it may be that a portion only of the plummet bulb has gelatine in it—just enough to fix the position of the needle.

CHAPTER XVI.

MISCELLANEOUS.

Surveying by Photography.—The photographic camera may often be useful to the surveyor, especially when collecting information in foreign countries, in helping to give a general idea of the configuration of the country, the shape of cliffs, and the situation of works; interesting views may also be got of underground workings by the aid of the magnesium, or the electric light. These views are rather for the adornment of a report, for its verification, and for the consideration of persons financially interested in mines, than for the practical use of the engineer.

The photographic camera can, however, be used for actual surveying, the process being known as "photogrammetry."[1] It is said to have originated with Colonel Laussedat, in 1850, and has since been largely developed in Germany, Austria, and Italy, and is also frequently used in Canada, America, etc., both for engineering and for military purposes.

The principle of the method is briefly thus: If a photograph is taken from a point whose position is already known, the direction of the axis of the object-glass and the focal length of the lens being also known, and the line of the horizon being marked on the picture, then the picture can be laid down on a sheet of paper on which it is desired to plot the survey, and will give the direction from the point of observation to all the points in the picture whose position is required. Two photographs of the same objects, taken from different points, define completely the position of each object, and also enable altitudes to be calculated or graphically determined.

The method is the same as that of the plane table, to be referred to later on, with the difference that most of the work which, with the plane table, has to be done in the field, is, with the photographic method, done in the office.

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[1] "The Application of Photography to Surveying," by E. Mouet (*Inst. C E. Proceedings*, vol. cxix. p. 414).

The camera used may be an adaptation of the ordinary photographic camera, or may be specially designed for photographic work.

This method of surveying is described by Mr. H. M. Stanley, in a paper read before the American Institute of Mining Engineers;[1] and there is no doubt that, by the careful use of cameras, a map may be produced from which the relative distances and altitudes of objects may be scaled, though this system would not be used where anything more than a rough approximation to the actual distance was required.

Referring to Mr. Stanley's paper, the properties to be surveyed comprised about six square miles of broken, mountainous country.

Mr. Stanley used an 8 × 10 Eastman camera; he used for the negatives celluloid films, with a "matt" surface on the back; he also used celluloid films for the positives, as being less likely to shrink than paper positives. Attached to the camera were four cylindrical levels, by means of which the optical axis could be set in a level line, and the base of the ground-glass also made level. The centre of the ground-glass was marked, and vertical and horizontal lines drawn through it. The position of these centre-lines was photographed on the negative by marks fixed on the carriers. The front board, carrying the lens, was adjusted till the optical axis passed through the centre of the ground-glass; the position of the front board was then noted, and a scale marked upon it, by which its movement above or below the central position could be measured.

In taking the photograph, the height of the front board above or below the centre-mark, and the height of the optical axis above the ground, were measured.

Mr. Stanley's method was to use the camera as an adjunct to a system of triangulation, the camera being fixed at a known point, and the view including some other known point (see Fig. 177). Here the camera is fixed at **X**, and pointed in the direction **A**, and includes a known point, **Y**.

On examining the negative, or the print from it, the distance of **Y** to the right of the centre-line **XA** can be measured with a

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[1] Glen Summit Meeting, October, 1891.

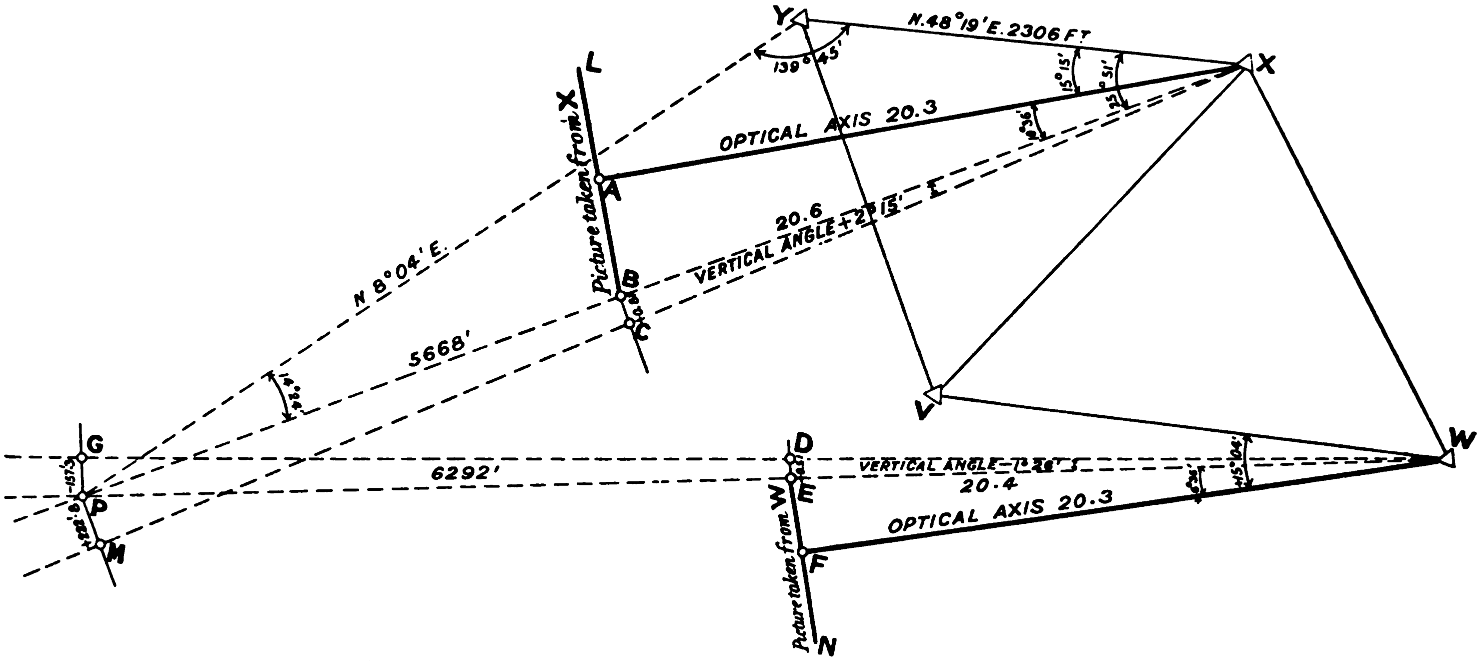

FIG. 177.—Diagram explanatory of the principle of photographic surveying.

scale. This distance is the altitude of a right-angled triangle whose base is the focal distance of the lens. In this case the drawing was made to a scale of one-half inch to 150 feet, and the focal distance of the lens used was 20·3 half-inches; the altitude of the right-angled triangle measuring 5·53 half-inches.

The angle at the base of the right-angled triangle can therefore be calculated. Tangent of angle $= \frac{\text{perpendicular}}{\text{base}} = \frac{5{\cdot}53}{20{\cdot}3}$ $= 0{\cdot}27241$, therefore angle $= 15° 15'$.

The centre line of the picture, that is, the optical axis, may now be drawn on the plan at an angle of 15° 15′ from the line **XY**, and the length of the line is 20·3 units. A line perpendicular to this, **LAB**, is the position of the picture.

In the same manner, the position **EFN** of the picture taken from the station **W** may be plotted by means of the known station **V** in the picture. A point, **P**, in both pictures is now observed, and with a scale the horizontal distance of this point to the left of the line **XA** is measured 3·8 units, from which the angle 10° 36′ can be calculated. Similarly, the angle 6° 36′ is obtained from the picture taken at **W**, and if these two angles are then plotted on the plan, the point of intersection gives the position of the point **P**. Instead of calculating the angles, the point **P** could have been located graphically thus, the distance **AB**, 3·8 as measured on the photo, may be plotted on the plan, and the distance **FE** is also plotted on the picture taken from **W**; it is 2·35 units to the right of the vertical axis. The lines **WE** and **XB** may then be produced to their point of intersection at **P**, and thus the position **P** is marked on the plan.

The elevation of **P** may also be measured. In the picture taken at **X**, **P** is 0·81 above the horizontal axis. To find the altitude of **P**, draw **BC** at right angles to **BX**, and equal to 0·81 units; prolong **XC** to **M**; let **PM** be at right angles to **PBX**; measure the distance **PM**, which is equal to 222·8′, which is the height of **P** above **X**. In the same way the height of **P** above **W** may be measured.

Instead of measuring the altitude, it may be calculated. In the right-angled triangle **XAB**, the side **XB** $= \frac{20{\cdot}3}{\cos 10° 36'} = 20{\cdot}6$ units; in the right-angled triangle **XBC** the tangent of the angle at the base $= \frac{0{\cdot}81}{20{\cdot}6} = 0{\cdot}03932$, the angle of elevation =

2° 15′. In the triangle **XPM** the side **PM** = 5668 (the length from **X** to **P**) × tan 2° 15′ = 222·8 feet.

Mr. Stanley says the best results are obtained when the sun is low, the lens being shielded from the direct rays of the sun, as there is a greater alternation of light and shade.

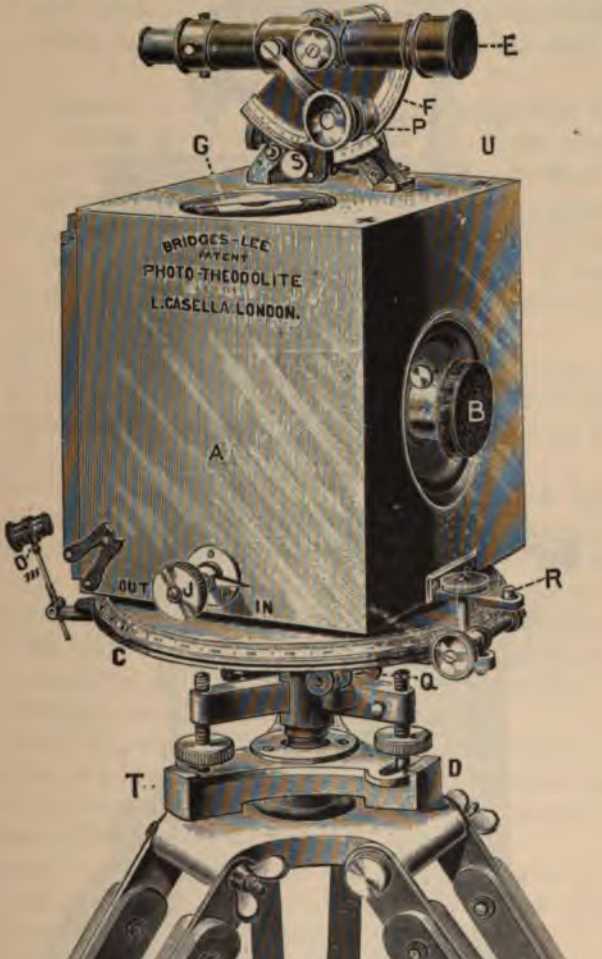

FIG. 178.—Bridges Lee photo-theodolite (outside elevation).

Bridges Lee Photo-Theodolite.[1]—This instrument, shown in Figs. 178 and 178A, is an ingenious combination of photographic camera and theodolite, specially designed for accurate photographic surveying by J. Bridges Lee, M.A., F.G.S. It will be

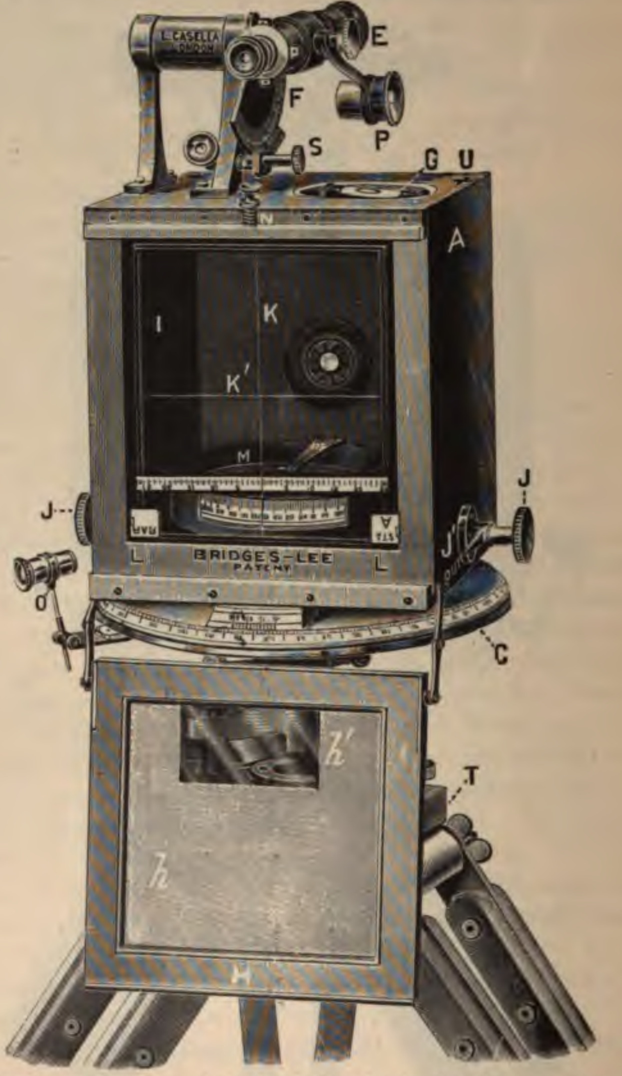

FIG. 178A.—Bridges Lee photo-theodolite, showing interior arrangements.

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[1] Maker, L. Casella, 147, Holborn Bars, London, E.C.

seen that the instrument consists of a rectangular box of aluminium, **A**, fitted with a rectilinear photographic lens, **B**, with iris diaphragm. The camera is mounted on a vertical axis, and can revolve round a horizontal graduated circle, **C**, a vernier reading on to this circle being fixed at the back of the camera.

The instrument is carried on a tribach stage with locking-plates and levelling-screws, **D**. On the top of the camera is fixed a telescope, which is free to rotate in a vertical plane when the instrument is accurately levelled; and, by means of a graduated arc, **F**, and vernier, vertical angles can be read to minutes. A revolving bubble-tube, **G**, is let into a recess on the top of the camera, which enables it to be levelled without disturbing its position.

The camera is provided with the usual ground-glass shutter **H**, and in the ground glass is a window of polished glass, *h'*, through which the inside arrangements of the camera may be seen.

Inside the camera box is a strong rectangular frame, **I**, which carries the compass-box **M**, and the vertical and horizontal hairs **K** and **K'**; by means of a rack and pinions, **JJ**, this frame can be moved along the bottom of the box, and is of such a size that when the dark slide is in position, and the shutter is raised, the hairs **K** and **K'** can be brought into actual contact with the sensitized plate; there are pointers attached to the pinion which indicate the position of the frame.

The magnetic compass **M** is provided with a vertical transparent scale, which passes quite close to the vertical hair **K**, and by this means the magnetic bearing of the centre line of the instrument is recorded on the photograph.

Attached to the frame which carries the hairs is a transparent horizontal scale of angular distances, which is photographically prepared by the makers.

Other parts of the instrument are: **S**, clamp and tangent screw for telescope; **Q**, clamp and tangent screw for horizontal graduated circle; **R**, clamp and tangent screw for camera; **P**, adjustable microscope for reading vertical angles; **O**, microscope with universal joint, to permit of its being used either for reading horizontal angles on the horizontal circle, or for reading the compass-bearings through the window in the ground-glass back; **LL** are small transparent tablets, on which the place, date, time, etc., can be written, and so recorded on the photograph.

The instrument is supplied with six double dark slides, to carry a dozen 5 × 4 plates either horizontally or vertically.

Fig. 178B shows a photograph taken with this camera, on which will be seen faint vertical and horizontal lines representing the images of the vertical and horizontal hairs in the camera.

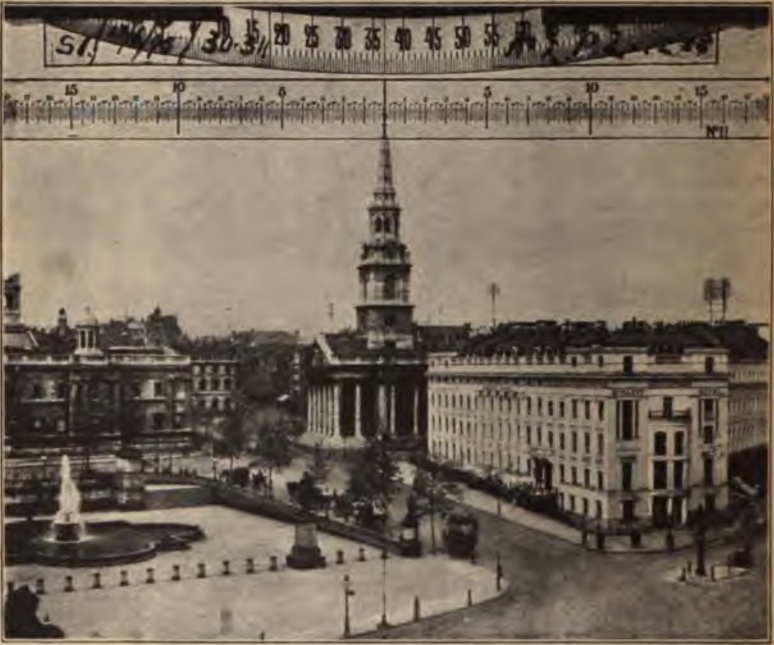

FIG. 178B.—Photograph taken with the Bridges Lee photo-theodolite.

The vertical hair intersects the compass scale at 37°; thus the magnetic bearing of the line joining the station where the camera was fixed, and any point on the vertical line on the picture is 37°. By means of the scale of angular distances the magnetic bearing of other points in the picture can now be ascertained. Whilst using the instrument for photography, it can also be used as a theodolite, and positions in the picture (or out of it) fixed in the same way as in ordinary surveying.

If a tacheometer is used, the distances to points in the picture can also be determined approximately, independently of observations from other stations.

Pillars of Coal or other Mineral left to protect Buildings.—As it is well known that the excavation of coal or other minerals leads to a subsidence of the surface, it is usual to leave a pillar of mineral ungot, or only partially got, for the support of buildings and works of a valuable kind. The conditions which decide whether or no it is necessary to leave a pillar come rather within the province of the engineer than the surveyor. When it has been decided to leave a pillar, it is the surveyor's business to set it out in the correct position, and to see that it remains unworked.

Size of Pillar.—The pillar is always larger than the building to be supported, in the same way as the foundations of a house or the pedestal of a column are larger than the house itself or the column erected.

As, however, the base of the pillar is many times deeper than the height of the building, ordinary architectural considerations do not govern the size of the pillar; this, indeed, is usually governed by experience gained in the locality. There is, however, a general consensus of opinion that the deeper the bed of mineral below the surface the larger the pillar required, if any pillar is left at all; thus, if a seam of coal 50 yards deep and 5 feet thick is left to protect the building, a margin of 25 yards would be considered ample; at a depth, however, of 150 yards a wider margin will be required, and at 300 yards a wider margin still. For ample security[1] the margin should be about one-third of the depth; thus at a depth of 300 yards the pillar should extend on every side a distance of 100 yards from the building; it is, however, only in the case of very important buildings indeed that such a large margin is left. Railway companies, who have to buy pillars for the support of important viaducts and tunnels, and frequently pay at a very heavy rate for the mineral left, usually consider a much smaller margin sufficient, and will probably in no case allow a margin of more than 40 yards, whilst in others the margin is very much less.

In a recent paper read before the Institution of Civil Engineers,[2] the following rule is laid down by Mr. S. R. Kay, Assoc. M.I.C.E., as to the size of pillar required under normal conditions. Let d denote the depth in yards; t, the thickness excavated in feet; and r, radius of support in yards. Then—

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[1] Even this margin does not give perfect security.

[2] Vol. cxxxv. *Proceedings of the Inst. C E.*, pp. 114, *et seq.*

$$r = \frac{\sqrt{3d} \times \sqrt[3]{t}}{0·8}$$

For example, an arched bridge for a road or railway requires support. The depth of the seam is, say, 400 yards; the thickness of the material excavated, say, 4 feet. Then—

$$r = \frac{\sqrt{3 \times 400} \times \sqrt[3]{4}}{0·8} = 68 \text{ yards}$$

or a pillar extending 68 yards beyond the structure on each side, will give the support necessary. The following table is taken from Mr. Kay's paper:—

TABLE XVI.

TABLE OF MINIMUM RADIUS OF PILLAR (IN YARDS) ACCORDING TO DEPTH OF SEAM AND THICKNESS OF EXCAVATION, CALCULATED FROM THE FOREGOING FORMULA

$$\left(r = \frac{\sqrt{3d} \times \sqrt[3]{t}}{0·8}\right)$$

Depth in yards.	THICKNESS OF EXCAVATION.							
	2 feet.	3 feet.	4 feet.	5 feet.	6 feet.	7 feet.	8 feet.	9 feet.
50	19·3	22·1	24·3	26·2	27·8	29·3	30·6	31·8
100	27·3	31·2	34·4	37·0	39·3	41·4	43·3	45·0
150	33·4	38·2	42·1	45·3	48·2	50·7	53·0	55·2
200	38·5	44·1	48·5	52·2	55·5	58·4	61·1	63·5
250	43·1	49·4	54·3	58·5	62·2	65·5	68·5	71·2
300	47·3	54·1	59·5	64·1	68·1	71·7	75·0	78·0
350	51·0	58·4	64·3	69·3	73·6	77·5	81·0	84·3
400	54·6	62·5	68·7	74·0	78·9	82·8	86·6	90·1
450	57·9	66·2	72·9	78·5	83·5	87·9	91·9	95·5
500	61·0	69·8	76·9	82·8	88·0	92·6	96·8	100·7
600	66·8	76·5	84·2	90·7	96·4	101·5	106·1	110·3
700	72·2	82·6	90·9	98·0	104·1	109·6	114·6	119·2

In each case the half-diameter of the piece of ground to be supported must be added to the calculated radius, to give the actual radius of the circle within which the coal is to be left.

Another rule sometimes recognized is that the pillar should extend on all sides to a distance equal to $\frac{1}{10}$ of the depth of the seams plus 20 yards. This rule would apply to horizontal seams in which the thickness does not exceed 5 or 6 feet.

The shape of the plan of the coal-pillar left is generally an enlargement of the plan of the plot of ground to be supported.

Pillars in Inclined Seams.—In case the seam is level, the pillar of coal, by universal consent, should extend equally on each side of the building or ground to be protected; but if the seam is inclined, then all unanimity of opinion ceases. A majority are of opinion that the pillar of coal should be moved so that it extends further from the building on the rise side, and does not extend so far on the dip side; in other words, the pillar of coal is moved uphill; the total acreage of the coal left (as measured on the plan) being the same as if the coal-seam was horizontal. Some mining engineers have gone so far as to say that the line of fracture of the strata (which occurs when the ground subsides to fill up the place from which the coal has been extracted) is at right angles to the dip, and that therefore a pillar of coal should be placed not vertically under the building, but around a spot fixed by drawing a line from the building at right angles to the dip of the seam. It is, of course, evident that this theory cannot be carried very far, as, in the case of highly inclined seams, it would lead to an absurdity. Other mining engineers deny that the inclination of the seam has to be considered, and would set out the coal-pillar for inclined and horizontal seams in the same form. Each side has cases in support of its theory. The author of this treatise has never taken either side, but would state that it is an undoubted fact that the line of fracture is sometimes vertical, sometimes inclined; and would suggest that a very little irregularity in the fracture might cause the subsidence to "pull" a long way over the edge of the pillar.

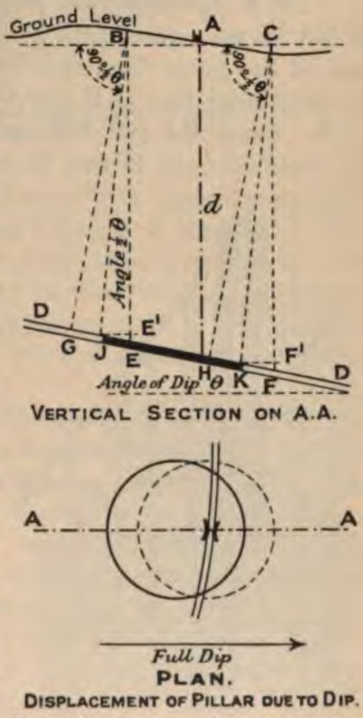

FIG. 179.—Position of pillar of coal in inclined seam.

On this subject Mr. Kay makes the following remarks:—

"It is often assumed that subsidence takes place at right angles to the dip, as in horizontal mines; and again, that it takes place vertically as being directly due to gravitation. The author (Mr. Kay), however, believes that a line midway between the two gives the more general line of break—that is to say, supposing the angle of dip to be θ, then the angle that the line of break makes with the horizon is $90^\circ - \frac{1}{2}\theta$, as shown in Fig. 179.

"In laying out upon a plan a pillar for the support of a bridge, as in Fig. 179, to be left in inclined seams up to 30°, the size of pillar necessary may be calculated by the formula given. Marking this upon the plan, round the bridge, the position of the pillar is given supposing the measures to be horizontal. Its lateral displacement to the high side due to the inclination, at a depth d, is $d \tan \frac{1}{2}\theta \cos \theta$. This lateral displacement may be graphically determined as shown in the vertical section on the line of dip.

"Let **A** represent the bridge to be supported by a pillar to be left in the coal **DD** at a depth of d yards. Calculate the size of the pillar from the formula given, and mark it upon the horizontal line at **B** and **C** drawn through the ground-level at **A**. Draw **BJ** and **CK**, making an angle of $90^\circ - \frac{1}{2}\theta$ with the horizon from **B** and **C** respectively. Then **JE** and **KF** represent the lateral displacement due to the dip in the coal, and **JE′** and **KF′** the displacement in plan. This is shown in the plan, where the dotted circle represents the position of the pillar when the coal is horizontal, and the circle in a full line its position corrected for dip."

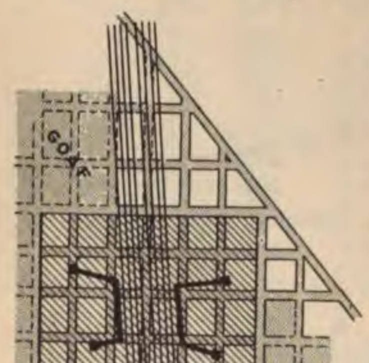

FIG. 180.—Pillar of coal left in pillar-and-stall workings.

It is frequently necessary to make roads through the pillar for the purposes of haulage and ventilation; the size and number of these roads is a matter of agreement, and they must be carefully limited to the authorized dimensions. Provision should be made for these roads by a corresponding enlargement of the pillar. In case it is decided that a solid pillar of coal is not required, then partial support may be given, as shown in Figs. 180 and 181.

A pillar is set out in the following manner: An accurate plan is made of the surface and of the underground workings, surveyed in each case from the shaft. The pillar as agreed upon is drawn on the plan as shown in Fig. 182. The surveyor gives the underground steward the lengths that he may drive from station **A** on the plan. When the road reaches the point **B**, the surveyor hangs lines by which headings are driven round two sides of the pillar; at **C** and **D** the surveyor again hangs lines, by means of which the boundary heading is completed. The steward now knows that the coal inside these headings is not to be got, whilst the coal outside may be removed.

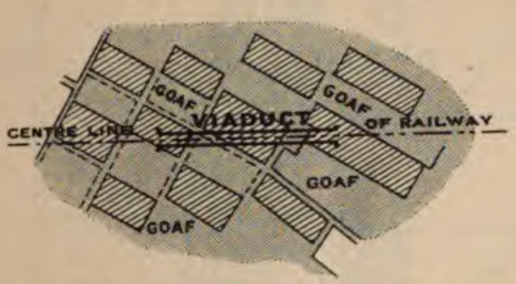

FIG. 181.—Viaduct supported by a number of pillars left in the ordinary course of working.

Setting out.—In addition to preparing plans of mines as they exist, the surveyor has to set out the works that are designed by the engineer.

Setting out Surface Works.—The engineer marks on the plan the position of the shafts, engine-houses, offices, etc.; the surveyor, by means of pegs, trenches, etc., marks on the ground the actual position, so that the contractor or other workmen can proceed with the necessary excavations. When once the place for the excavation of the shaft or foundations has been marked out, it becomes rather the province of the engineer and architect to set out with minute accuracy the actual lines of masonry.

Fig. 183 shows a portion of an estate on which the engineer has marked the position of a shaft which is to be sunk, and some buildings to be erected. In this particular case it happens that the exact position of the shaft to within a few inches, or even perhaps to within 2 or 3 feet, is not a matter of prime importance; it is, therefore, easily marked out in the following manner. The surveyor draws on the plan lines similar to those

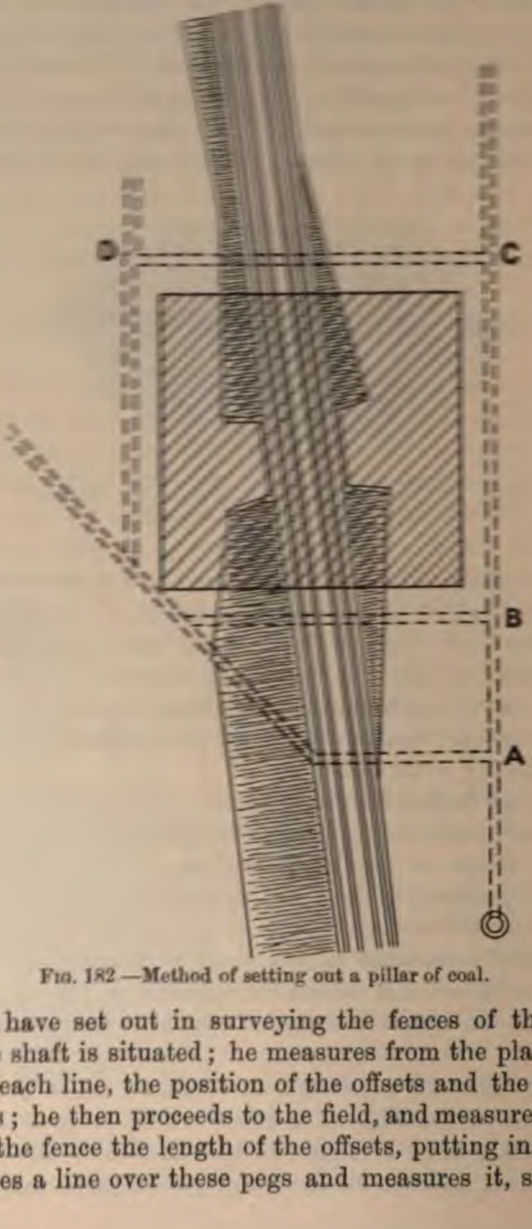

FIG. 182.—Method of setting out a pillar of coal.

he would have set out in surveying the fences of the field in which the shaft is situated; he measures from the plan the total length of each line, the position of the offsets and the length of the offsets; he then proceeds to the field, and measures from the centre of the fence the length of the offsets, putting in pegs; he then ranges a line over these pegs and measures it, setting out all the lines in a similar manner. As the lengths of the offsets as measured will probably not give a line that is quite straight, he will set out a straight line running past the pegs as put down from the offset measured, but correcting the irregularities. Having set out the four lines parallel with the four sides of the field, he will then measure a diagonal, and if this does not agree

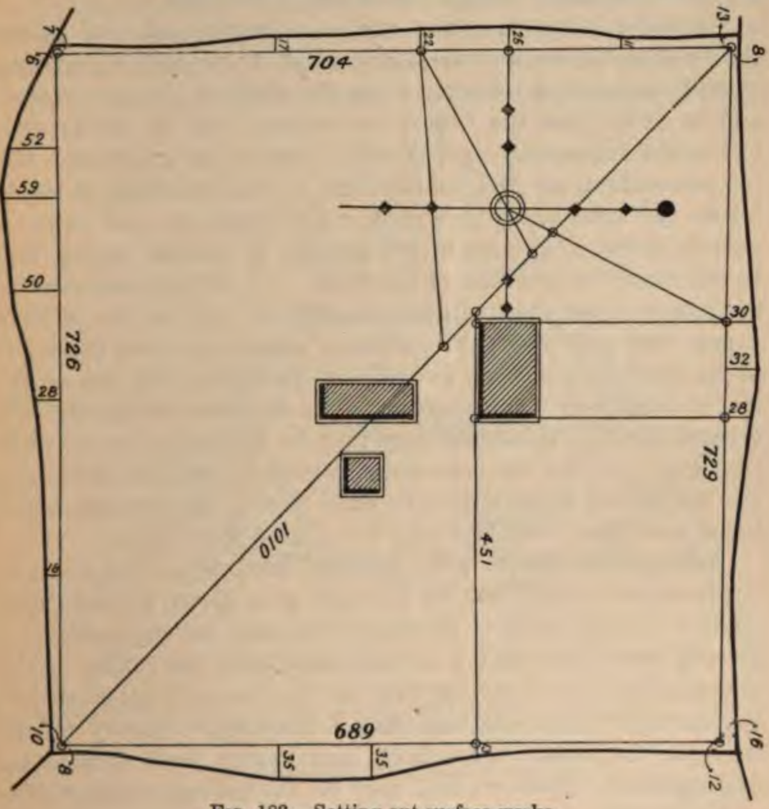

FIG. 183.—Setting out surface works.

with the length on the plan, it will indicate a corresponding error in the plan or in the setting out of the lines by means of offsets. As measurements to and from the centre of a hedge are necessarily very rough and cannot ordinarily be taken nearer than a link, and in the case of a thick hedge to 2 or 3 links, the lines as first set out may be susceptible of little adjustments, and the real position of the line can only be approximated by means of a great many offsets, the average length of which should agree pretty nearly with the average length as measured from the plan. Having poled out these lines, the surveyor can then take measurements from them with as much accuracy as the case requires to fix the position of the shaft by setting out lines which form sides of triangles, or lines measured from a fixed point on one side of a triangle to a fixed point on the side of another triangle, as shown in the figure.

In order that, when the work is pegged out, the labour shall not be lost by the careless removal of the pegs, the surveyor should cut holes or trenches along the sides of the excavation; and in order that the trench so cut may not be prematurely lost in the subsequent operations, it should be understood that the excavation for the foundations of the buildings is to be inside the trench, say one foot. The pegs are put down at corners of the excavation in the trench; a circular trench may be cut round the position of the shaft. As the peg representing the centre of the shaft will be removed as soon as the work is begun, four pegs should be put down equidistant from the centre of the shaft, say 20 feet; two 20-feet lines from the top of any two of these pegs will then meet in the centre of the shaft. For greater security, these four pegs may be duplicated, as shown in the figure. If the two cross-lines over these four (or eight) pegs are set out at right angles to each other, the winding-engine house and other buildings may be squared from them.

Setting out Shaft with Extreme Accuracy.—It sometimes happens that a shaft has to be sunk at a given distance from another existing shaft or precisely over some underground works already made, and that it is important that the centre of this new shaft should be set out with all the accuracy attainable by the surveyor's art. In this case it may be necessary for the surveyor to prepare an entirely new survey, both surface and underground. However that may be, the survey, whether old or new, must of course be exact, and the new shaft must be set out by measurements taken from the main lines of survey as shown in Fig. 184, and it is assumed in this figure that the stations on these main lines can be found; if not, the setting out of the original survey-lines is tantamount to making a new survey. In the case shown in Fig. 184 portions of three main survey-lines are measured on the plan, triangles set out and measured, so that six lines meet in the centre of the shaft; the angles made by the intersection of the lines are calculated, the cross-lines being set out at the proper angle from the main survey-lines by means of the theodolite. The accuracy of the original survey as shown on the plan will be tested by the remeasurement of the parts shown in the figure and cross-lines which form ties; then, assuming the necessary accuracy of the plan, the centre of the shaft can be set out to the twentieth part of an inch.

Of course it will be necessary to erect permanent stations round the shaft, from which cross-lines can be stretched as shown in Fig. 183. These permanent stations must consist either of strong posts of timber, say 12 inches square, securely fixed into holes 6 feet deep and tightly rammed, or else they must be masonry pillars, say 3 feet square. On the top of these pillars must be a cap of stone or iron, on which a centre-line can be chiselled, or, in the case of wooden posts, the centre-line may be cut in the wood itself. These posts must each be on lines connected with the main survey; the exact centre of the shaft can then at any time be found by stretching two lines across the four posts; the intersection of these lines will be the centre of the shaft.

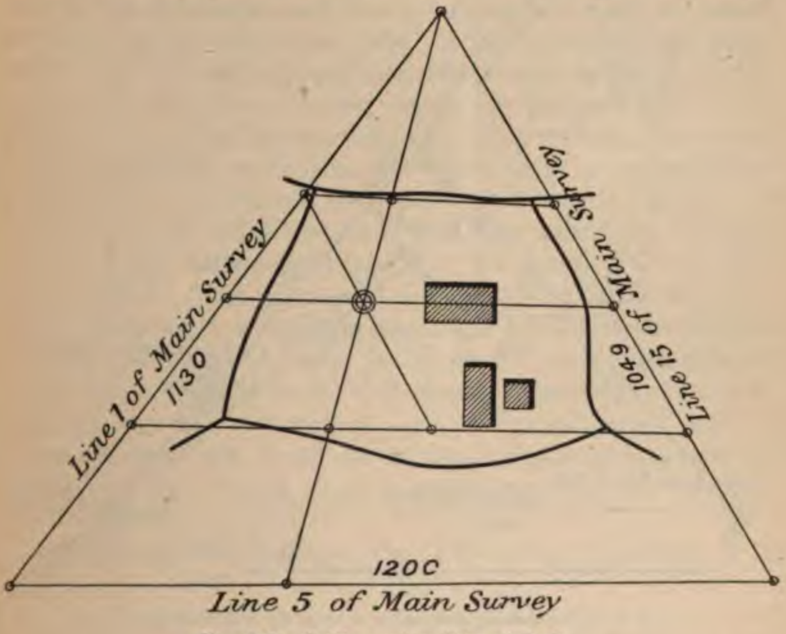

FIG. 184.—Setting out position of shaft.

Setting out Centre of Shaft by Meridian Line.—It may be that the position of the new shaft is to be set out entirely with regard to an existing shaft at a given distance and bearing. If the geographical meridian is shown on the surface by carefully placed marks, it will be easy to set out the bearing from this line. In case the meridian line is not so marked out, it may be that the stations of some main survey-line are accessible, and that the bearing of this line has been carefully ascertained and checked against the bearings of the other main survey-lines.

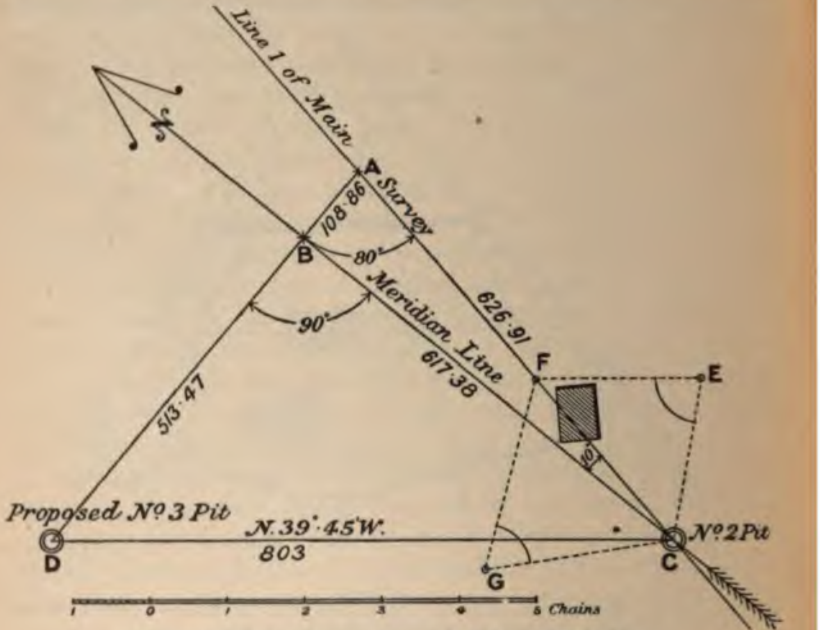

FIG. 185.—Setting out centre of shaft by meridian line.

Such a case is illustrated in Fig. 185. In this case the proposed new pit is N. 39° 45′ W., 803 links from the centre of No. 2 pit; No. 1 line of main survey passes through the No. 2 pit; the bearing has been ascertained to be N. 10° E. It is then ascertained by calculation that the No. 3 pit is 617·38 links north (cos 39° 45′ × 803) and 513·47 links west (sin 39° 45′ × 803) of No. 2 pit. If a line is drawn from the centre of the No. 3 pit perpendicular to the meridian, it cuts the No. 1 line at the station **A**, crossing the meridian at **B**. The line **BA** is a tangent of the angle of 10° to the radius 617·38, and by calculation is found to be 108·86 links long. The length from **A** to the centre of No. 2 pit is the secant of the angle **ACB**, and by calculation is found to be 626·91 links long. This length is then measured on the No. 1 line, and the point **A** marked with a peg. The angle **ABC** is a right angle, the angle **ACB** is 10°, therefore the angle **BAC** is 80°; then with the theodolite a straight line **ABD** is set out, making an angle of 80° with the line **AC**. The distance of the No. 3 pit is 108·86 + 513·47 = 622·33 links.

In order to test the accuracy with which the position **D** is set out, the theodolite may be set up over this peg; the angle **BCD** is 39° 45′, the angle **DBC** is 90°; therefore the angle **BDC** is 50° 15′, and this angle should be read by the theodolite if the work has been correctly done; the distance **DC** is 803 links, and it should measure this length precisely. In case of buildings existing on the line **AC** which make it impossible to measure the line without deviation, the distance **FC** may be set out by fixing up the theodolite at **E**, and measuring the angle **FEC** and the distances **EC** and **FE**; the length **FC** can then be calculated. This length can be checked by fixing the theodolite again at **G**, measuring the angle **FGC** and the lengths **CG** and **FG**, by which again the length **FC** can be ascertained. In the same way, the length **DC** may be measured round any building or other obstruction. No. 1 line of survey will, of course, be poled out, if not for its whole length, at any rate for a length of half a mile, so as to make sure that the real line has been taken.

Setting out Underground.—Fig. 186 shows a heading in a

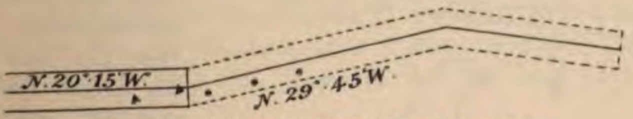

FIG. 186.—Setting out a heading underground.

mine proceeding N. 20° 15′ W.; the direction of this heading has to be changed to go N. 29° 45′ W. The surveyor proceeds to the place with his dial, and puts a chalk mark or other mark in the heading to guide the miners along the given bearing. When the heading has been driven a sufficient distance for permanent marks to be conveniently put in, the surveyor returns to the place and makes three marks in the roof in the bearing as shown by his dial; at these three marks holes are drilled, and into the drill-holes wooden pegs are firmly driven. The dial is now turned on to these pegs, and a pencil-mark carefully made on each peg, so that these three marks shall all be in one straight line, which has a bearing of 29° 45′. Into the pegs at these marks a small hole is bored, and into this small hole is screwed a small screw with a little brass hook at the lower end, care being taken that the centre of the hook corresponds with the pencil-mark on the peg. From each of these hooks a plumb-bob line is suspended. The line of pegs is generally on one side of the heading, say 1 foot from the right-hand side. The miner must now drive the heading so that a light held at the face 1 foot from the right-hand side shall be in the line of these three strings. The reason for having three strings instead of two is to detect any variation in the position of the pegs or hooks; if there were only two, the position of one might be changed without being noticed. At every fresh turn in the heading the surveyor must repeat the operation.

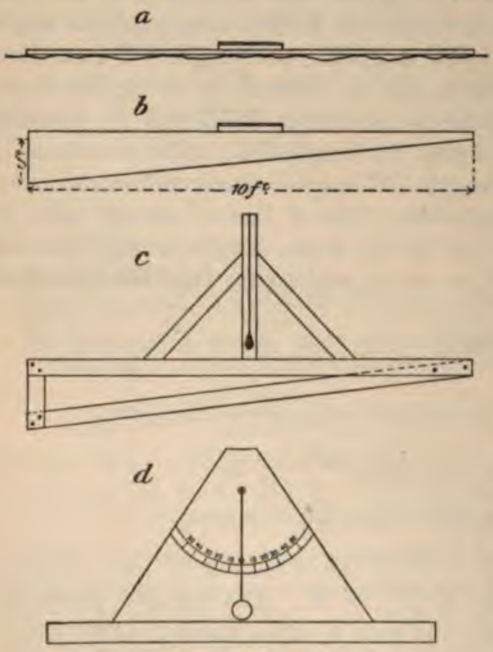

Fig. 187.—Method of setting out gradients.

Setting out Gradient.—It is not only necessary to set out the direction of a heading, but to set out also the gradient. This may be done in various ways. Suppose the road is to be driven level; a straight-edge and spirit-level must be provided, and the miner or officer in charge must from time to time see that the floor of the heading is level (see *a*, Fig. 187). Suppose, however, it is intended to drive at an inclination of 1 in 10, then the same straight-edge and spirit-level will do; but underneath the straight-edge should be fastened a piece of wood, the under side of which is cut to the required slope; thus if the straight-edge were 10 feet long, the piece of wood on the under side would be 1 foot deep at one end and taper away to nothing at the other (see *b*, Fig. 187). For steeper inclinations a shorter straight-edge must be used.

Instead of a spirit-level, a T-square and plumb-bob may be used (see *c*, Fig. 187). The writer has designed a modification of the T-square for setting out a gradient by the angle instead of at the rate per cent. (see *d*, Fig. 187). In this case a graduated arc is fixed to the T-square, with the degrees marked on; a pin is fixed at a point corresponding with the centre of the circle of which the arc is a part; a small ring fits over the pin, and from this is suspended, by a fine string, a plumb-bob; the plumb-bob is shaped like the weight in the pendulum of a clock, and just hangs clear of the frame. If the heading is driven at a gradient of 10°, the string will hang over the tenth degree on the graduated circle when the straight-edge is laid on this slope.

A clinometer on a somewhat similar principle has been designed and invented by Colonel G. P. Evelyn, in which he uses a bubble of compressed air in a curved tube; the tube is curved to the sweep of a circle. Adjoining the curved tube is a scale graduated in degrees. When the straight-edge is level, the bubble is in the centre; when the straight-edge is inclined, the bubble comes to rest opposite the degree on the graduated circle that corresponds with the inclination of the straight-edge.

Where there is water, it is easy to drive a level; if the water flows from the face of the heading, it is evident that it is rising; if it flows towards the face, it is evident that the heading is falling; if it is stagnant, it shows that the floor of the drift is level.

It is obviously difficult to ascertain the precise inclination of a road by means of a straight-edge laid upon the rough floor. It may be more accurately done by raising the straight-edge above the floor, either by building temporary supports or fixing two side-posts, and then, looking along the edge, adjust it so as to point to a light at the end of the heading, which is held at the same height above the floor; the angle of the straight-edge can then be observed by means of a clinometer. A straight-edge may be permanently fixed to posts at the required angle, and by looking along the top of the edge from time to time, the miner or the official in charge can see if the inclination of the heading is parallel to this fixed straight-edge.

Instead of a straight-edge, a metal tube, say $\frac{1}{2}$-inch gaspipe, may be suspended from the roof, and adjusted to the desired inclination by means of suspending strings or wires. By looking through the tube, a light can be fixed at the end of the heading which will be in a line with the tube (see Fig. 188).

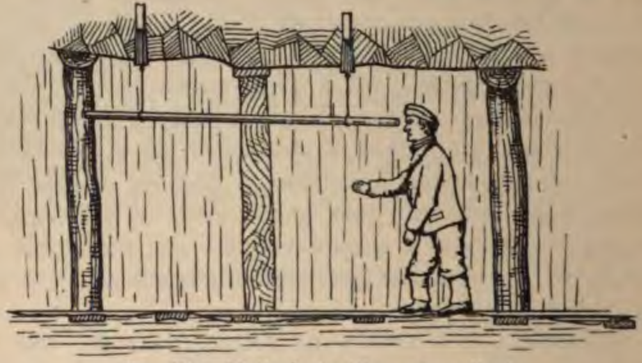

FIG. 188.—Method of setting out gradients.

A tube is sometimes used in a similar manner for giving the direction as well as the inclination.

Setting out Cuttings and Embankments on the Surface.—In addition to pegging out the position of surface-works, it is often necessary to mark the depth of cuttings and the height of embankments. This must be done by means of measurements taken from the drawn section. Fig. 189 is a section showing cutting and embankment. In order to show the height of the embankment, poles are fixed in the ground; if they are long,

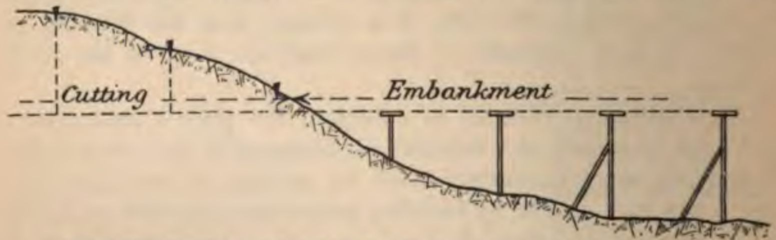

FIG. 189.—Setting out heights of embankments and depths of cuttings.

they are stayed with diagonal struts. On to these poles are nailed cross-bars, which indicate the height of the embankment; the poles are placed, say at chain-lengths, along the centre-line of the embankment, and their correct position is measured from the nearest fence or other fixed point shown on the plan. In the absence of fences or other fixed points in the neighbourhood, the position of the line has to be set out by bearings or angles from some base-line.

In the case of a cutting, the level and gradient of this is fixed by sights taken from two or more cross-bars or poles fixed in the ground near the commencement of the cutting. By taking sights from these cross-bars the cutting can be made at the required level and inclination.

In the case of an excavation on the top of a hill or in a flat, and not on the side of a hill, two posts must be fixed, one on each side of the excavation, and a cross-bar nailed to each at a given elevation above the bottom of the intended excavation; the cross-bar is, say, 3 feet above the ground, and the excavation is to be 20 feet below the ground; then the cross-bar is marked 23 feet, that being the depth to which the excavation is to be carried below that level. A string can be stretched from cross-bar to cross-bar over the excavation, and the depth measured from these strings by means of a pole. The posts carrying the cross-bars must be ranged along each side of the excavation, say at chain-lengths.

Tunnel Shafts.[1]—In driving a tunnel, either for railway or drainage purposes, it is frequently necessary to sink shafts along the line of the tunnel, in order that the work may be carried on simultaneously at a number of points. Taking the case of a tunnel 2000 yards long, if the rate at which the heading advances, having regard to the nature of the ground and the tools used, is, say, 10 yards a week, and the tunnel is started at each end simultaneously, the total rate will be 20 yards, and it will take 100 weeks to get the heading through. If, however, two intermediate shafts are sunk, each at a distance of 666 yards from one end of the tunnel, two headings can be driven from the bottom of each shaft, so that the total number of headings in progress at one time will be six, and thus the tunnel may be put through in about 33 weeks. It is, of course, of the utmost importance that the intermediate lengths of tunnel should be in the correct position both as regards situation, direction, and level.

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[1] An interesting paper on this subject will be found in the *Proceedings of Civil Engineers*, vol. xcii. p. 259, "The Alignment of the Nepean Tunnel, New South Wales," by Thomas William Keele, Assoc. M. Inst. C.E.

As regards the position of the shafts, they can be set out in the method already described (Figs. 183, 184), in case there happens to be a sufficiency of marks on the surface and an accurate plan from which the measurements can be taken. If, however, as is frequently the case, the ground between the ends of the tunnels contains but few land-marks, the position of the intermediate shafts must be set out by lines specially ranged between the two entrances of the tunnel, supposing the tunnel to be in a straight line between the entrances; this straight line must be carefully poled out with the aid of a theodolite, and at convenient places stations are built on which the theodolite can be fixed. These stations may be of masonry or of timber, and must be sufficiently firm, so that the theodolite is not affected by wind or the movements of the persons observing it. The distance of the shaft from the entrance can then be set out by ordinary chaining, the errors incidental to chaining not being of importance in this case. The depth of the shaft is measured from the section on which the height of some fixed mark near the top of the shaft is shown. By means of the spirit-level, the level of this mark is transferred to the walling of the shaft, and the depth to the bottom of the tunnel can be set out either by chaining down the side of the shaft or by a measured length of wire. The direction of the headings is set out by means of plumb-lines suspended down the shaft, and fixed on the surface in the direction of the centre-line of the tunnel, or the line can be transferred to the bottom of the shaft by means of the transit telescope, as described in Chapter XI.

It sometimes happens, however, that the shaft is sunk on one side of the centre-line of the tunnel, which has to be reached by a cross-drift (see Fig. 190). In this case the surveyor hangs his lines down the shaft at right angles to the direction of the tunnel, or, if he uses the transit instrument, sets out marks at the bottom of the shaft by means of it at right angles to the direction of the tunnel. He then fixes his theodolite in the drift in the direction of these lines, and sets out an angle of 90°, which is, of course, the direction of the tunnel. Or, instead of fixing the lines at right angles, he may fix them at any other angle that the circumstances of the case make convenient. The inclination at which the head is driven can be set out by any of the methods already described.

In driving large tunnels, it is often convenient to let the men at the face be able to see their position in regard to the centre-line. In this case lamps may be suspended from cross-bars in the line, and a man at the face, placing himself in a line with two lamps, gets the centre approximately. Similarly, candles fixed in triangular stirrups may be suspended in place of lamps. For the exact setting out of the brickwork, stations are fixed at intervals of say 50 feet along the tunnel, and at these stations are fixed spikes with small eye-holes, placed exactly in the line by means of the theodolite; through these eye-holes strings can be stretched, and from these the requisite measurements can be taken.

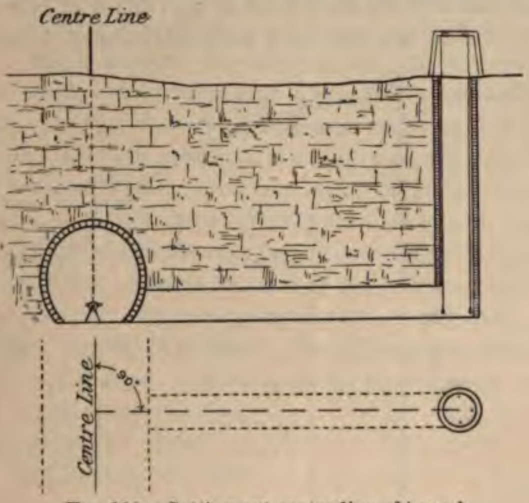

FIG. 190.—Setting out centre line of tunnel.

Setting out Curves.—No railway, canal, or culvert should ever change its direction except by means of a curve. In ordinary canals and culverts the exact ranging of the curve is sometimes not of serious importance; in railways the accuracy with which the work is set out is of the very highest importance, as without that accuracy it is impossible to run a train with safety. The surveyor has therefore frequently to set out curves both on the surface and in the mine.

Curves may be roughly set out by means of chain and poles in the manner shown in Fig. 191. In this case the straight portions are shown by the pieces marked *ab* and *pq*. The line *ab* is prolonged indefinitely, and at a distance of 1 chain from *b* the offset *cd* is marked off by swinging the chain in the direction of the curve a distance equal to *cd*; the line *bd* is now set out, and at a distance of 1 chain from *d* the offset *ef* is marked off; the line *df* is now set out, and at a distance of 1 chain from *f*

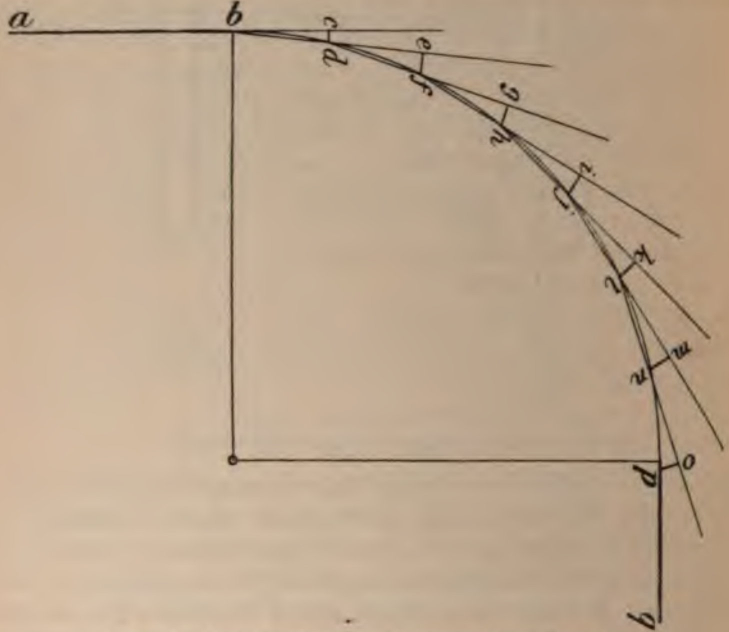

FIG. 191.—Laying out railway curves by the chain only.

the offset *gh* is marked off; the line *fh* is now set out, and at a distance of 1 chain the offset *ij* is marked off; the line *hj* is now produced, and the offset *kl* set out; and so on to *p*, being the commencement of the straight portion *pq*. The lengths of the offsets *cd*, *ef*, *gh*, *ij*, *kl*, *mn*, and *op* may be calculated by well-known rules (see following pages). But in case the surveyor should have forgotten the rule, and fail to have at hand any pocket-book or text-book to remind him, he can easily find the offsets with approximate accuracy by measuring them from the plan—the accuracy, of course, of these measurements depending on the scale of the plan and the care with which the drawing is made and scaled.

There is a difficulty, however, in making the drawing to a large scale when a large radius is required; for instance, if the radius of the curve were 80 chains, then to draw the curve with that radius to a scale of 1 chain to an inch, would require the arm of a compass 80 inches in length; but the scale of 1 chain to the inch would be too small to give the measurement with any approach to accuracy, and even a compass arm of 80 inches and the requisite table surface for setting out the curve might be difficult to obtain, but by means of a box of curves which are cut in cardboard, pearwood, or vulcanite, to radii varying from 1½ inches to 240 inches, a small portion of a curve may be set out on a small piece of paper. If, however, the radius is not large, say 10 chains, then the drawing may be made to a large scale, especially with the aid of the wooden curves; by taking a curve of 120 inches radius a scale of 12 inches to a chain might be used, on which the offsets could be measured with considerable accuracy.

Setting out Curves by Offsets from Tangent.—The required offsets for a curve may, however, be measured with much less liability to error from the tangent to the curve, as shown in Fig. 192. In this case the first offset is similar to *cd* in Fig. 191; the second, third, and succeeding offsets are also measured from the same tangent line. By this method, any error made in fixing the first stations *d*, *f*, *h*, merely affects the accuracy of those particular points, and the long offset at *j* can be measured with substantial accuracy from the plan.

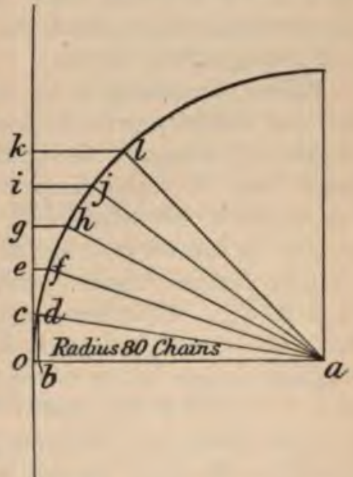

Fig. 192.—Laying out railway curves by offsets from the tangent to equidistant points on the curve.

Assuming the case shown in Fig. 192, here is a curve with a radius of 80 chains (a radius much greater than is to be expected on a branch railway to a mine); this is drawn on paper to a scale of 1 chain to an inch; the tangent is drawn, and chords, *od*, *df*, *jh*, *hj*, etc., each 1 chain in length, are marked off along the curve from the point *o*, where it commences; the length of the offset to the tangent from each of these points can be measured, and will be as shown in the following table:—

RADIUS OF CURVE 80 CHAINS.

Distance measured along curve in chain-lengths.	Offset. Inches.	Deflection angle, or angle at centre.
1	(*cd*) 4·9	(*oad*) 0° 42′ 58″
2	(*ef*) 19·8	(*oaf*) 1° 25′ 57″
3	(*gh*) 44·5	(*oah*) 2° 8′ 55″
4	(*ij*) 79·2	(*oaj*) 2° 51′ 53″
5	(*kl*) 123·6	(*oal*) 3° 34′ 52″
6	178·0	4° 17′ 50″
7	242·3	5° 0′ 48″
8	316·5	5° 43′ 47″
9	400·5	6° 26′ 45″
10	494·4	7° 9′ 43″

At the tenth chain (which corresponds to an angle of 7° 10′ nearly) the offset is 494·4″, or 62·4 links, which is a length that can be scaled off the drawing with approximate accuracy.

If, instead of 80 chains, the radius of the curve was 20 chains, and a curve of 80 inches is marked out on the paper, the scale of the drawing would be four times as great, and the measurements taken from the drawing would be more accurate in a corresponding degree.

Before proceeding to set out the curve on the ground, the surveyor should provide himself with a sketch-plan showing the lengths of tangent, offset, and chord. An offset of greater length than 50 links cannot be set out with accuracy by eye, and unless a cross-staff, dial, or theodolite is employed, a new tangent to the curve must be set out as a base from which to continue further offsets. Suppose that after setting out the offset *kl* (Fig. 192) a new tangent is required, the surveyor can put a pole in at the point *h*, 2 chains back along the curve; then the tangent to the curve from the point *h* will have an offset to the curve at the point *l* of 19·8 inches; and if this length is set out from *l* in a direction perpendicular to the new tangent, it will give the point through which the tangent can be drawn.

This method of setting out curves from the tangent can only be practised in the pit when the length of the longest offset does not exceed half the width of the road, and is, therefore, only applicable to very short curves. On the surface the method will do very well for ordinary ground where there are no cliffs, rivers, or buildings to interrupt the measurement of the tangent or of the offsets.

Setting out Curves by Angles. — It is also possible for the surveyor to set out the curve with his dial or theodolite, by measuring the angle of each successive chord (see Fig. 193). In this case a drawing is made showing the curve connecting two straight portions of a railway, **AB** and **XY**. Chords of 1 chain are marked off on the curve with a scale, and drawn, **BD**, **DE**, **EH**, etc. Taking the tangent **AB** as the meridian, produce it to **C**; the chord **BD** is also produced, and the angle **CBD**, which it makes with the meridian, is measured by means of the protractor; the angle that the chord **DE** makes with the meridian is also measured with the protractor, and the angle of all the other chords. For instance, if the protractor is laid on the line **ABC** so as to read 360°, then the bearing of the chord **BD**, as read off the protractor, would be, in the case of a 25-chain curve, 1° 8′ 46″. The bearing of the chord **DE** would be 3° 26′ 18″; the bearing of the chord **EH** would be 5° 43′ 50″, and so on.

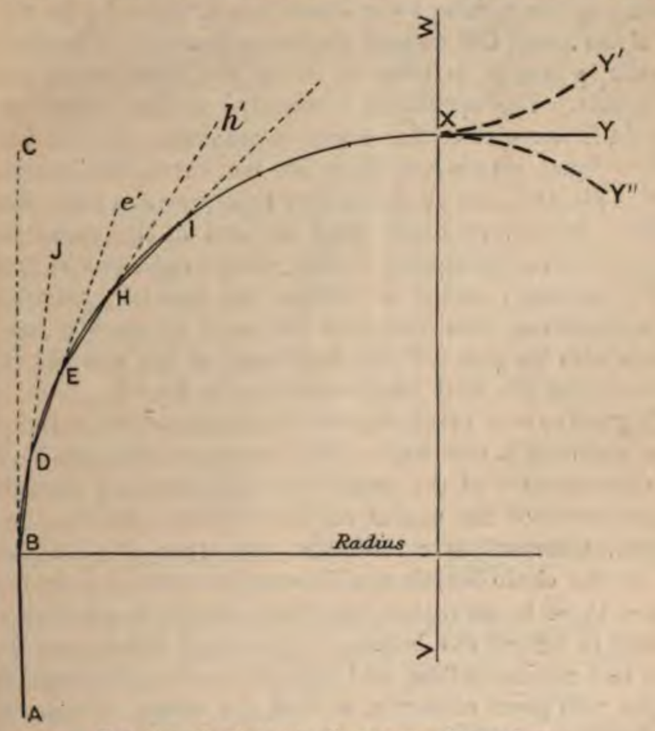

Fig. 193.—Laying out railway curves by angles.

To set the work out, the dial or theodolite is fixed at **B**, and the sights clamped in the direction **ABC**, the vernier being at 360°. By means of the rack the sights are then turned through the angle **CBD** and clamped with the vernier at 1° 8′ 46″, and a chain-length measured from **B**, and a peg put in at the point **D**; the instrument is now fixed at the point **D** in the direction **BD**, the vernier reading as before 1° 8′ 46″, and the sights moved by the rack through the angle **JDE**, the reading on the vernier being made to correspond with the bearing of the chord **DE** as read by the protractor. The chord **DE**, 1 chain in length, is then set out in the direction as given by the sights. The operation is repeated at the length of every chain till the end of the curve is reached. It will be found that if equal chords are taken on the curve, the angles **JDE**, *e*′**EH**, *h*′**HI**, etc., are all equal, and that they are twice the angle **CBD**. It is very likely that the end of the curve will not coincide with a chain-length. The exact length can be measured off the drawing; and if on setting out this last length at the angle measured from the plan the point so marked out corresponds with the point **X**, the beginning of the straight portion, it shows that the work has been correctly done.

If great care is taken, the curve may be set out with approximate accuracy in this way. The sources of error will be, firstly, the measurement of the lengths on the drawing; secondly, the measurement of the angles on the drawing; thirdly, the fixing of the instrument over the pegs. An error of ½ inch at each end of the chain-length would cause an error of 1 in 792. Of course, there is no reason why there should be an error of this amount in setting out, because three tripod stands may be used, as in fast-needle dialling, and with the vernier the angle may be set out with great accuracy, so that the errors in the work will be chiefly those due to an incorrect drawing or inexact measurements from the drawing.

Another method of setting out the curve by angles will be found on referring to Fig. 194. Let **AB** be the straight portion of the line, the curve beginning at **B**. **C, D, E, F** are chain-lengths measured as chords of the curve; **B**′ is an extension of the tangent **AB**. The point **C** may be found by fixing the theodolite or dial at **B**, clamping it in the direction **BA**, and then turning the sights in the direction **BC**.

The angle **CBD** is the same as the angle **CBB**′; the angle **DBE** is also the same as the angle **CBD**, and so is **EBF**; so that the sights can be turned in succession upon **C**, **D**, **E**, and **F**. To mark out **C**, one end of the chain is held at **B**, and the other swung into the line of sight of the theodolite or dial, and the peg put down at the chain-end at **C**. The chain is now drawn on; the follower holds one end at **C**, and the other is swung into the line of sight, and a peg put down at **D**, and so on.

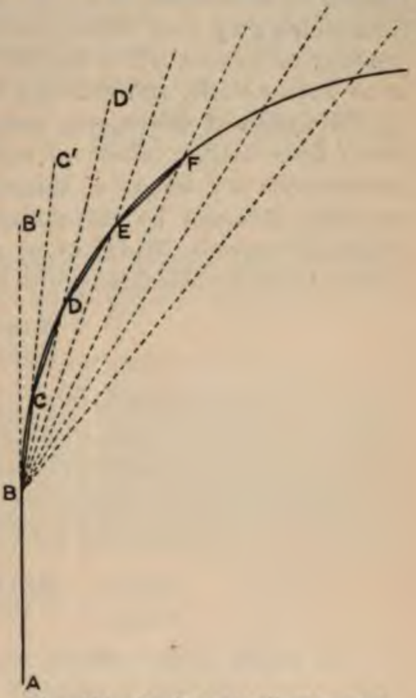

FIG. 194.—Laying out railway curves by angles.

This method is, of course, only suitable on the surface, because the line of sight **BF** would be obstructed by the solid ground, if in a tunnel. It will, therefore, be necessary underground to be constantly moving the instrument forward along the line of the curve, as previously described.

Of course, if there happens to be room in the tunnel, and the curve is of large radius, a number of points may be set out from once fixing the instrument; or, if the curve is of small radius, it may be desirable to set out points nearer together than 1 chain, possibly every 10 links, in which case once fixing the instrument may be sufficient for setting out a number of points on the curve.

It frequently happens that a curve does not end in a straight line, but in another curve, either curving in the same direction with a greater or less radius, or in a reverse direction, as shown by the dotted lines **XY′**, **XY″** (Fig. 193). In this case the point **X** is simply the ending of the first curve and the beginning of a new curve. The radius of the new curve must be struck from a centre on a line drawn through **X**, which line, if produced, will pass through the centre of the circle of which the first curve is an arc. This line is shown on Fig. 193, **VW**. In setting out large curves, the wooden curves previously referred to are used instead of compasses, and they must be held as if the centre from which they were struck were on this line. The curves, if properly laid down, will never cut each other, when produced so as to form a circle, and will only touch at one point, **X**.

Calculation of Offsets.—In order to avoid the errors likely to result from scaling offsets off a plan, or measuring angles with a protractor, the length of the offsets and also the angles are generally obtained by calculation. Referring to Fig. 191, in which the curve is divided up by equal chords, the length of the offset cd may be calculated from the rule—

$$\frac{\text{chord}^2}{2\text{R}}$$

where R is the radius of the curve. The length of the chord may be 100 links, in which case the radius will be expressed in links, and the offset will be in links.

The length of the offset ef is twice the length of the offset cd, and can be calculated from the rule—

$$\frac{\text{chord}^2}{\text{R}}$$

The length of the offsets gh, ij, etc., are equal to ef. The offset cd is called the tangential offset, as it is measured from the tangent; the offset ef is measured from the extension of the chord bd at e.

Referring to Fig. 192, the length of the offsets cd, ef, etc., may also be calculated as follows:—

The chords od, df, fh, etc., are each equal to 1 chain in length, and the radius of the curve being 80 chains, it will be seen that the natural chord of the angle oad is equal to—

$$\frac{\text{chord}}{\text{radius}} = \frac{od}{oa} = \tfrac{1}{80} = \cdot 0125$$

On reference to the table of natural chords, this is found to correspond with an angle of 0° 42′ 58″. The required distance cd is equal to ob, which is the versed sine of the angle oad. On again referring to the tables the versed sine of 0° 42′ 58″ is found to be 0·0000781; multiplying this by the radius in inches (80 × 792) gives the length 4·9 inches for the first offset cd.

The other offsets are obtained in a similar manner, and a rule might be stated as follows:—

$$cd = \mathrm{R}\,.\ \text{versed sine } oad$$
$$ef = \mathrm{R}\,.\ \text{versed sine } oaf$$
$$gh = \mathrm{R}\,.\ \text{versed sine } oah, \text{ etc.}$$

If the length on the tangent is required, it can be calculated; for instance—

$$oc = \mathrm{R}\,.\ \sin oad$$
$$oe = \mathrm{R}\,.\ \sin oaf, \text{ etc.}$$

As this calculation is a somewhat tedious one, tables are published giving the lengths of the offsets for curves from radii of 5 chains to 3 miles.[1]

Calculation of Angles.—The angles necessary for setting out curves with the dial or theodolite can also be obtained by calculation. Referring to Fig. 193, the angle **CBD**, or tangential angle (so called because it is the angle made by the chord with the tangent), is equal to half the deflection angle, or angle subtended at the centre of the circle by the chord **BD**, and can be found by the following rule:—

$$\text{Tangential angle (minutes)} = \frac{\text{chord}}{\text{radius}} \times 1718{\cdot}873$$

Example.—What is the tangential angle for a chord 1 chain in length of a circle whose radius is 80 chains?

$$\begin{aligned}\text{Tangential angle in minutes} &= \tfrac{1}{80} \times 1718{\cdot}873 \\ &= 21{\cdot}486 \text{ mins., or } 21'\ 29''\end{aligned}$$

The angle **JDE** (Fig. 193) is double the angle **CBD**, provided that the chord **BD** equals the chord **DE**. The angles made by successive chords of equal length are also equal to each other, and a rule might be expressed as follows:—

$$\text{Angle between equal chords (in minutes)} = \frac{\text{chord}}{\text{radius}} \times 3437{\cdot}746$$

Referring now to Fig. 194, one of the fundamental properties of the circle is that equal chords subtend equal angles at the centre of a circle, and also at the circumference, if the angles are contained in similar segments; thus, having calculated the tangential angle **B′BC** by the above rule, the succeeding angles are all equal to it, and their sum might be found at once, as they are all tangential angles.

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[1] Kennedy and Hackwood's Tables for Setting out Curves. London, E. and F. N. Spon.

It is sometimes convenient to calculate the radius of the curve that will connect two straight portions of line. Referring to Fig. 195, two straight portions of line, **AB** and **XY**, are shown. What is the radius of the curve, commencing at **B**, that will connect these straight parts? This may be found geometrically as follows: Produce **AB** and **XY** till they meet at **O**; on **OX** make **OC** = **OB**, and at **B** and **C** erect perpendiculars cutting in **D**; then **D** is the centre of a curve that will join **AB** and **XY**, and the radius of this curve can be measured from the drawing.

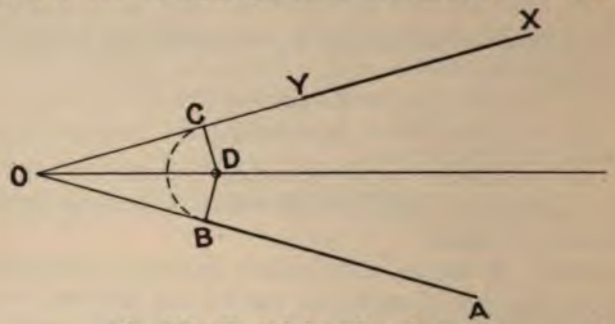

FIG. 195.—To find the radius of a curve.

The radius can also be found by calculation, if the length **OB** from the tangential point at which it is required to strike the curve to the point of intersection of, and also the angle **AOX** between, the two tangents, are known.[1]

Thus, in Fig. 195 it will be seen that $\frac{\mathbf{DB}}{\mathbf{OB}}$ is the tangent of the angle **DOB** (which is half the angle **AOX**).

$$\text{Radius of curve} = \mathbf{OB} \times \text{tangent of angle } \mathbf{DOB}$$

Calculation of Average Dip or Inclination of Measures.—It is often desired to measure the angle and direction of dip, but "it is not always possible to ascertain the true dip by one observation; it often happens that it must be ascertained from two observations, neither of which is on the line of greatest dip.

---

[1] The length **OB** can be calculated, if the length of a straight line **BC** perpendicular to **OD** is known, by the rule—

$$\mathbf{OB} = \frac{\mathbf{BC}}{\text{natural chord of the angle } \mathbf{AOX}}$$

Fig. 196 shows in plan two lines along which the dip has been observed: **CD**, direction south-east 50°, dip 1 in 10; **EF**, direction north-east 30°, dip 1 in 20. These two lines must be plotted on paper to scale, showing their direction and position correctly, and prolonged till they meet in **G**. On the line **GD** must then be marked out a length of 10, **GH** (because the dip is 1 in 10); and on the line **GF** a length of 20, **GI** (because the dip is 1 in 20); **H** and **I** must be connected by the line **HI**, and a perpendicular to this line drawn from the apex **G**. **GK** is then the direction of the greatest dip, and represents the amount of dip. The length **GK** is 8¼, and therefore the inclination is 1 in 8¼.

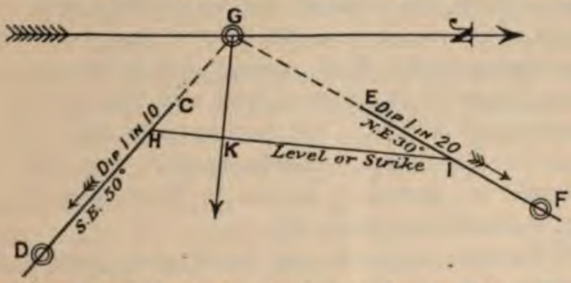

FIG. 196.—Method of finding the true dip from two observations.

"In a similar manner the true dip may be ascertained from the depth of three pits, represented in Fig. 196 by the letters **G**, **D**, and **F**. It is first necessary to reduce the actual depths to their relative depths above or below the sea-level. Thus suppose **G** is 150, **D** 220, and **F** 250 yards deep, all down to the same coal; if the top of **G** pit is 300 feet above the sea-level, **D** 360 feet, and **F** 390 feet,—then 60 feet must be taken off **D**, and 90 off **F**, reducing the depth of **D** to 200 yards, and of **F** to 220 yards. Then, if the distance between **GD** and **GF** is known, the rate of inclination on those lines can be calculated, and the true dip set out in the manner given in the first instance.

"If the dip is ascertained by means of a clinometer, and is recorded in degrees, the rate of inclination can be quickly ascertained by bearing in mind that it is equal to the ratio of the radius of the circle to the cotangent of the angle of inclination. If, for instance, the radius is 1 and the cotangent of the angle 10, the inclination would be 1 in 10; for an angle of 6° the (natural) cotangent is 9·5, and the inclination is therefore 1 in 9·5."[1]

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[1] "Mining," by Arnold Lupton (Longmans, Green, and Co.).

Levelling to ascertain Subsidence of Surface due to Underground Workings.—In order to ascertain the subsidence due to underground workings, there are two methods which can be adopted. The first is to observe the subsidence under a canal, reservoir, railroad, high-road, wall, or building, the levels and gradients of which are known; then, if one portion has been lowered by the underground workings, the amount can be measured. Thus in a canal the water from lock to lock is maintained at one level, and this level is, say, 1 foot below the top of the towing-path; if the towing-path is lowered by underground workings till it becomes level with the top of the water, the subsidence is 1 foot. The towing-path will, of course, be raised, and it may again subside another foot; the amount of subsidence is therefore known to those who have raised the towing-path from time to time.

The subsidence, however, may have taken place under some bridge crossing the canal, which is lowered by the extraction of minerals from below. The crown of the arch of this bridge was, say, 6 feet above the surface of the water. If after the extraction of the minerals it is found that the crown of the arch is only 5 feet above the water, the crown has been lowered 1 foot; and until the bridge is rebuilt, the height of the arch above the water will continue to give a measure of the total subsidence.

In a similar manner, in the case of a railway which is level for a long length, or of which the gradient is known, if the ground subsides there will be a hollow where the line was previously level, and the amount of subsidence can be measured. The plate-layers will, of course, raise the rails from time to time as they fall; the amount of subsidence is therefore known to the plate-layers, who know the extent to which the rails have been raised.

In a similar manner with a turnpike road which is well kept at a uniform gradient, subsidence of the ground will cause a hollow, the amount of which can be measured. Also in the case of a long wall, the coping-stones of which have been laid on a level or on a uniform slope, any subsidence of the ground beneath would be shown by a breach in the regularity of the surface-line of the coping-stone.

In the absence, however, of any such marks as those above mentioned, the amount of subsidence cannot be determined unless, previous to the working of the coal, the ground has been levelled and accurate cross-sections of the surface of lines which are marked on the plan are made; subsequent levellings will show any variation in the surface. It is necessary, of course, that the levels should start from some permanent mark which is not altered by the subsidence. For very accurate observations of the subsidence, bench-marks should be made on gate-posts, walls, trees, posts, and rails, or, in the absence of a sufficiency of these, on posts specially fixed in the ground.

Candles and Lamps.—In ordinary mine surveying no means of illumination is more convenient than the candle, both for reading the instrument and for sighting. There are many places, however, where the candle cannot be used, on account either of wind or gas. In these cases an oil-lamp is generally used; and for reading the dial a small lamp made of copper, and provided with a burnished reflector and side handle (similar to a bicycle lamp), is very convenient. Where there is gas, safety-lamps are used exclusively. For reading the dial the lamp must be made exclusively of brass or copper, or of aluminium; the latter is a great improvement, as the weight of the lamp often becomes irksome. A swinging handle, as shown in Fig. 197, is also useful, because the lamp generally becomes too hot to be held with comfort without the aid of a handle, and it is impossible to hold it in the right place by the suspending ring. The lamp shown in the figure can be held by grasping the two sides of the handle, and so held the lamp may be above the hand.

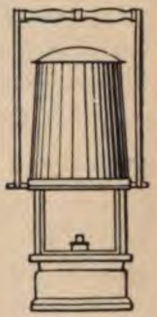

FIG. 197.—Surveyor's safety-lamp.

Coloured Lights.—In order to avoid mistakes through multiplicity of lamps near the object sighted, some surveyors adopt the plan of coloured lights, so as to distinguish the lamp at the station from other lamps in the vicinity. This is a good plan, especially where the theodolite is used, but where the dial is used the coloured glass diminishes the light, and makes it less easy to see at long distances. When coloured lamps are not used, the surveyor waits till all the lamps but one are hidden, and then takes that as the station lamp.

It is often difficult to read a finely graduated theodolite with the ordinary safety-lamp, and a lamp with a reflector and condensing lens would be a great advantage for this purpose.

For fast-needle and theodolite work care must be taken to have a lamp to fit the lamp-cups on the tripod, and that the wick-tube is exactly in the centre of the lamp; unless the lamp-flame is precisely over the centre of the tripod, it will lead to inaccuracy.

Plane Table.—The plane table shown in Fig. 198 is an instrument much used in some countries for preparing maps. It is considered specially useful for contouring. The instrument consists of a drawing-board, mounted on a tripod stand; there are levelling screws, by which the board can be levelled, and it can be turned round on a brass ring, supported by the levelling screws, and revolving on a centre pin, with coned or special head. The surveyor is also provided with a rule (termed an alidade) with sights placed at its ends (in Fig. 198 a telescope is shown instead of sights), and carrying a trough compass. A loose spirit-level is also provided with which to level the board.

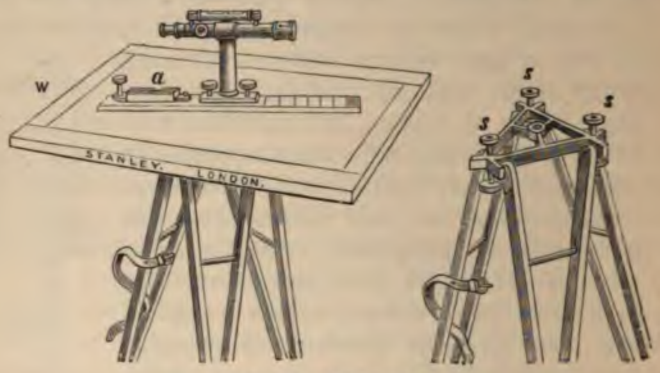

Fig. 198.—Plane table.
(*Kindly lent by Messrs. W. F. Stanley and Co., Ltd.*)

The intention is to make a drawing or sketch upon this board, showing the salient features of the landscape; if it is on a small scale, these will be mountains, hills, churches, towns, clumps of trees, rivers, lakes, roads, etc.; if it is on a large scale, more detailed objects, such as corners of buildings, fences, brooks, outcrops of minerals, position of shafts, etc., may be sketched. In the first place, it might be considered that the drawing on the plane table is a picture representing the landscape, such as would be seen from the top of a very tall tower infinitely high, so that all the objects in the landscape would appear in their correct relative positions, and be so sketched upon the plan.

In an ordinary landscape the near objects appear large, and the distant objects small; but this would not be the case if the artist were at the top of a tower infinitely high, and were sketching a limited landscape. From this great height he would see the buildings, hills, rivers, and lakes separated from each other by an apparent distance, which would in each case be proportional to the real distance.

Method of working the Plane Table.—The surveyor, using the plane table, stands on an elevation, so that his line of sight passes over hedges, walls, and other obstructions. He sees two villages, say 2 miles distant from him, and 2 miles distant from each other; he cannot tell what distance they are apart or from himself, but having fixed a clean sheet of paper to the table, he puts a mark (*a*) upon it, representing the place where he is standing (see Fig. 199), which is, perhaps, near to a village church, which he sketches on the paper, and puts on the name of the village: this is station **A**. He then takes the ruler or straight-edge, lays it on the paper with its edge on the point *a*, and directs it towards the church spire in village **B**, and rules a light line over the paper, or, to avoid too many lines, rules a short line at the edge of the paper, writing on it **A** to **B**. He then turns the ruler into the direction of the church at village **C**, and rules a similar line, writing on the end of the line **A** to **C**. He may proceed to rule any number of other lines in the direction of buildings, hills, and other objects, which he desires to place on the map.

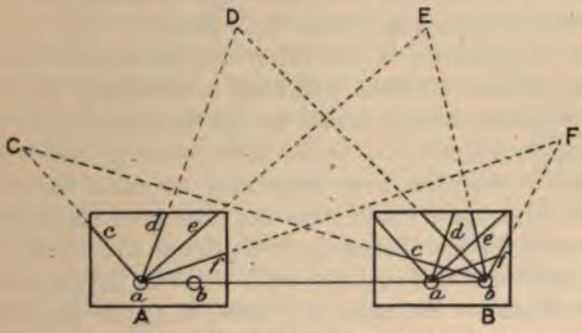

Fig. 199.—Method of working the plane table.

He then, leaving a flag to mark the station at **A**, proceeds with his instrument to station **B**; he measures the distance from **A** to **B** as he walks by counting paces, or, as this is the first line and may be the base of a system of triangulation, by accurate chaining. It may be that he can take the distance **AB** from some existing map with accuracy; if the distance **AB** is not accurately placed on the map, then any distances calculated from that base will, of course, be inaccurate. This measurement enables him to mark the point *b* on the line *ab* previously ruled.

He now fixes the tripod with the point *b* on the paper over the station **B**, and turns the plane table until the point *a* on the paper is exactly in the direction of the station mark at **A**. He now takes the ruler, and places it on the paper with its edge on the point *b*, sights to **C**, and rules a line. The intersection of this line *bc* with the line *ac* gives the exact position of the point **C**. If the base-line **AB** has been accurately measured, the position **C** is accurately fixed, and the distances **BC** and **AC** can be scaled off the plan which is so made.

From the position **B** the ruler may be directed towards all the other stations previously sighted from **A**, and by the intersections so given their positions are all fixed upon the plan, and if the angles are not too acute, their positions are accurately fixed, and in addition lines may now be ruled towards other buildings, hills, etc., which were not seen from station **A**.

The surveyor then proceeds to **C**, fixing the mark *c* over the station, and turning the plane table till *b* on the paper is in the direction of station **B**; and, of course, *a* on the paper is at the same time in the direction **A**, if the work has been correctly done. He now lays the ruler in the direction of the stations sighted from **B**, and marks the intersections by which the position of all these places is accurately fixed. The work may thus be continued without fresh measurements from station to station.

This is a system of triangulation exactly similar to that described in Chapter VII. for use with the theodolite; but instead of booking angles in a note-book, and subsequently plotting them with a protractor, the angles are all drawn out by sight upon the plan, and no booking is necessary. If the survey is started with an accurately measured base-line, and sufficient care is taken, the result will be an approximately accurate plan suitable for a preliminary survey.

To get the table level, a small pocket-level is placed upon it; a fine pencil is used in ruling the lines.

Sketching in Contours, Roads, etc.—When the instrument is set up at the first station, only radial lines from the station **A** can be drawn; but when the instrument has been set up at **B**, and intersections at **C** and other places marked on the plan, details of the landscape may be sketched in. The position **C** having been accurately fixed on the plan, shading may be added to show that it is a hill; the position of a river running past **A** and **B** may also be sketched with approximate accuracy; and a road or railway going from **A** to **B** may be sketched on the plan in the same way.

When a telescope is used instead of plain sights, the plane table becomes a much more precise instrument. The telescope is fixed upon a ruler which has a broad base, so that it is not easily upset. The plane table being fixed level, the vertical axis of the telescope is, of course, vertical. The telescope can be moved through an arc, so as to measure elevations and depressions. A spirit-level on the top of the telescope is a check upon the levelling of the plane table.

By means of parallel hairs the telescope can be used for tacheometry. By this means positions can be marked upon the plan, when there are no intersections, by simply ruling a line in the direction of the object, and measuring the distance by the readings of a staff seen through the telescope.

Advantage of the Plane Table.—The chief advantage of the use of the plane table is the facility for sketching in the contours of hills, and the course of streams and rivers from a vantage-ground. With the main stations accurately fixed by intersections and tacheometrical measurements, the details may be sketched in with considerable accuracy in a short time; whereas to do this from measurements and angles recorded in the note-book would require a great deal of measuring and note-taking.

Plane Table and Trough Compass.—The plane table is often used with the trough compass. The compass is placed on the table and turned until the needle is parallel to the centre-line of the box. This direction may be ruled on the board, and at every fresh station the board may be turned until this line so marked comes into the meridian. If this is done, it obviates the necessity for taking a back sight as a base-line for fresh intersections, as every line ruled upon the plan makes an angle with the meridian. It is better, however, to use the needle as a check upon the accuracy of the work done without it than as a substitution for it.

In the United States it is the practice to make the field-map twice the scale of the map to be published; thus any errors made in the original survey are much reduced.

In the topographical land survey of Wurtemberg the drawing on the plane table was $\frac{1}{2500}$, or $25\frac{1}{3}$ inches to the mile, and the scale of the published plan was $\frac{1}{50000}$; thus 400 plane-table sheets were required for one published map of the same size.[1]

Plane-table work is most suitable for countries where a continuance of fine, dry weather can be expected.

For rapid work the plane table may be used strapped to the arm, and in this case a magnetic compass is often fixed to the table, so that it can always be held in the meridian, and the bearings of the various points sketched, as the angles could not be taken correctly from a fixed base without the use of a tripod.

Simultaneous Use of Two Plane Tables.—By the simultaneous use of two plane tables the intersections of lines of sight can be obtained with increased rapidity without the need of permanent stations being fixed, or for refinding stations observed from the first position of the table.

In some cases a flag may be carried by a horseman; at every place where he stops the line of his direction is marked on the plan, and their intersections found by subsequent comparison of the two plans.[2] A surveyor using the plane table has constant opportunities of correcting his drawing as he changes his station, and ultimately arriving at a fairly correct representation of the chief features which are important for his purpose. Such a plan would be useful for many mining purposes, especially in conjunction with photographs.

Portable Boards.—Boards are sometimes made to roll up with light folding tripods, so as to be easily carried by a horseman.

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[1] J. Pierce, Junr., M.A., A.I.C.E., "Economic Use of the Plane Table," *Inst. C. E.*, p. 187, vol. xcii.

[2] Pierce, on the Use of the Plane Table.

CHAPTER XVII.

PROSPECTING FOR MINERALS BY MEANS OF THE MAGNETIC NEEDLE.

THE properties possessed by the magnetic needle have enabled it to be used to advantage in searching for ore deposits. Instruments for this purpose have reached a high state of perfection in the country of Sweden, and Professor G. Nordenström, of the Stockholm School of Mines, gives an account of these instruments and the method of using them, in a valuable paper read before the Iron and Steel Institute, at their Stockholm meeting.[1]

Magnetic instruments have been employed in Sweden for more than two hundred years in exploring for ore. This fact can doubtless be ascribed to the interest for exploring for ore among the mining engineers, and also among the inhabitants of the mining districts in general, the Government encouraging this interest by rewards to such as discover new deposits.

The ores occurring most frequently in the country are the magnetite iron ores, which are strongly magnetic; the next commonest, the hematites, are also magnetic, but in a lesser degree, since they are always more or less mixed with magnetite.

Other ore deposits, such as copper, zinc, cobalt, and nickel, have also been and may be found by the aid of the needle, since these ores contain a greater or lesser proportion of magnetite or magnetic pyrites.

The miner's dip compass was introduced in 1770, and by its use all the Swedish iron ores have been explored. It was constructed as follows: In a round brass box a magnetic needle is suspended in such a way that it can move freely on a horizontal plane and on a vertical plane to an angle of about

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[1] Published in *Engineering*, September 30 and October 17, 1898, from which this description and illustrations are taken, with the Editor's kind permission.

70° from the horizon. It is compensated for the earth's magnetism, so that it takes a horizontal position in districts void of ore, or where there are no magnetic ores. As a rule, miner's compasses without graduation are used; the horizontal plane of the needle is only indicated by a ring inside the compass. The dip of the needle is estimated only by the eye, and is not actually measured.

The miner's compass is still used, and with success, for exploring for ores, but more particularly for the preliminary exploring work in ore fields.

In later times, however, the demand for more accurate results has grown, and during the past thirty years there have been introduced magnetic instruments by means of which a still more exact knowledge of the magnetic conditions of our iron-ore fields can be obtained.

Thalén's Magnetometer.—This instrument, constructed by Professor Thalén, of the Upsala University, is a modification of Lamont's theodolite.

It consists of a declination compass **A** (Fig. 200) of about 80 millimetres ($3\frac{3}{16}$ inches) in diameter, which is provided with a

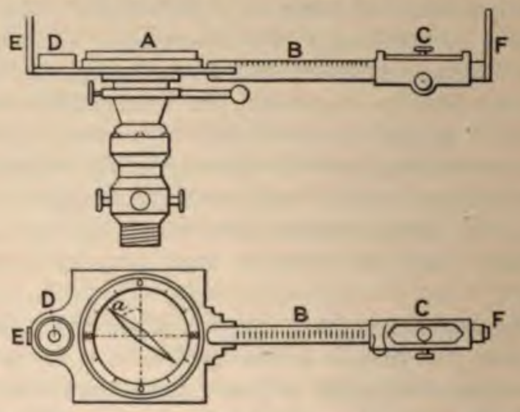

FIG. 200.—Plan and side elevation of Thalén's magnetometer.

scale graduated to degrees and half-degrees from 0° to 90°. At right angles to the diameter, which passes through the zero point of the scale, there is attached an arm **B** from 200 to 220 millimetres ($7\frac{7}{8}$ to $8\frac{5}{8}$ inches) long.

On this arm, which is graduated in millimetres, is placed the bar magnet **C** for the deviation measurements, which can, at the will of the operator, be given a certain fixed distance from the centre of the needle.

The instrument is rotated on a vertical axis, whose central line passes through the centre of the magnetic needle. It is provided with a spirit-level D, sights E and F, and levelling screws, and is placed on a tripod.

This instrument, which has been in use for more than twenty-five years, is now used principally for measuring horizontal intensity. In so doing two methods may be used—the tangent method and the sine method.

In using the tangent method, the magnetic needle is first placed at zero, after the instrument has been levelled, and the bar magnet has been removed from its place. Then the bar magnet is put in its proper place on the arm, and the angle of deviation a is read.

In using the sine method, the bar magnet is put in place on the arm. Then the magnetic needle is placed at zero, and, after the bar magnet has then been removed, the angle of deviation a is read. This latter method gives the more accurate results, but in practice the tangent method is generally used, partly because it is more convenient, and partly because it is everywhere applicable, which is not the case with the sine method in certain points of the ore field north of the ore mass.

Method of using the Magnetometer.—Before the measurements are begun, the instrument is adjusted at a place where there are no magnetic ores, and consequently no other magnetic force than the earth's magnetism. The angle of deviation found here is noted a_0, and is generally so arranged that it is equal to 25° or 30°.[1] Then begins the measurement of the ore field, which for this purpose is divided into squares with sides 10 metres in length.

By the aid of the tangent method the angle of deviation a is afterwards obtained in each corner of every square. These a values are noted on a map (see ideal map, Fig. 201), and the points for which equal angles have been obtained are joined. This gives two systems of curves, which in a more or less regular manner are grouped round their centres. One of these is situated north of the ore, and where the a values are greatest, and is therefore noted with a maximum; the other is situated either directly above the greatest mass of ore, or somewhat to the south of it, and represents the smallest a value, being therefore noted with a minimum. Between these two sets of curves there is a wavy line, whose angle of deviation is the same as obtained where there is no ore, and it is noted with a_0; this curved line is called a neutral line.

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[1] The angle of deviation means the angle between the magnetic meridian and the position of the compass needle, and the deviation is caused by the bar magnet.

The line which unites the maximum point and the minimum point indicates the direction of the magnetic meridian of the ore field. The centre of the greatest mass of ore is situated either at the point of intersection of the magnetic meridian and the neutral line, or else directly under the point marked a minimum. In order to get correct results, the levelling of the ore field which is being measured should be known.

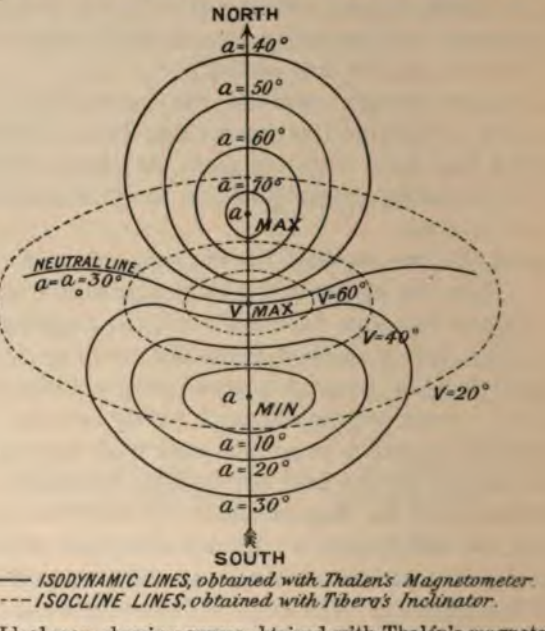
NORTH
a=40°
a=50°
a=60°
a=70°
a
MAX
NEUTRAL LINE
a=a₀=30°
V MAX
V=60°
V=40°
V=20°
a
MIN
a=10°
a=20°
a=30°
SOUTH
——— ISODYNAMIC LINES, obtained with Thalén's Magnetometer.
------- ISOCLINE LINES, obtained with Tiberg's Inclinator.

FIG. 201.—Ideal map, showing curves obtained with Thalén's magnetometer and Tiberg's inclinator.

Tiberg's Inclinator.—This instrument has been in use since 1880, when it was invented by E. Tiberg. It consists of a dip compass 80 millimetres in diameter, graduated from 0° to 90°, and a magnetic needle so hung that it cannot move except in the plane of the graduated circular scale. The instrument furthermore differs from other dip compasses in that the centre of gravity of the magnetic needle is a little below its horizontal axis when the compass is in a vertical position. The needle is compensated for the vertical force of the earth's magnetism by a piece of wax or by a counterbalance of aluminium fixed to it. For some years this instrument has been generally used in combination with Thalén's magnetometer, and by means of this combination measurements according to both Thalén's and Tiberg's methods may be quickly made. The combined instrument is illustrated in Figs. 202 and 203. Fig. 202 shows the instrument furnished with Tiberg's compass, but in Fig. 203 Thalén's compass is substituted. In order to make it possible to use first the one and then the other of these compasses, they are provided with axle-pins fitting into the bearings

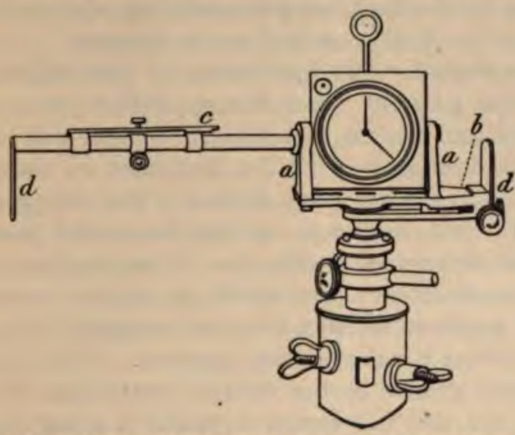

Fig. 202.—Combined instrument fitted with Tiberg's compass.

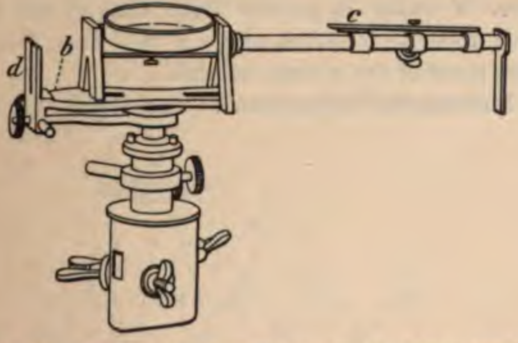

Fig. 203.—Combined instrument fitted with Thalén's compass.

in the standards *a*. The centre-lines of the axle-pins in the Tiberg compass run through the zero points, but in the Thalén compass through the 90° point. The instrument is furnished with a spirit-level *b*, a transverse arm *c*, and sights *d*. The arm *c*, secured on one of the standards, serves to receive the bar magnet for the deviation measurements, when measurements according to the Thalén method are to be made.

Tiberg's Method.—The instrument is first adjusted on perfectly neutral ground. After the ore field to be explored has been divided into squares with sides 10 metres long at the most, observations are made with the inclinator in each corner in every square, in the following manner: The compass is placed horizontally, and is turned on the horizontal plane till the central line through the axle-pins of the compass is at right angles to the direction of the needle, or, in other words, so that the needle is placed at 90°; then the compass is turned on its axle-pins so that it has a vertical position. In this position the needle is only affected by the vertical component of the attraction of the ore, and this causes a greater or lesser inclination of the needle. If the magnetic force of the ore is **P**, and the angle of inclination is **V**, then we have $P = K \tan V$.

If we mark the value of **V** on a map, and the points for which equal values are obtained are united, we get a system of curves which are more or less regularly grouped round a certain centre whose **V** value is greater than that of all the others. Immediately under this centre, where $V = V$ max. (see Fig. 201), the greatest mass of ore always occurs.

Use of Instruments Underground.—Besides for surveys at the surface, both these instruments are used for surveys underground. For this purpose the sine method is generally used.

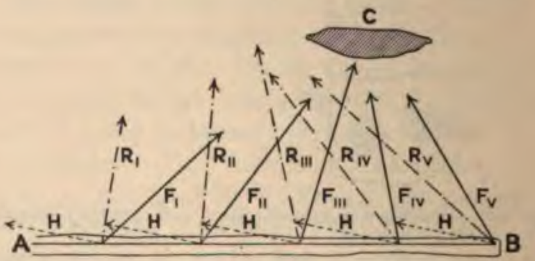

FIG. 204.—Method of prospecting underground.

If **H** (Fig. 204) is the horizontal component of the earth's magnetism, and **F** that of the ore, and **R** is the resultant of both, we obtain for each point of observation $\mathbf{R}_1$, $\mathbf{R}_2$, $\mathbf{R}_3$, etc., according to the formula $R = H\frac{\sin a_0}{\sin a}$, where a_0 is the angle of deviation found when there is no magnetic ore present, and a is the angle of deviation as read in the gallery of the mine. If we give an arbitrary value to **H**, which is considered to be a constant, we get the lengths $\mathbf{R}_1$, $\mathbf{R}_2$, $\mathbf{R}_3$, etc., and also their direction. The length and direction of the component **F** is then obtained by construction. The position of the centre of the ore sought for, *c*, is indicated by the direction of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, etc., all of which converge more or less to this centre.

Professor Nordenström concludes his valuable paper by expressing his opinion as to the value of magnetic measurements in all countries where magnetic ores are known to occur.

CHAPTER XVIII.

METHODS OF FINDING TRUE NORTH, OR GEOGRAPHICAL MERIDIAN.

In connection with every important mining survey it is highly desirable that a line in the direction of the geographical meridian, that is to say, a north-and-south line, should be set out and fixed by permanent marks for a considerable length, say 10 chains. These marks should be on pillars of brick, stone, iron, or oak, and the centre line indicated with great accuracy. There are many ways in which the direction of the north pole or of the south pole may be ascertained. The sun is the best indicator of direction in the temperate regions. In the northern hemisphere, north of the tropics, the sun is always due south at noonday; and in the southern hemisphere, south of the tropics, the sun is always due north at noonday. On the northernmost tropic the sun is vertically overhead at noonday on June 21, and on the southernmost tropic the same is the case on December 22, while at the equator the sun is in the zenith at the equinoxes.

The following are some of the methods used by surveyors for ascertaining the north-and-south line:—

By Equal Shadows of the Sun.—At apparent noon the sun, in the northern hemisphere north of the tropics, is due south, and the shadow thrown by a vertical pole[1] would represent the direction of a line joining the north and south poles; that is to say, the true meridian. At equal times before or after apparent noon, the shadows thrown by the pole would be of equal length. This method is applied in practice as follows: A vertical pole, shown in plan at O (Fig. 205), is erected on the south side of a level surface. A few hours before noon a mark is made at the end of the shadow cast by the pole, and a circle is described having its centre at the foot of the pole O, and with radius equal to the shadow **OA**. At an equal interval of time after noon the shadow will be again equal to **OA**, and the position of the end of the shadow is marked at the exact point, **B**, where it touches the circle already described. The arc **AB** is then bisected at the point **C**, and the line **OC** represents the direction of the true meridian. This direction may be produced and pegged out on the surface. If two or three circles are drawn at different hours before noon, and the two points marked in which each is touched by the shadow of equal length in the afternoon, a number of arcs are obtained; these may all be bisected, and a more accurate result obtained by taking the average. This method is only perfectly correct at the time of the solstices (June 21 and December 22). To get accurate results the ground should be quite level and white, and the circles of large diameter, so as to minimize the effect of any error in fixing the exact position of the end of the shadow.

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[1] Where there is shelter from the wind, a plumb-line might be substituted for the pole.

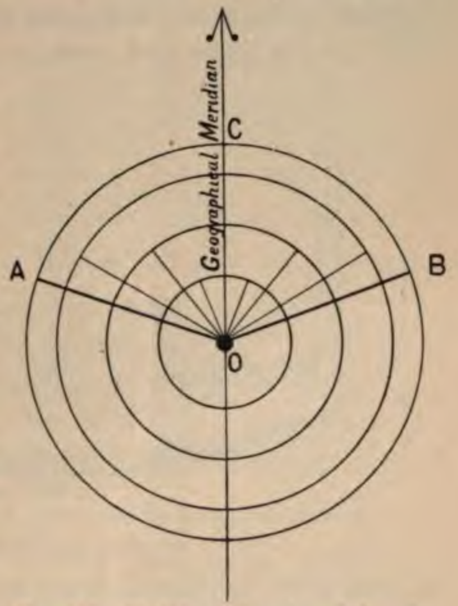

FIG. 205.—Finding north-and-south lines by shadows of sun.

Meridian Dial.—Mr. E. T. Newton of Camborne has utilized the principle, and constructed a special instrument for obtaining the true meridian. This consists (as shown in Fig. 206) of a brass ring 10 inches outside diameter, and 2 inches wide, with four arms and central boss; this fits on to a dial from which the sights have been unscrewed. The ring is provided with an alidade working round a centre pin in the boss, with plain sights at each end. A pillar 3½ inches high is screwed into a hole exactly in the centre of the boss. This pillar may either end in a needle-point, or may have a small plate fastened to the end, in which a small hole is pierced. If the former, a shadow is cast; if the latter, a small bright spot.

As the position of the sun alters during the day, the shadow or spot crosses different circles drawn on a white celluloid disc, which is secured to the brass plate; the circles are struck from the centre of the boss. The points where the spot or end of the needle shadow touch each circle are marked. The two points on each circle equidistant from the centre are joined by a straight line, which is bisected in each case; a line drawn from the centre of the boss through these bisections (which, if correct, will be in the same straight line) will be a north-and-south line.

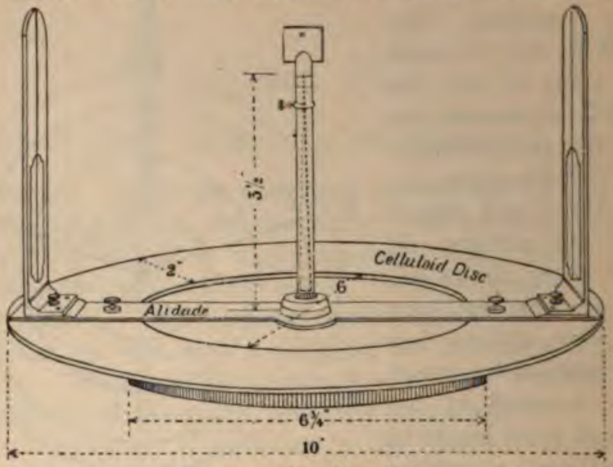

FIG. 206.—Meridian dial.

The alidade can now be fixed in the direction of this line, which may then be set out by means of the sights.

The declination can be read off from the compass-needle of the dial, which can be seen through the openings between the arms connecting the ring with the centre boss.

Neither of the above methods provides for accuracy, and they are unsuitable for important surveys.

By Equal Altitudes of the Sun, or a Star.—If a theodolite be substituted for the pole (or plumb-line), the altitude of the sun (taking, say, the lower edge) may be observed very accurately before noon (at, say, 10 a.m.), reading the azimuth circle simultaneously. The observer then waits until the sun reaches the same altitude after noon (at, say, 2 p.m.), and again observes the azimuth. The mean of the two azimuth readings would be the south point, very nearly. A small error will arise from the sun's motion in declination during the interval between the observations. But the amount of this error may be very easily calculated. It would not, as a rule, amount to more than four or five minutes of arc. If, instead of the sun, any bright star be observed, this method admits of great accuracy.

In order to calculate the error due to the sun's motion in declination, the following facts must be considered. From December 21 to June 21, the sun is rising higher in the heavens at noonday in the northern hemisphere (north of the Tropic of Cancer), and of course is falling in the southern hemisphere; and from June 21 to December 21, the sun is falling lower in the heavens at noonday in the northern hemisphere, and of course is rising for the same period in the southern hemisphere (south of the Tropic of Capricorn).

This rising and falling is due to the variation of the declination of the earth's axis of rotation, towards or from the sun.

At the equinoxes, that is, about March 21 and September 23, there is no declination (these dates are for 1902).

After March 20, the declination (see Nautical Almanack) is north, and increases each day about 23 minutes or about 59 seconds per hour; but the rate of increase or variation gradually diminishes as midsummer approaches, until the maximum northern declination of 23° 27′ is reached on June 21.

The rate of variation on June 1 is 21 seconds per hour. On June 20 it is 2 seconds per hour; after June 21 the rate of variation gradually increases to about 59 seconds per hour at the September equinox, after which the rate of variation gradually decreases, until the maximum southern declination of nearly 23° 27′ is reached on December 22 (1902).

If, then, the observation is made on June 21 or December 22, no correction for variation of declination is necessary.

If the observation is made on March 21 or September 21, a correction must be made, due to a variation in declination of 59 seconds per hour between the times of the first observation (say 10 a.m.) and the second observation (say 2 p.m.). Between the above dates a proportional correction must be made, the exact variation per hour being obtained from the Nautical Almanack. At the latitude of Greenwich on August 18, 1901, the variation of declination per hour is 48 seconds, and the second observation at 2 p.m. will be 10 minutes too far halfway between the two observations would be 5

In afternoon at 2 p.m.,
316° 49′ 38″.

Sun's declination August 18, 13° 21′.

Difference of declination = −48 seconds × 4 hours = −192 seconds.

Diff. dec.	= −192″ log is	2·2833
Declination	= 13° 21′ log cosine	9·9881
Azimuth	= 43° 20′ log cosecant	0·1634
Altitude	= 44° 52′ log secant	0·1495
Latitude	= 51° 28′ log secant	0·2055
		2·7898

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[1] Rule given by Mr. C. R. Davidson, Royal Observatory, Greenwich.

which is the log of $-616'' = -10' \, 16''$.

$$
\begin{array}{lcr}
\text{Second azimuth} & = & 316^\circ \, 49' \, 38'' \\
\text{Correction} & = & -\, 10' \, 16'' \\
\hline
 & & 316^\circ \, 39' \, 22'' \\
\text{First azimuth} & = & 43^\circ \, 20' \, 40'' \\
\hline
\text{Mean} & & 180^\circ \ \ 0' \ \ 1''
\end{array}
$$

Observation of the Sun at Noon.—Mr. S. A. Warburton of Moira, near Ashby-de-la-Zouch, has sent the author the following description of his method. The instruments he uses are a good theodolite, and a good watch set to Greenwich mean time :—

"The method which I employed was to ascertain, by observation, the actual passage of the sun's centre over the meridian of the place of observation, which seems to me to be the best method, only there are a number of calculations to be made, and it is necessary to have a good watch set exactly to Greenwich mean time; this must be got by setting the watch exactly at 10 a.m., or other telegraphed hour, on the day of observation by a time-ball.

"Now, Greenwich mean time is not apparent time, the latter being solar time, such as would be given by a sun-dial, and Greenwich mean time noon is not apparent noon, the difference being about 16 minutes at one time of the year and varying to nothing; these differences are given for every day of the year in the Nautical Almanack. Take, for instance, a certain day when from the Nautical Almanack you find the solar time at Greenwich to be 10 minutes 5 seconds earlier than Greenwich mean time; this would mean that a theodolite pointed to the centre of the sun at Greenwich at 11 hours 49 minutes 55 seconds, would give the true Greenwich meridian; but as our place of observation is not likely to be at Greenwich, but some place east or west of it, another factor is brought in, and a correction for our longitude east or west of Greenwich must be made; again, we are unable to observe the sun's centre with accuracy, therefore his right or left limb is observed; then adding his semi-diameter (if we observe his left limb), we find his centre. The sun's apparent diameter varies with his distance from the earth, and his semi-diameter is given in the Nautical Almanack for every day in the year at mean noon. First, then, we must know our longitude east or west of Greenwich, and in order to do this take an Ordnance sheet and draw a vertical line through the point of observation, and the longitude east or west will be given on the top and bottom of the sheet. Take the following example :—

"Place of Observation. Hyde Park Corner, Leeds.

"Date, October 2, 1899.

"Take the 1″ Ordnance sheet,[1] and drawing a vertical line through Hyde Park Corner, we shall find that it is situated in long. 1° 33′ 40″ W.

"Referring to the Nautical Almanack (or Brown's), we see that on October 2, the equation of time is 10 minutes 38·3 seconds to be added to mean time; in other words, that 10 minutes 38·3 seconds must be added to Greenwich time to find the apparent time at Greenwich.

Thus when it is mean noon at Greenwich by the clock, the real or apparent hour by the sun is 12 hours 10 minutes 38 seconds, and the apparent noon is 12 hours less 10′ 38·3″ = 11 hours 49 minutes 21·7 seconds.

"We must next reduce our 1° 33′ 40″ W. long. into time. Now, 15° longitude = 1 hour of time; therefore 1° = 4 minutes, and 1′ = 4 seconds of time; and 1° 33′ 40″ = 6 minutes 15 seconds very nearly; and as it is west longitude, we must add this to the time we know the true sun passed the meridian of Greenwich, which we have already found to be 11 hours 49 minutes 21·7 seconds.

Hrs.	Mins.	Secs.	
11	49	21·7	= true sun passes the meridian of Greenwich.
	6	15·0	= time taken by sun to arrive at long. 1° 33′ 40″ W.
11	55	36·7	= time (Greenwich mean time) of sun passing long. 1° 33′ 40″ W.

"We can now set up our theodolite, fitted with coloured eye-piece for solar observations, and we will observe the left limb of the sun, and, with the vernier clamped at 360°, follow the sun with the tangent screw carrying the whole instrument (as the telescope will probably be an inverting one, we shall appear to be observing the right limb, and the sun to be moving from right to left); say we begin this at 11 hours 54 minutes by our correctly timed watch, then at 11 hours 55 minutes 36 seconds we stop, because at that moment the sun is on our meridian.

"Referring to the Nautical Almanack, we find that the sun's semi-diameter on October 2 is 16′ 0·8″; we advance our vernier to read 16′ 0·8″, and our telescope is now in the true meridian for our place of observation. Had the right limb of the sun been observed, we should have had to bring the vernier back 16′ 0·8″.

"If we cannot obtain our longitude east or west from an Ordnance map, we must do it from an atlas, only, as this will not give it very accurately, we must mind and use the same figure in any future observations at the same place in order to avoid discrepancies, but of course the accuracy of the result will be affected by any inaccuracy in ascertaining the longitude.

"To convert longitude into time, multiply the degrees by 4, and this will give you minutes; multiply the minutes by 4, and this will give seconds; and multiply the seconds by 4, and this will give sixtieths of a second.

"To reduce 6° 10′ 20″ to time.

		Mins.	Secs.
6° × 4	=	24	0
10′ × 4	=		40
20″ × 4	=		1⅓
		24	41⅓″

By Observation of the Pole Star.—The pole star (Polaris *α* Ursæ Minoris) is 1° 15′ from the north pole of the heavens, and moves in a circle round it; twice in 24 hours (more precisely, 23 hours 56 minutes) it is in the true meridian. Another star known as Alioth (ε Ursæ Majoris) comes into the meridian on its right ascension about half an hour before the pole star reaches the meridian on its lower transit. Thus if the pole star is sighted with the vertical hair of the theodolite telescope, and followed till the vertical line through it cuts the star Alioth, then, if at the moment when this happens the theodolite is clamped, we obtain a line approximately in the meridian. If the observation is made when Alioth is below the pole the line is 17 minutes east of north, and if when Alioth is above the pole the line is 17 minutes west of north. The north star is exactly in the meridian some 31[1] minutes after the above observation has been made, and if the telescope is then directed to the north star, it will be exactly in line with the true meridian. Fig. 207 shows the relation to each other of Polaris and Alioth and some other stars of the Great Bear, and shows the north star vertically over the pole and the star Alioth. The "upper transit"—that is to say, when the pole star is above the pole—is the most convenient, because at the lower transit the star Alioth (ε Ursæ Majoris) is at its upper transit and too high to be conveniently observed.[2]

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[1] The 6-inch Ordnance Map gives greater accuracy in fixing the longitude.

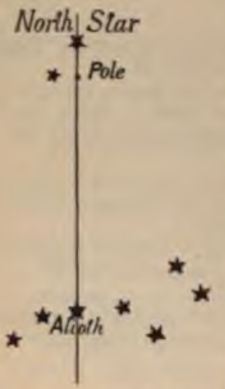

FIG. 207.—Finding north by pole star and others.

A second method is as follows :—

On referring to Fig. 208 it will be seen that there are two points, **B** and **D**, which represent the extreme easterly movement and the extreme westerly movement of the pole star. If observations be taken of these two points with a theodolite, and the angle bisected, then the bisecting line would pass exactly through the pole, *i.e.* would represent the true meridian. Unfortunately, one or other of these two positions occurs usually in daylight, when it will generally be invisible except with the aid of a very powerful telescope.

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[1] This figure of 31 minutes is correct for the year 1901, but the time increases at the rate of about 23 seconds a year, and in 1911 it will be about 35 minutes. The correction for any year may be found on reference to the Nautical Almanack.

For the year 1901, the right ascension of Polaris is 1 hour 22 minutes 57 seconds, and the R.A. of Alioth is 12 hours 49 minutes 41 seconds, the difference between the upper transit of Polaris and the lower transit of Alioth being 33 minutes 17 seconds. Alioth and Polaris are in the same vertical line 2 minutes after the transit of Alioth; deducting these 2 minutes leaves the interval of 31 minutes above given.

[2] In England, with the ordinary telescope of a theodolite, the upper transit (right ascension) of Polaris may be observed at night in the months of September, October, November, December, and the first twelve days of January, and the lower transit may be observed between January 1 and May 12.

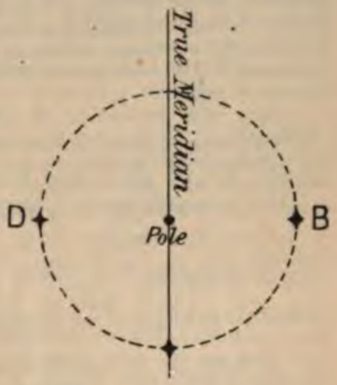

Fig. 208.—Finding north by pole star only.

Third method. In the months of December and January it is possible to observe the pole star at equal distances from the upper and lower transit in the same night. Thus the first observation may be made early in the evening, when the pole star would be near its upper transit. A mark should then be fixed at a convenient distance in the direction given by the star. The observation is then repeated after an interval of 11 hours 58 minutes, when the star would be near its lower transit; the theodolite being fixed at the same centre, and a distant mark put as before. Thus, whatever might be the deviation of the star from the meridian at the evening observation, there would be the same deviation in the opposite direction in the morning observation; and, accordingly, if we took the middle point between the two distant marks, this point, as seen from the instrument, would give the direction of the meridian line near enough for all practical purposes.

By Observation of Various Stars.—The north-and-south line may be ascertained by reference to many other stars, the apparent places of which are given in the Nautical Almanack and other almanacks. Mr. A. L. Steavenson, in a paper read before the Federated Institute of Mining Engineers,[1] describes a method of ascertaining the north-and-south line as follows:—

"The meridian of any place is represented by a line drawn through it from the north to the south pole, and as the sun or stars cross this line, they reach their greatest elevation, and are said to transit. The times of right ascension or transit are given for the principal stars, and the sidereal time each day at noon in the Nautical Almanack and also in Whitaker's Almanack each year. Perhaps the shortest and best way to describe the ***modus operandi*** is to take a case, and suppose that the writer wishes to describe a meridian line at his own house.

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[1] Vol. x. p. 53 (August, 1895).

"Referring to the Ordnance Survey map, and with a parallel ruler drawing lines vertical and horizontal through the point in question, he finds that the latitude is 54° 43′ 49″ N., and the longitude 1° 36′ 41″ W., and as there are 360 degrees of longitude which the revolution of the earth performs in 24 hours, he finds that the allowance of time to be made for the position west of Greenwich is 6 minutes 26·7 seconds, that is to say, local time at Holywell is so much behind Greenwich time.

"Now, with respect to the altitude of a star, the elevation is given as 'declination.' Declination is measured vertically above or below the equator, and corresponds to latitude on the earth's surface; and the height of the equator corresponds with the co-latitude of a place, that is to say, its height on the meridian is equal to our co-latitude; thus—

	Degs.	Mins.	Secs.
Constant	90	0	0
Latitude of Holywell	54	43	49
Co-latitude or height of the equator on the meridian at Holywell	35	16	11

"Now we are in a position to find our star to-night, say, August 3. On referring to Whitaker's Almanack, p. 42, we find that on August 3 the sidereal time at mean noon is 8 hours 47 minutes 16 seconds, and as we want to do our work as soon after dark as possible, we will take a star passing about 10 o'clock. On p. 80 we find that the star Vega (α Lyræ) has a right ascension of 18 hours 33 minutes 23 seconds—

	Hrs.	Mins.	Secs.
	18	33	23
From this deduct sidereal time of August 3	8	47	16
Difference	9	46	7
From this we must deduct the difference which has occurred between our mean time and sidereal time since noon, at the rate of 9·85 or 10 seconds per hour	0	1	37
Due at Greenwich ...	9	44	30
And add to this the time allowance required by our longitude ...	0	6	27
The star Vega passes our meridian on August 3 at	9	50	57

"But we must next find the elevation, thus—

To our co-latitude	35	16	11
We add the declination of Vega	38	41	10
And we get the altitude ...	73	57	21

"If, however, this is too high to be seen in the theodolite, we might take the star μ Sagittarii, the declination being −21° 5′ 10″; in this case it must be deducted from the co-latitude, being a minus, or south declination.

"Having, then, a good watch carefully set by Greenwich time—say by the gun at Shields—we proceed about 9.30 p.m. to put the theodolite in position to observe the star, which the instrument very soon indicates, for a few minutes before the watch reaches the time of 9 hours 50 minutes 57 seconds p.m., and at the exact moment the instrument is pointing true south. Before making permanent marks, it will be well to repeat the observation, both on other stars and on other nights, and take the average or mean of them.

"In conclusion, it seems only desirable to point out, having once got this base-line or meridian, how interesting and valuable a means it affords for afterwards checking and regulating clocks and watches. To set a transit instrument for a given star on a fine clear night, watch it appear in the field of observation, exhibiting as it does the incomprehensible regularity of the heavenly bodies, is a delightful recreation, which has served to amuse and occupy the writer for many years, and induces him to encourage his hearers and readers to try it."

In the discussion Mr. Steavenson added the following note: "A slight correction was required for latitude to find the elevation, but it was so very small that it was not sufficient to carry the star outside the range of the instrument, and it was an easy matter to raise or lower it to the position required."

One of the obvious objections to the method described by Mr. Steavenson is the difficulty of having a watch set to the correct time. The star (and the same observation applies to the sun) is apparently moving at the rate of 1 minute of angle in 4 seconds of time; therefore, if the watch is 4 seconds wrong, there may be an error of 1 minute in the angle; for that reason surveyors prefer the observation of a star like the pole star, whose apparent movement is much slower, so that in the case of the pole star an error of 1 minute in the watch of the observer would only affect the accuracy of the observation to the extent of half a minute of angle. The following is Mr. Beanlands' description[1] of his methods of ascertaining the north-and-south line:—

"1. The pole star might be observed on the meridian either at its upper or lower transit, the time being determined in the manner explained by Mr. Steavenson. He (Mr. Beanlands) thought it would be more convenient, however, to obtain the time of transit from the data furnished by the Nautical Almanack. If they referred to p. iii. in each month of that almanack, they would find a column giving for each day the 'mean time of transit of the First Point of Aries.' The time of transit of the pole star would be found by adding to this the right ascension of the star as given on p. 290. The lower transit would take place about 12 hours, or more correctly 11 hours 58 minutes, after the upper transit.

"2. A meridian line might be determined by observing the pole star, in conjunction with another star having the same, or nearly the same, right ascension, or differing from it by 12 hours in right ascension. Perhaps the most convenient star for the purpose was ζ (*zeta*) in the constellation Ursa Major—in other words, the middle star in the tail of the Great Bear. The time of transit[2] must first be ascertained approximately; and

[1] August, 1895.

[2] Upper transit or right ascension of Polaris.

the theodolite being previously adjusted, the telescope must be pointed to the pole star, which must be bisected with the cross-wires. The instrument being then clamped in azimuth, the telescope must be lowered nearly to the horizon, when the star ζ Ursæ Majoris would be seen at an altitude of about 5°.[1] Without altering the horizontal position of the instrument, the star must then be watched until it appeared in the centre of the field of view. The telescope should then be raised and directed to the pole star, which should be again bisected, if necessary, by means of the tangent-screw. In this way we could obtain the direction of the pole star when the other star ζ Ursæ Majoris was in the same vertical plane. This method would give the meridian line with considerable precision. This observation, however, could only be made in the autumn and winter,[2] when the pole star would be visible at its upper transit during the hours of darkness.

"There are two other stars, however, which might be conveniently observed in conjunction with the pole star at its lower transit. These were δ (*delta*) Cassiopeiæ in the northern hemisphere, and Spica, or α (*alpha*) in the southern constellation Virgo. These stars would both be seen on the meridian almost precisely at the same time as the pole star; and by directing the transit instrument so as to observe all the three stars in the same vertical plane, a very good determination of the meridian line might be obtained.

"The constellation Cassiopeia could be easily recognized, as it was always visible in the northern hemisphere, being about the same distance from the pole star as Ursa Major in the opposite direction. The principal stars were five in number, arranged in a zigzag form (Fig. 209), and the star in question was the fourth in order, counting from east to west when the constellation was below the pole.

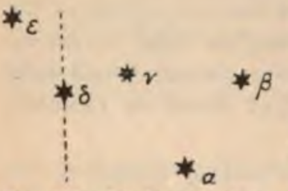

FIG. 209.—Finding north by observation of δ Cassiopeiæ.

"The star Spica would be at once recognized towards the south, at an elevation of about 25°.

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[1] Note by author. It appears to the author that this is a misprint, and should be 20°.

[2] See footnote [2] to p. 362, as to months for this observation. When the observation is made at the lower transit of ζ Ursæ Majoris the deviation from true north is 2 minutes 14 seconds east; at the upper transit the deviation is the same amount west.

There was no other very bright fixed star near it, but occasionally one of the brighter planets—Mars, Jupiter, or Saturn—would appear in this quarter of the heavens, and might possibly be observed by mistake. These stars, Spica and δ Cassiopeiæ, might be conveniently observed in conjunction with the pole star, during the earlier months of the year, from about January 15 to May 10.

"3. Another method was to observe the pole star six hours before or after the upper transit, when at its greatest distance east or west of the meridian. This observation might be made with considerable precision, as the star would then be apparently moving in a vertical direction. During the winter months it would be most convenient to observe the star six hours after the upper transit at its farthest distance to the west. The instrument should be fixed and adjusted somewhat before the time specified, and pointed to the star which would then appear, with an inverting telescope, to be moving slowly upwards, and diverging slightly to the right. The star should then be followed by means of the tangent screw, until it has reached its farthest point westward, and apparently to the right. If the telescope was now lowered to the horizontal position, it would point in a direction inclined at an angle of 2° 10′ west of the true meridian, and accordingly it was simply necessary to move the instrument in azimuth towards the east, through this small angle as shown by the verniers. The correction, 2° 10′, was calculated for the present year (1895), and for the latitude 54° 45′. For places somewhat north of Newcastle it might be stated as 2° 11′. Owing to the slow progressive increase in the star's declination, this angle would be slightly reduced in future years, the change being at the rate of about 1′ in two years. It was scarcely necessary to remark that if this star was observed in this way six hours before its upper transit, as it might be during the summer months, it would then be at its greatest distance east of the meridian, and the correction of 2° 10′ would have to be made towards the west.

"He (Mr. Beanlands) considered that each of the foregoing methods was suitable for the purpose required, and might be recommended for general adoption. They were all sufficiently accurate, and, with one exception, they required no special determination of the time. Moreover, they might be employed, one or other of them, almost at any period of the year."

By Observation of Stars in the Southern Hemisphere.—In the southern hemisphere, the stars α Crucis and β Hydri can be used for setting out a north and south line. When they are both in the same vertical line they are almost due south; if β Hydri is above the pole, then the line is 2 minutes[1] west of south, and when β Hydri is below, the line is 2 minutes east of south; the upper transit of β Hydri is 12 hours 32 seconds in advance of the upper transit of α Crucis. This observation can be made all the year except from about the middle of November to the end of January (the period depending on the latitude).

The writer is indebted to Professor Liveing, of the Yorkshire College, for the following statement:—

"There are several methods of finding the true meridian. One of the best for common purposes is to place the transit instrument or transit theodolite carefully levelled approximately in the meridian, and observe the meridian passage of some star near the zenith (that is, one with a north declination about equal to the latitude of the place). At the moment the star passes the central wire, set a watch, carefully regulated to gain 4 minutes per day, to the right ascension in hours and minutes of the star from the Nautical Almanack: your watch now shows approximate sidereal time. Note the next upper transit or meridian passage of the pole star, which should occur in the present year (June, 1901) at 1 hour 23 minutes sidereal time, or, for the lower transit, at 13 hours 23 minutes. If the passage does not occur at this time, shift the azimuth of your instrument to make it occur at this time, and the line of sight will be in the meridian. Repeat once or twice on different nights to obtain an average.

"The most exact method, however, is to observe the time-intervals between the upper and lower and lower and upper transits of the pole star. If these intervals are equal, the line of sight is in the meridian; if not, the line of sight lies to the side of the shorter interval. This method is employed for astronomical purposes, but needs a telescope of sufficient aperture to show Polaris in daylight. The clock employed need not be regulated or show correct time, but only needs a uniform rate."

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[1] The deviation given is correct for about latitude 50° south. If the latitude is smaller the error is smaller; thus for latitude 40° it is 1·6′, and for latitude 30° it is 1·4′.

Setting out North Line from Ordnance Map.—Where Ordnance maps are obtainable, the north-and-south line can be set out from them with sufficient accuracy for most purposes. It is necessary to set out a line of considerable length, say one mile or more (the longer the line the greater the accuracy). The sides of the map are all north-and-south, and any line parallel to the sides is also north-and-south, and such a parallel line can be ruled on the map at some place convenient for staking out a line and fixing permanent marks on some part of it; the position of the line on the map can then be measured from the fences and buildings and other marks, and set out on the ground by means of poles. If these are not all in an exactly straight line, the average must be taken, and if this is carefully done, great accuracy may be obtained.

Accurate tracings may be obtained from the Ordnance Survey Department, and so the errors due to the shrinkage of the paper on which the maps are usually printed is avoided.

Corelation of Various Plans.—It is usual in mining districts to make a separate survey of each particular leasehold or ownership, and of each particular mine; and if a number of these plans were put side by side, it might be impossible to place them in their true relative positions unless the boundary fences of neighbouring collieries happened to be included in each survey.

If, however, each plan has its latitude and longitude marked upon it, it can be transferred to an Ordnance map, and if all the plans were so transferred, they could be seen in their true relative positions to one another.

If the exact latitude and longitude of each mine shaft is marked on its own plan, then the latitude and longitude of any point underground or on the surface can be calculated, and the distance of any point on that plan from any point on one of the neighbouring plans can also be calculated, assuming that the latitude and longitude of the mine shaft is given on the neighbouring plan. In any country where there is a Government Survey corresponding to our Ordnance Survey, the latitude and longitude of any place on the map can be easily ascertained with accuracy corresponding to that of the map.

In England the Ordnance Survey is published, for many parts of the country, on three scales as follows: 1 inch to the mile, 6 inches to the mile, and 25·344 inches to the mile. If the latter scale is used, the position of a mine shaft may be marked upon it with great accuracy; but this map does not show the latitude and longitude, which must be ascertained by reference to the 6-inch map of the same district. The 6-inch quarter sheet covers an area which, on the scale of 25·344 inches, is covered by four maps. On the sides of the 6-inch quarter sheet the latitude is given in degrees, minutes, and half-minutes, and the longitude is given in degrees and minutes. By measuring from the top or bottom of one of the 6-inch maps, which corresponds with the top or bottom of a 25-inch map, the position of the degrees, minutes, and half-minutes of latitude can be transferred, by the use of a pair of dividers set to the required scales, to the 25-inch map; and by measuring from the side of the 6-inch map, which corresponds to the side of the 25-inch map, the degrees and minutes of longitude can be transferred to the 25-inch map. As many points of latitude and longitude as are on both the similar maps should be transferred to the 25-inch map, so that any inaccuracy in one measurement can be corrected. One minute of latitude is equal to approximately 6076 feet. A minute of longitude varies with the latitude, from 6086 feet at the equator to nothing at the poles, and, of course, covers a different length, going north or south on every map (and it can easily be measured on the map). The length covered by 1 minute of longitude varies approximately with the cosine of the angle of latitude. Thus at a latitude of 0° one minute of longitude is 6086 feet, then the length at a latitude of, say, 55° is equal (approximately) to $6086 \times \text{cosine } 55^\circ = 6086 \times 0{\cdot}5735764 = 3490{\cdot}7$ feet. On referring to the Smithsonian Geographical Tables (published by the Smithsonian Institute, Washington), it will be found that the length of 1° of longitude in latitude 55° is given as 39·766 miles, which makes 1 minute of longitude = 3499·4 feet.

In a similar manner the length of 1 minute of longitude can be got approximately for any latitude.

On the 25-inch map it is quite possible to scale a distance with an error not exceeding 2 feet, assuming the perfect accuracy of the map. It would be therefore possible to ascertain the latitude to say $\frac{1}{3000}$ of a minute, or $\frac{1}{50}$ of a second. With mining maps thus referred, with care, to the latitude and longitude, it would be possible to calculate, with considerable accuracy, the distance from each other of places in different mines.

APPENDIX

EXAMINATION QUESTIONS—VARIOUS.

1. How many tons of coal are there in an estate of 672 acres containing one seam 3 feet 7 inches thick, assuming the specific gravity of the coal to be 1·27, and the weight of a cubic foot of water 62·5 lbs.? (Colliery Managers, Newcastle, 1900.)

Answer. 3,716,893 tons.

2. In consulting old plans, what source of error must especially be guarded against? (Colliery Managers, Newcastle, 1900.)

3. A road driven east in a seam rising 2 inches per yard cuts a trouble, an upthrow of 6 fathoms with a vertical hade, beyond which the seam rises to the east 3 inches to the yard. It is desired to connect the two portions of the seam, one on each side of the trouble, by means of a stone drift rising 6 inches per yard. At what distance from the trouble, measured in the seam on the west side, must this drift be set away in order that it may cut the seam on the east side at a distance of 40 yards from the trouble measured in the seam? (Colliery Managers, Newcastle, 1900.)

Answer. 79 yards (nearly).

4. What is the diameter of a circular shaft having the same area as an oblong shaft 12 feet 6 inches long by 9 feet 6 inches wide? (Colliery Managers, North Staffordshire, 1900.)

Answer. 12·3 feet.

5. Give the value of 3 acres 2 roods 17 perches of coal, 4 feet 5¼ inches in thickness, at £25 per foot per acre. (Colliery Managers, North Staffordshire, 1900.)

Answer. £400 1s. 4½d.

6. How would you connect an underground and surface survey? (Colliery Managers, North Staffordshire, 1900.)

7. How would you survey a field without the aid of any instrument for measuring angles? (Colliery Managers, Newcastle, 1899.)

8. Describe the vernier; make a sketch of one, and describe what it is used for. (Colliery Managers, Newcastle, 1899.)

9. Plot the following survey, and give the direction and distance of the last set (No. 10) so as to tie into the starting-point:—

Bord.	164.
Bord.	107.
Bord.	53.
.	N. 88° E.

	(9)	
Headways.	148.	
	S. 8° W.	
	(8)	
Bord.	58.	
	N. 82° W.	
	(7)	
Headways.	162.	Headways.
	S. 6° W.	
	(6)	
Bord.	54.	
	N. 88° W.	
	(5)	
	Bord.	
Headways.	154.	Headways.
	S. 3° W.	
	(4)	
Bord.	200.	
Bord.	144.	Bord.
Bord.	93.	Bord.
Back bord.	40.	Back bord.
	N. 87½° W.	
	in (2).	
	From 297.	
	(3)	
	going Bord.	
	Face of	
	339.	
Headways.	297.	
Headways.	140.	
	N. 3° W.	
	(2)	
Headways.	152.	
	N. 2½° E.	
	(1)	

Second west 287 from engine plane.
Survey from mark in Mothergate bord.

(Colliery Managers, Newcastle, 1899.)

10. The *débris* from the sinking of two shafts, each 500 yards deep, 16 feet diameter inside the brickwork of 9 inches thick, has to be deposited on an area of 4 statute acres. What will be the average thickness? (Colliery Managers, Lancashire, 1898.)

Answer. 4·14 feet (assuming that the débris will occupy the same volume as when in the solid).

11. A colliery reservoir is 100 feet long, and 60 feet wide at the bottom

10 feet in perpendicular height to the surface of the water when full; the sides are at an angle of 45°. How many gallons of water will it contain when filled? (Colliery Managers, Lancashire, 1898.)

Answer. 483,112·5 gallons.

12. Describe a surveying compass, Gunter's chain, protractor, and drawing-scales; and state what they are used for. (Colliery Managers, Newcastle, 1898.)

13. What method would you adopt for ensuring that a drift below the ground was driven in a given direction, and at a given gradient? (Colliery Managers, Newcastle, 1898.)

14. Plot the following survey :—

No. 1.	...	N. $86\frac{3}{4}$° W.	...	474 links
,, 2.	...	N. $44\frac{1}{4}$° W.	...	163 ,,
,, 3.	...	N. $11\frac{1}{2}$° E.	...	322 ,,
,, 4.	...	N. $83\frac{3}{4}$° E.	...	291 ,,
,, 5.	...	S. $7\frac{1}{2}$° E.	...	515 ,,
,, 6.	...	S. $3\frac{1}{2}$° W.	...	171 ,,
,, 7.	...	S. $86\frac{3}{4}$° E.	...	169 ,,

,, 8. Give bearing and distance to tie the survey.

(Colliery Managers, Newcastle, 1898.)

Answer. N. 2° 17′ 27″ E.; length, 200·15 links.

15. Why do you need to make allowances in the measurements of lengths in steep mines? (Colliery Managers, Liverpool District, 1898.)

16. State briefly the requirements of the Coal Mines Regulation Acts, 1887 and 1896, with regard to plans. (Colliery Managers, Liverpool District, 1898.)

17. Describe and give sketch of how you would make a section of surface between two points, A and B, 1000 yards apart, with undulating ground between. (Colliery Managers, Liverpool District, 1898.)

18. What are the provisions of the Mines Regulation Act with regard to plans of workings? (Colliery Managers, South-Western District, 1898.)

19. A level course of road extends 7 chains from the centre of a pit; the direction of the road is 64° 20′ east of north. At 575 links from the pit is a branch which extends 850 links in the direction of 25° 40′ west of north. Plot the two drives to a scale of 100 links to an inch. (Colliery Managers, South-Western District, 1898.)

20. On a plan drawn to a scale of 4 chains to an inch, how many perches are represented by a circle of 1 inch diameter? (Colliery Managers, South-Western District, 1898.)

Answer. 201·06 perches (square measure).

21. How would you test the adjustment of a theodolite? (Colliery Managers, South-Western District, 1898.)

22. The workings of two collieries are separated by a barrier of coal 400 feet wide. The barrier extends on the line of dip. It is necessary to drive on the level course 200 feet into the barrier from each colliery, so that the drives shall meet at the middle of the barrier, and shall be 50 feet vertically above the downcast pit-bottom of one of the collieries. How would you determine the correct starting-points for both drives? (Colliery Managers, South-Western District, 1898.)

23. A seam of coal and ironstone lies at an angle of 45°. The stone is 3 feet thick, and the coal is 2 feet 8 inches thick, measured at right angles to the dip. The royalty on the coal is £25 per acre per foot thick, measured vertically; the

royalty on the ironstone is 6*d.* per ton, calcined. What is the royalty value of one surface acre of coal? and what is the value of one surface acre of stone, supposing the yield to be 1800 calcined tons per acre per foot thick, measured vertically? (Colliery Managers, North Staffordshire District, 1898.)

Answer. Coal, £94 5s. per acre; ironstone, £190 18s. per acre.

24. State briefly what precautions you would take in making (1) a loose-needle survey where the conditions are favourable; (2) a fast-needle survey with outside vernier dial under favourable conditions; (3) in taking a meridian on the surface. (Colliery Managers, North Staffordshire District, 1898.)

25. The base-line *ab*, 1000 feet long, is measured along a straight bank of a river; *c* is an object on the opposite bank; the angles *bac* and *cba* are observed to be 65° 37′ and 53° 4′ respectively. What is the breadth of the river at *c*?

Answer. 829·87 feet.

26. *a* and *b* are two positions on opposite sides of a mountain; *c* is a point visible from *a* and *b*; *ac* and *bc* are 10 miles and 8 miles respectively; and the angle *bca* is 60°. What is the distance between *a* and *b*?

Answer. 9·165 miles.

27. The sides of a triangular field are 1250 feet, 790 feet, and 585 feet. What is its area in acres, roods, and perches?

Answer. 4 acres 0 roods 8 perches.

28. Find the sixth root of 16,777,290. (City and Guilds of London Institute. Mine Surveying, 1891.)

Answer. 16·00001.

29. Find the cube of 649. (City and Guilds of London Institute, Mine Surveying, 1891.)

Answer. 273,359,449.

30. Find the value of the seventh root of 78,125. (City and Guilds of London Institute, Mine Surveying, 1891.)

Answer. 5.

31. Find the angle of which the logarithmic sine is 9·7382412. (City and Guilds of London Institute, Mine Surveying, 1891.)

Answer. 33° 11′.

32. Find the radius of an arc of which the angle is 28° 26′, and of which the logarithm of the natural sine is 2·1122998. (City and Guilds of London Institute, Mine Surveying, 1891.)

Answer. 272.

33. *abc* is a triangular plot of ground, of which the side *ab* measures 1200 links; the angle at *a* equals 39°; the angle at *b* equals 68°. Find the area in acres, roods, and perches. (City and Guilds of London Institute, Mine Surveying, 1891.)

Answer. 439,312 square feet, or 10 acres 0 roods 13·6 perches.

34. Under the plot of ground in Question 33 is a seam of coal dipping at an angle of 15°. The thickness of the seam, measured at right angles to the dip, is 7 feet 3 inches. A cubic foot of this coal weighs 80 lbs. The royalty is £200 per acre of surface. Of the total area, 5 per cent. is occupied by faults. Of the remaining coal, 10 per cent. is lost in working. Find the tonnage of coal to be sent out of pit, and the royalty per ton in pence to two places of decimals. What is the specific gravity of the coal? (City and Guilds of London Institute, Mine Surveying, 1891.)

Answer. Specific gravity, 1·28; 100,098 tons; £2017 royalty = 4·83d. per ton.

CITY AND GUILDS OF LONDON INSTITUTE.

The City and Guilds of London Institute holds Annual Examinations in Mine Surveying. There are two grades, Ordinary and Honours. There is also a Preliminary Examination, which it is necessary to pass before becoming a candidate in the Ordinary Grade. Candidates for Honours also must have previously passed in the Ordinary Grade.

The programme of Examinations contains information as to the subjects included in the syllabus.[1] The fee for the Ordinary Grade Examination is 1*s.*; and both the Preliminary and Ordinary Examinations are held about the 1st of May of each year. The Honours Examination is a two-days' Examination, and the fee is 10*s.* It is held at any centre at which a sufficient number of candidates undertake to attend, and is of a practical nature, candidates having to make actual surveys in the mine.

The Examination Papers in the Preliminary and Ordinary Examinations for 1900 and 1901 are given as a guide to intending candidates.

MINE SURVEYING.

PRELIMINARY EXAMINATION.

MONDAY, APRIL 30TH, 1900, 7 TO 10 P.M.

Drawing instruments and mathematical tables may be used.

Not more than seven questions to be answered.

1. The three sides of a triangle measure 370, 295, and 466 yards respectively. Draw the triangle to a scale of 100 feet to the inch, and calculate its area in acres, etc. (50 marks)

Answer. 11·27 acres.

2. An embankment is 30 chains long; the top is 10 feet wide; one side has a slope of 55° and the other of 50° to the vertical; the ground and top of the embankment are level, and the embankment is 13 feet high at the centre. Calculate its contents in cubic yards. (50)

Answer. 25767·8 cubic yards.

3. Draw a scale of $\frac{1}{27}$ to show feet, and long enough to measure 20 feet. (30)

4. Plot the following traverse lines, all starting from one central point; scale, 25 feet to the inch:—

No. 1.	...	N. 77° E.	...	64 feet	
„ 2.	...	N. 21° 30′ W.	...	1 chain 12 links	
„ 3.	...	N. 15° 15′ E.	...	15 yards	
„ 4.	...	S. 29° 20′ W.	...	187 links	
„ 5.	...	S. 56° 45′ E.	...	14 fathoms	(30)

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[1] It can be obtained from Messrs. Whittaker & Co., Paternoster Square, price 1*s.* 4*d.* post free.

5. How many tons of coal per acre will there be in a seam 3 feet 9 inches thick, dipping at an angle of 9°, allowing 20 per cent. deduction for faults, etc.? (30)

Answer. (Coal taken as 80 lbs. to the cubic foot) 4725·8 tons.

6. In an ordinary miner's dial the E mark is to the left of the N. Why is this? (30)

7. Explain the terms "diurnal variation," "dip," "declination," and "secular variation" of the magnetic needle. (30)

8. You have to measure the width of a deep river about 150 yards wide, and your only measuring instrument is an ordinary chain. How would you proceed? (30)

9. A vertical shaft is 400 feet deep. Halfway down it an incline starts from it, which meets a drift dipping towards the shaft-bottom at a grade of 2 inches to the yard at a distance of 4 chains from the shaft, this distance being measured along the floor of the drift. Find the length and inclination (in degrees and minutes) of the incline. (50)

Answer. Length, 322 feet; angle of inclination (from the vertical), 54° 53′.

10. A theodolite is set up in line with two telegraph-poles, 150 feet from the nearer pole, and 420 feet from the further pole. The top of the further pole subtends an angle of 18°, the line of sight passing through a hole exactly halfway up the nearer pole. Required the heights of the two poles, the theodolite standing 5 feet above the ground. (50)

Answer. First pole, 107·47 feet; second pole, 141·4 feet.

11. A seam of mineral dipping 12° is thrown down 200 feet by a vertical fault; an inclined drift is started from the top of the downthrow, and cuts the seam 400 feet horizontally from the fault. Required the length and dip of the inclined drift. (40)

Answer. Length, 491·16 feet; angle of dip from vertical, 54° 32′.

ORDINARY GRADE.

THURSDAY, MAY 3RD, 1900, 7 TO 10 P.M.

INSTRUCTIONS.

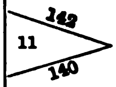
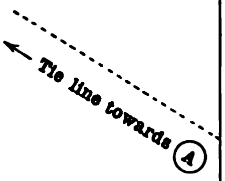
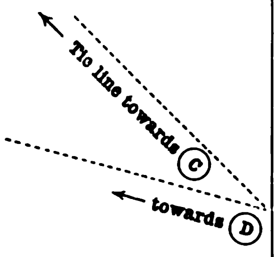
A sheet of drawing-paper is supplied to each candidate.

Candidates may use protractor, parallel ruler, T-square, set-squares, scales, compasses to span 16 inches, drawing instruments, tables of logarithms, logarithmic and natural sines, tangents, etc.

[The working out of all answers must be shown.]

Question 1 must be attempted by all candidates, and not more than five others in addition. The maximum number of marks obtainable is affixed to each question.

1. Plot the following chain survey of a field to a scale of 2 inches = 1 chain. All dimensions are in links; all offsets are to the boundary:—

	Ⓒ	
	399	
	Ⓐ	
Tie Line		**going about N.W.**
	Ⓐ	
0	506	
0	442	
17	401	
0	385	0
	348	44
	297	24
0	261	0
26	221	
0	199	0
	175	36
	93	73
	Ⓓ	0
Line 4		**going about E.S.E.**
	Ⓓ	
	277	0
	261	10
	227	34
	196	
	192	11
	59	
	42	78
	Ⓒ	0
Line 3		**going about S.W.**
	Ⓒ	
0	348	
18	298	
0	268	0
	192	36
0	106	0
13	86	
0	77	0
	59	12
	Ⓑ	0
Line 2		**going about W.N.W.**
	Ⓑ	
	221	0
	200	11
	151	25
	61	7
	42	11
	Ⓐ	0
Line 1		**going about N.N.E.**

2. Calculate the area of the field (survey of which is given in Question 1) in acres, etc. (45)

Answer. 1 acre 0 roods 36 perches.

3. Calculate the co-ordinates of the following traverse survey; calculate the length and bearing of the line GA, and plot by co-ordinates to a scale of 1 chain to the inch:—

> Traverse survey of polygonal area made by double foresight method[1] with a right-handed theodolite reading to 30 seconds; the theodolite was originally set in the true meridian, true north reading 360° 00′ 00″.

Line.	Observed angle.	Length in links.
AB	14° 48′ 00″	245
BC	198° 06′ 30″	310
CD	284° 01′ 30″	480
DE	200° 12′ 30″	709
EF	271° 33′ 30″	430
FG	268° 01′ 30″	607
GA		

(60)

Answer. Bearing of GA is N. 61° 25′ 43″ W.; length, 220·6 links.

4. Calculate the area of the above polygon in acres, etc., by the method of co-ordinates. (60)

Answer. 4 acres 2 roods 18 perches.

5. A bed of mineral dips 58° (to the horizontal), the direction of full dip being S. 24° 56′ E. What will be the dip of a road running N. 80° 20′ W.? (45)

Answer. Angle of dip, 42° 15′ 42″, or 1 in 1·100473.

6. Two horizontal levels are driven in a vein dipping 77° towards N. 56° E., the levels being 200 feet apart vertically. A flat winze in the vein connecting the levels is 446 feet long. What is its dip and bearing? (45)

Answer. Bearing, S. 40° 39′ 5″ E.; angle of dip, 26° 38′ 34″.

7. How would you proceed to level along an inclined drift about 3 feet 6 inches high, and inclined about 40°? (30)

8. Draw a section of the telescope used in the ordinary dumpy level, showing clearly the path of the rays of light through it. (30)

9. Under what circumstances must a correction for the earth's curvature be applied in levelling? State a formula for this correction. (30)

10. Sketch and explain the action of the tangent screw and clamp, as applied to any part of a theodolite. (30)

11. Describe a method of connecting underground and surface traverses through a single shaft, the use of the magnetic needle being inadmissible. (30)

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[1] Note by author.—By the "double fore sight method" is meant taking the exterior angle between each sight and the next. Thus at A the theodolite is at a place where there is no attraction, therefore the bearing of AB is N. 14° 48′ E. The theodolite is then moved forward to B, and the angle between AB and BC is observed to be 198° 6′ 30″.

To get the meridian bearings of BC and the following sights, the following rule is used: "Add the observed theodolite reading to the last meridian bearing, and subtract 180° from, or add 180° to, the sum, according as the sum is greater or less than 180°."

PRELIMINARY EXAMINATION.

MONDAY, APRIL 29TH, 1901. 7 TO 10 P.M.

INSTRUCTIONS.

No certificates will be given to candidates on the results of this Preliminary Examination, but their successes will be notified.

The number of the question must be placed before the answer in the worked paper.

Drawing instruments and mathematical tables may be used.

Not more than seven questions to be answered.

Three hours allowed for this Examination.

The maximum number of marks obtainable is affixed to each question.

1. If a plan is drawn to the scale of 2 inches to the chain, what is the proportion between the actual area in the field and the area as shown on the plan? 30

Answer. As 156,816 : 1.

2. Draw a scale of 1½ fathom to the inch, long enough to measure 1 chain, and a corresponding scale of metres to read to decimetres. 30

3. Draw the plan of the following field to a scale of 2 chains to the inch, and calculate its area. The measurements are given in links:—

	Ⓑ	
	2,165	
815	1,787	
	1,463	336
719	1,100	
217	987	
	654	508
415	219	
	Ⓐ	

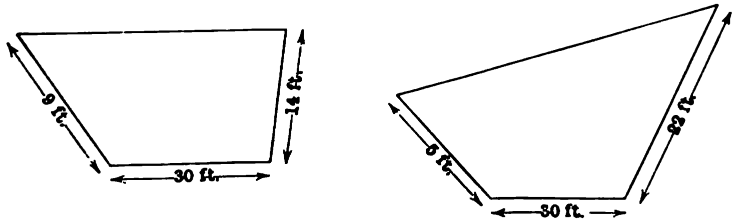
45

Answer. 16 acres 1 rood 37 perches.

4. A right-angled triangle has a base 27 yards long, and the angle between the base and the hypothenuse is 27° 19′. Find its area in square feet. (40)

Answer. 1694·4 square feet.

5. Determine the volume of a railway cutting 3 chains long, the end sections being as given below, the ground sloping uniformly, and the slopes of the sides of the cutting being 1 in 2:—



(NOTE.—The drawings are not to scale.) 45

Answer. 42,566 cubic feet.

6. How can you set out a right angle by means of a chain alone? (30)

7. From two points, A and B, 1500 feet apart, the bearings of a point, C, are found to be respectively N. 67° E. and N. 4° E. B bears S.E.—exactly from A. Required, the lengths AC and BC. (45)

Answer. AC = 1270·54 feet; BC = 1560·90 feet.

8. A vein of mineral, of specific gravity 3·7, is 4 feet 8 inches thick, and dips 70° to the horizontal. A drift along the vein, the full width of the vein, is 6 feet 3 inches high vertically, and 110 yards long. How many tons of mineral will it yield? (40)

Answer. 1056 tons.

9. Write a brief description of the plain miner's dial. (30)

10. A drift rising 1 in 27 cuts a seam of coal dipping 49°, the dip of the seam and of the drift being in opposite directions. The width of the seam, as measured along the floor of the drift, is 12 feet. What is the true width of the seam? (40)

Answer. 9·4 feet.

11. A shaft is sunk 20 feet in diameter and 200 yards deep; assuming the rock to occupy a volume 30 per cent. greater after excavation, and that the excavated material is piled in the form of a square pyramid, the sides of which are inclined 40° to the horizontal, calculate the area of the base of the pyramid. (45)

Answer. Area of base, 14,534·7 square feet.

ORDINARY GRADE.

TUESDAY, APRIL 30TH, 1901, 7 TO 10 P.M.

INSTRUCTIONS.

Candidates for the Ordinary Grade must have previously passed the Preliminary Examination.

If the candidate has already passed in this subject in the first class of the Ordinary Grade, he cannot be re-examined in the same grade.

The number of the question must be placed before the answer in the worked paper.

A sheet of drawing-paper is supplied to each candidate.

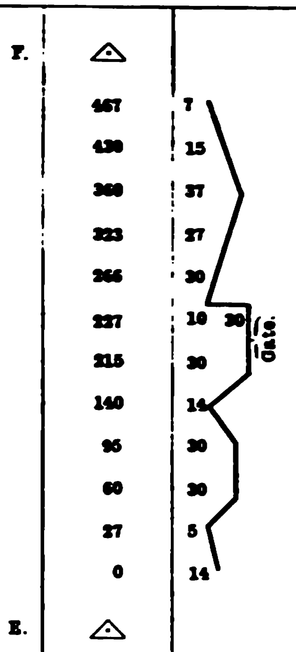
Candidates may use protractor, parallel ruler, T-square, set-squares, scales, compasses to span 16 inches, drawing instruments, tables of logarithms, logarithmic and natural sines, tangents, etc.

Three hours allowed for this paper.

[The working out of all answers must be shown.]

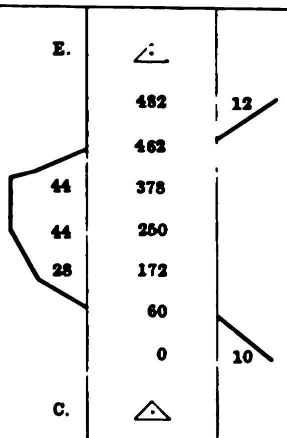
Question 1 must be attempted by all candidates, and not more than four others in addition. The maximum number of marks obtainable is affixed to each question.

Line No. 9.



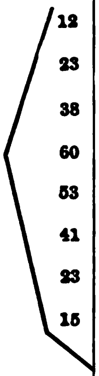
From E. go about W.S.W. to F.

Line No. 8.



From C. go about N.W. to E.

	△	C.
Line No. 7. Tie Line.	817	
	△	F.
From F.		go about S.E. to C.

		△	F
	12	423	
	23	377	
	38	345	
	60	267	
Line No. 6.	53	190	
	41	155	
	23	82	
	15	36	
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		△	B.
From B.			go about N.N.W. to F.

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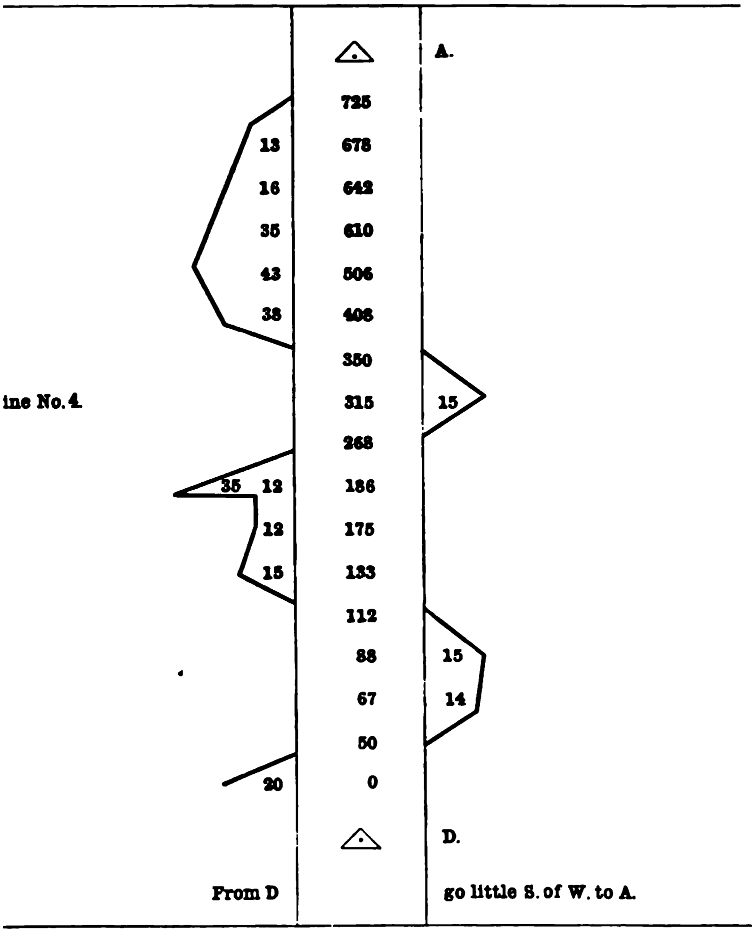
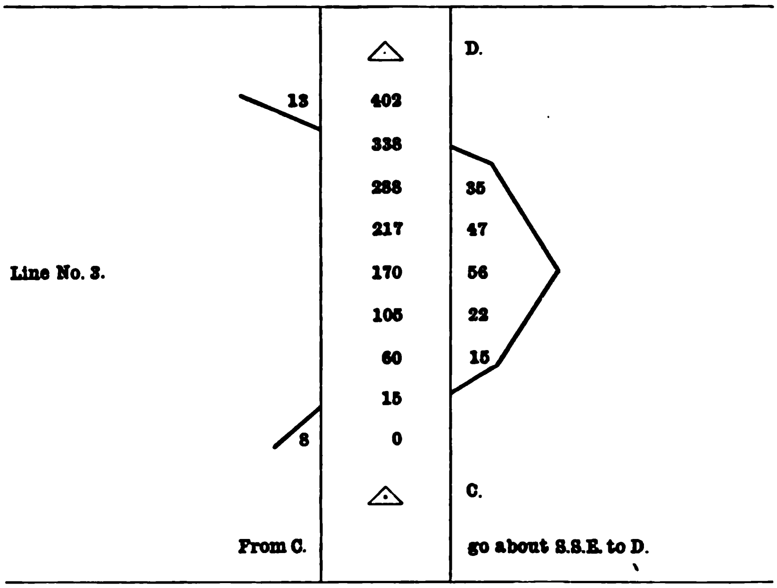
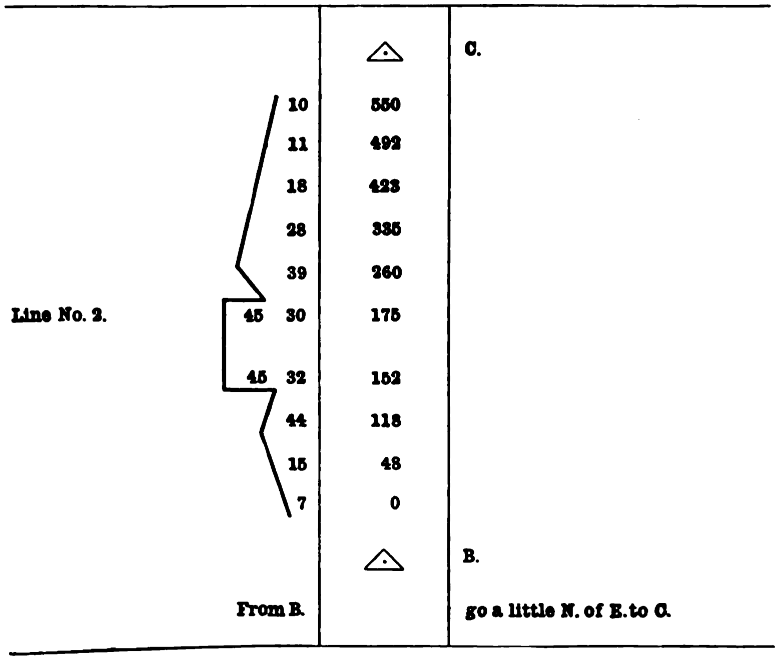
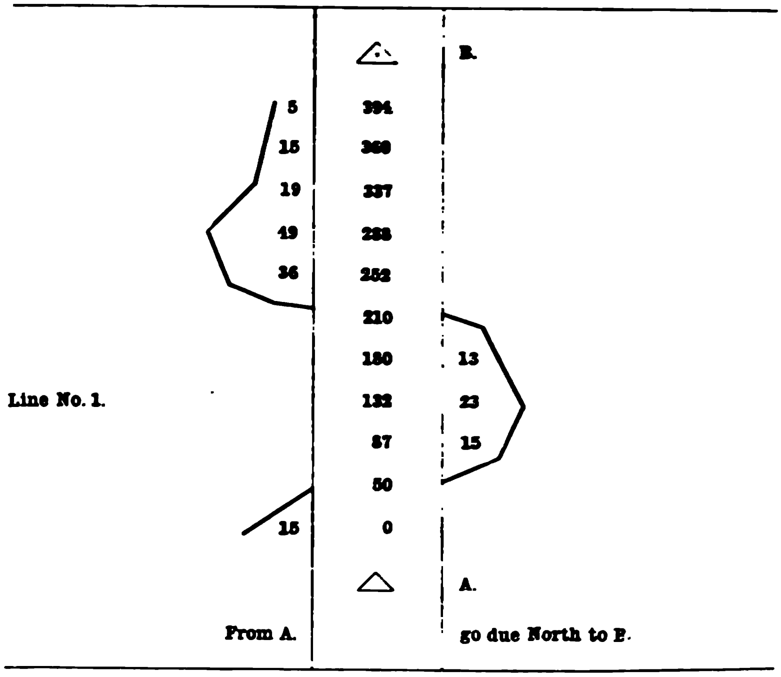
Line No. 5. Tie Line.	△	A.
From A.		go about N.E. to C.


A.
725
13 678
16 642
35 610
43 506
38 408
350
ine No.4
315 15
268
35 12 186
12 175
15 133
112
88 15
67 14
50
20 0
D.
From D go little S. of W. to A.


D.
Line No. 3.
C.
From C.
go about S.S.E. to D.


C.
Line No. 2.
B.
From B.
go a little N. of E. to C.



1. Plot the preceding survey to a scale of 1 chain to the inch. All measurements are in links. (100)

2. Calculate the area of the more southerly of the two fields in Question No. 1. (40)

Answer. 2 acres 2 roods 1 perch.

3. Make out an imaginary page from a level-book, showing twelve readings taken with four settings of the instrument over undulating ground. Work out and plot the section. (50)

4. A mineral seam dips 70°, outcropping in ground sloping 17° to the horizon; a shaft is to be started downhill from the outcrop so as to cut the seam at a depth of 400 feet below the shaft collar. How far horizontally must the shaft be from the outcrop? (50)

Answer. 163·81 feet.

5. Three bore-holes, A, B, and C, intersect a seam of coal; they are situated at the angles of an equilateral triangle whose sides are 300 yards in length. B is N. 17° 20′ E. of A, and C is to the westward of the line AB. A cuts the seam at a depth of 175 feet, B at a depth of 342 feet, and C at a depth of 240 feet. Determine the direction and amount of dip of the coal seam. (50)

Answer. Dip 1 in 5·3 (nearly); N. 24° 20′ E.

6. Describe in detail how you would set out underground a curve of 10 chains radius to connect a main travelling road with a branch road, the directions of the

two roads making an angle of 60° with each other. Draw a plan to a scale of 50 links to the inch. (50)

7. Describe the German miner's compass, and the method of using it. (30)

8. What is a plane table, and how is it used? (30)

9. Explain the principle of the vernier. (30)

SURVEYORS' INSTITUTION EXAMINATION PAPERS.

The Surveyors' Institution, Westminster, holds Annual Preliminary and Professional Examinations, which it is necessary to pass before being able to subscribe one's self as a Fellow of the Surveyors' Institution (F.S.I.). The Examinations include a great number of subjects, but the papers in Land Surveying, and Levelling, and Mensuration only are given here.

SURVEYING AND LEVELLING.

Morning Paper.

Time allowed, three hours.

Note.—*All candidates are required to attempt Questions Nos.* 1, 2, *and* 3.

Candidates other than Building candidates will receive full marks for any 10 *questions correctly answered.*

Building candidates will receive full marks for any 8 *questions correctly answered.*

Candidates omitting to leave figures by which results are arrived at will risk a loss of marks in case of a wrong answer being given through accident.

Questions 1, 2, 3, 6, 7, *and* 9 *carry higher marks than the remainder.*

1. On the plan given (see p. 387) draw in pencil the lines it would be necessary to run to enable you to make a complete survey with the chain only.

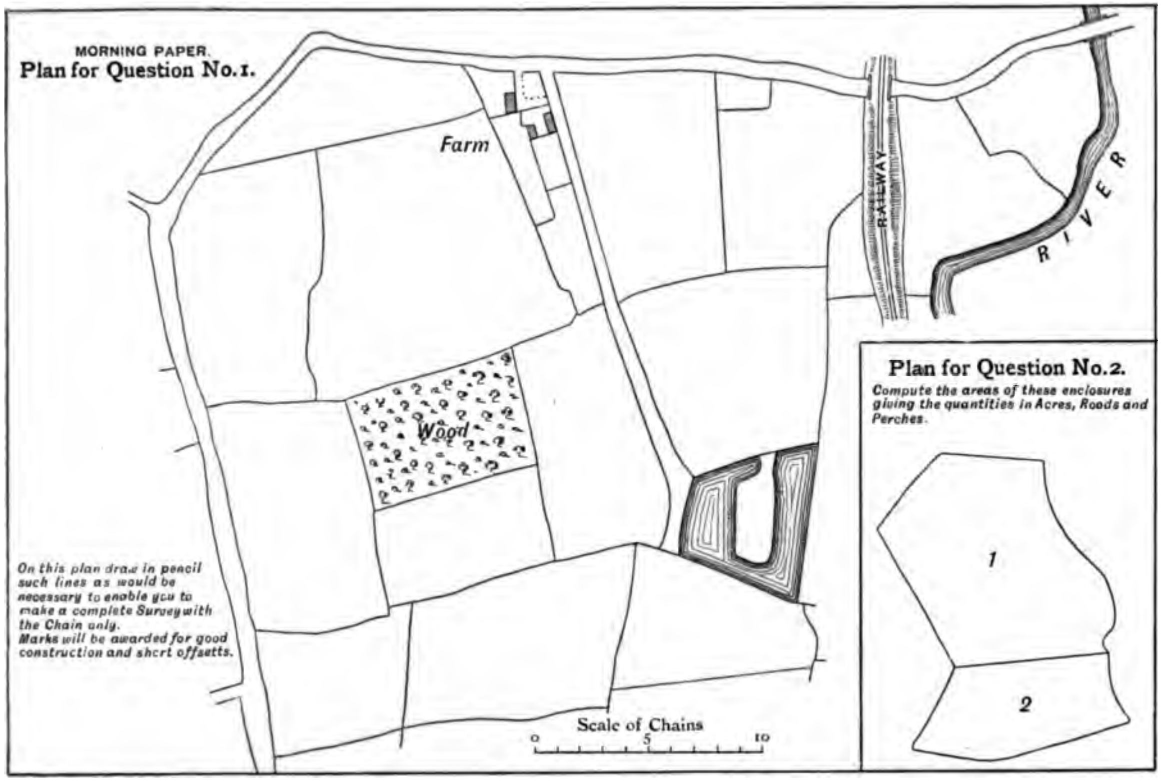
2. Compute the areas of the enclosures in the corner of the plan above mentioned, giving the results in acres, roods, and perches. One of these enclosures must be computed by means of the ordinary plotting scale, and the other in any way the candidate may elect. (Enclosure No. 1, if well done and a correct answer arrived at by the ordinary plotting scale, will carry full marks.)

Answer. Enclosure No. 1, 6 acres 3 roods 7 perches; No. 2, 3 acres 0 roods 16 perches.

3. From the field notes given lay down the survey lines, and plot a plan to a scale of 2 chains to an inch.

4. Required to set out a circular space for a reservoir to contain 1 acre 1 rood and 20 perches. Give the radius in links.

Answer. 209·2 links.



MORNING PAPER.

Plan for Question No. 1.

On this plan draw in pencil such lines as would be necessary to enable you to make a complete Survey with the Chain only.
Marks will be awarded for good construction and short offsetts.

Plan for Question No. 2.

Compute the areas of these enclosures giving the quantities in Acres, Roods and Perches.

Plan referred to in Question 1, p 386.

5. Divide the triangle ABC into three equal portions by lines parallel to the side AB. AB = 2500 links; AC = 2100 links; and BC = 1800 links. Give the area of ABC, and the distances A*a*, *ab*, and *b*C.

Answer. Area ABC = 185·73 square chains; Aa = 3·854 chains, ab = 5·022 chains, bc = 12·124 chains.

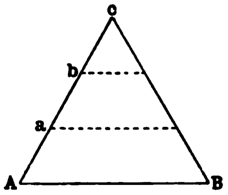


Figure referred to in Question 5.

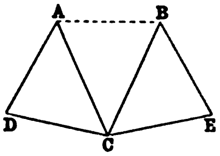


Figure referred to in Question 6.

6. The points A and B are only both visible from one point, C. Lines CD = 1260 links, and CE = 1040 links, were run, and the following angles were taken, viz.: ADC = 67° 30′, ACD = 45° 0′, ACB = 70° 20′, BCE = 39° 10′, and BEC = 81° 50′. Find the length AB in links.

Answer. 1418 links.

7. Plot the above figure to a scale of 1 chain to an inch, and give the distance AB as it measures upon your plan.

8. A traverse round a wood is as follows:—

A to B =	290 links,	bearing	255° 5′	A . . B
B to C =	1000 „	„	194° 10′	
C to D =	680 „	„	77° 12′	D . . C

Give the calculated distance D to A.

Answer. 905·1 links.

9. Protract and plot the above to a scale of 1 chain to an inch.

10. Convert 17 acres 1 rood and 20 perches, statute measure, into square yards.

Answer. 84,095 square yards.

11. How would you determine the latitude of any position (on land)? and what instrument would you require?

12. Illustrate and describe in what way you would produce a survey line obstructed by a large tree or building.

13. If a plan is plotted to a scale of 3 chains to an inch, what proportion does the area of the plan bear to the ground?

Answer. 1 : 5,645,376.

AFTERNOON PAPER.

Time allowed, two hours and a half.

NOTE.—*All candidates are required to attempt Questions Nos.* 1 *and* 2.

Candidates other than Building candidates will receive full marks for any 9 *questions correctly answered.*

Building candidates will receive full marks for any 7 *questions correctly answered.*

Candidates omitting to leave figures by which results are arrived at will risk a loss of marks in case of a wrong answer being given through accident.

Questions 1, 2, 8, 9, *and* 11 *carry higher marks than the others.*

1. Make up the following level-book :—

Level-book for Question No. 1.

Back sight.	Intermediate.	Fore sight.	Rise.	Fall.	Reduced levels.	Distance.	Remarks.
					Feet.	Chains.	
6·60					45·80	0	
	4·00		*2·60*		*48·40*	1·00	
	5·70			*1·70*	*46·70*	2·00	
0·80		12·20		*6·50*	*40·20*	3·00	
	6·90			*6·10*	*34·10*	4·00	
	11·20			*4·30*	*29·80*	5·00	
0·24		13·12		*1·92*	*27·88*	6·00	
	4·80			*4·56*	*23·32*	7·00	
	8·30			*3·50*	*19·82*	8·00	
1·10		13·75		*5·45*	*14·37*	9·00	
	6·70			*5·60*	*8·77*	10·00	
	5·70		*1·00*		*9·77*	11·00	
	8·10			*2·40*	*7·37*	12·00	
2·90		15·05		*6·95*	*0·42*	13·00	1st side of pond water-level
	7·10			*4·20*	*−3·78*	14·00	
	10·60			*3·50*	*−7·28*	14·30	
	11·70			*1·10*	*−8·38*	15·00	
	10·80		*0·90*		*−7·48*	16·00	
	7·10		*3·70*		*−3·78*	16·40	2nd side of pond water-level
13·75		6·85	*0·25*		*−3·53*	17·00	
	11·10		*2·65*		*−0·88*	18·00	
	8·60		*2·50*		*1·62*	19·00	
	2·30		*6·30*		*7·92*	20·00	
		0·85	*1·45*		*9·37*	21·00	
25·39		*61·82*	*21·35*	*57·78*			
		25·39		*21·35*			
		36·43		*36·43*			

(*The figures in italics are those required in answering the question.*)

2. Plot the following section to a horizontal scale of 2 chains to an inch, and to a vertical scale of 20 feet to an inch :—

Distances. Chains.	Height above base. Feet.
0·00	30·00
1·00	31·30
2·00	33·40
3·50	27·85
5·00	27·01
6·20	30·47
7·00	29·43
9·00	26·30
10·00	23·20
11·20	25·20
12·00	29·30
14·00	31·25
15·00	33·31
15·60	34·16
17·50	32·27
18·00	30·25
18·50	29·00

3. In setting out the centre line for a new road or a railway, illustrate and describe in what way you would proceed to connect two pieces of straight by a curve of, say, 10 chains radius.

4. Before commencing to take a series of levels, briefly describe how you would ascertain if your level was in adjustment.

5. The point A being inaccessible and at a considerable altitude above the surrounding country, illustrate and describe in what way you would ascertain its height above the point B (the nearest convenient point of observation), using a theodolite for the purpose.

Distance. Chains.	Height. Feet.
0·00	41·00
2·40	42·16
3·00	42·50
6 30	44·20
8·00	45·70

6. Give the rates of inclination between the given points of level taken upon a line chained along the invert of a water-course.

Answer. (1) 1 in 136·5; (2) 1 in 116·5 (nearly); (3) 1 in 128; (4) 1 in 74·8.

7. What is the rate per chain (in feet and decimals) of a gradient rising 1 in 250?

Answer. 0·264 feet per chain.

8. Give the levels of points B, C, and D on a continuous section, the level of point A being 25 feet, and the horizontal distances and angles as follows :—

A to B, 12 chains; angle of elevation, 3° 20′
B to C, 9 „ „ depression, 4° 25′
C to D, 15 „ „ elevation, 2° 15′

Answer. Level of B, 71·128 feet; level of C, 25·249 feet; level of D, 64·146 feet.

9. The telescope of a theodolite set 4·25 feet above the point A, having a level value of 25 feet, is directed towards the bottom of a staff at B, and shows an angle of elevation of 10° 4′; it is then directed to 10 feet on the staff, when it shows an angle of elevation of 10° 35′. Required the horizontal distance A to B in feet, and also the level of point B.

Answer. A to B = 1073·31 feet; level of B, 219·79 feet.

10. Illustrate by diagram the difference between "true" and "apparent" level, and give a rule for determining same.

11. Construct a triangle ABC, having its sides AB = 3 inches, BC = 2½ inches, and AC = 1½ inch. Suppose the points A, B, and C to be trigonometrical stations of a survey, and that from a point D of a traverse A bears 120°, B, 150°, and C, 165°. Find the point D by construction.

12. Explain and illustrate by diagram how you would obtain the distance to an inaccessible point, using only chain and poles.

MENSURATION.

Time allowed, two hours.

1. How many rods of brickwork are there in a circular pier 4 feet in diameter and 20 feet in height? (One rod of brickwork is equal to 306·2812 cubic feet.)

Answer. 0·82 rod.

2. A circular water-tank is 12 feet internal diameter, and is 10 feet deep. A drawing of it was made to a scale of ½ inch to a foot. Some one carelessly scaled it with a scale of ⅜ inch to a foot. What error would be made in calculating the number of gallons contained in the tank when full?

Answer. 1549 cubic feet; 9686 gallons.

3. A road rises with a gradient of 1 in 75 from its commencement to a point distant 1½ mile (on a horizontal datum). It then falls with a gradient of 1 in 100 to its termination at a further distance of 140 chains (on a horizontal datum). What is the difference of level between the beginning and the end of the road?

Answer. The end of the road is 13·2 feet above the beginning.

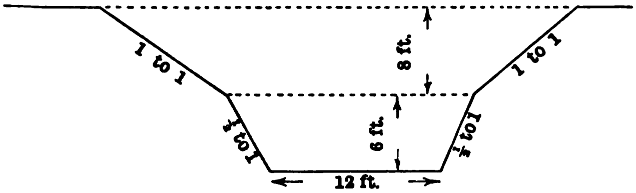
4. The air in a room 30 feet × 25 feet × 10 feet has to be changed three times in an hour by air conveyed through a pipe 6 inches in diameter. At what velocity must the air move in the pipe to do this?

Answer. 114,591 feet per hour.

5. A shower of rain is registered to give $1\frac{1}{8}$ inch. How many gallons would have fallen on a field containing 100 acres?

Answer. 2,552,343 gallons.

6. What is the sectional area of a cutting with slopes, as shown in the sketch? and how many cubic yards are there in 1 chain of this cutting?



Answer. Area 298 square feet; 728·4 cubic yards in 1 chain of cutting.
The sketch evidently shows that by ½ to 1 a rise of 1 in ½ horizontal is meant.

7. A railway bank is half a mile in length, and is 20 feet above the ground at one end, and 30 feet above the ground at the other. The slopes are 2 to 1 throughout. How many acres of ground does it cover?

THE LAW AND MINE SURVEYING.

Provisions of the Coal-Mines Regulation Acts, 1887 and 1896, in Regard to Plans and Sections of Mines.

Coal-Mines Regulation Act, 1887.

34. (1) The owner, agent, or manager of every mine shall keep in the office at the mine an accurate plan of the workings of the mine, showing the workings up to a date not more than three months previously, and the general direction and rate of dip of the strata, together with a section of the strata sunk through; or, if that may be not reasonably practicable, a statement of the depth of the shaft, with a section of the seam.

(2) The owner, agent, or manager of the mine shall, on request at any time of an inspector under this Act, produce to him, at the office at the mine, such plan and section, and shall also, on the like request, mark on such plan and section the then state of the workings of the mine; and the inspector shall be entitled to examine the plan and section, and, for official purposes only, to make a copy of any part thereof respectively.

(3) If the owner, agent, or manager of any mine fails to keep, or wilfully refuses to produce or allow to be examined, the plan and section aforesaid, or wilfully withholds any portion thereof, or wilfully refuses, on request, to mark

thereon the state of the workings of the mine, or conceals any part of those workings, or produces an imperfect or inaccurate plan or section, he shall (unless he shows that he was ignorant of the concealment, imperfection, or inaccuracy) be guilty of an offence against this Act; and, further, the inspector may, by notice in writing (whether a penalty for the offence has or has not been inflicted), require the owner, agent, or manager to cause an accurate plan and section, showing the particulars hereinbefore required, to be made within a reasonable time, at the expense of the owner of the mine. Every such plan must be on a scale of not less than that of the Ordnance Survey of 25 inches to the mile, or on the same scale as the plan for the time being in use at the mine.

(4) If the owner, agent, or manager fails within twenty days after the requisition of the inspector, or within such further time as may be allowed by a Secretary of State, to cause such plan and section to be made as hereby required, he shall be guilty of an offence against this Act.

38. (1) Where any mine or seam is abandoned, the owner of the mine or seam at the time of its abandonment shall, within three months after the abandonment, send to a Secretary of State an accurate plan, showing the boundaries of the workings of the mine or seam up to the time of the abandonment, and the position of the workings with regard to the surfaces, and the general direction and rate of dip of the strata, together with a section of the strata sunk through, or, if that is not reasonably practicable, a statement of the depth of the shaft, with a section of the seam. Every such plan must be on a scale of not less than that of the Ordnance Survey of 25 inches to the mile, or on the same scale as the plan used at the mine at the time of its abandonment.

(2) The plan and section shall be preserved under the care of the Secretary of State; but no person, except an inspector under this Act, shall be entitled, without the consent of the owner of the mine or seam, to see the plan when so sent until after the expiration of 10 years from the time of the abandonment.

Coal-Mines Regulation Act, 1896.

3. Plan of mine in working. The plan required to be kept in pursuance of section 34 of the principal Act shall show the position of the workings therein mentioned with regard to the surface, and the position, extension, and direction of every known fault or dislocation of the seam, with its vertical throw.

4. Plan of abandoned mine. (1) For sub-sections (1) and (2) of section 38 of the principal Act shall be substituted the following sub-sections:—

"(1) Where any mine or seam is abandoned, the person who is owner of the mine or seam at the time of its abandonment shall, within three months after the abandonment, send to a Secretary of State

"(i.) An accurate plan of the mine or seam, being either the original working plan or an accurate copy thereof made by a competent draughtsman, and showing

"(*a*) The boundaries of the workings of the mine or seam, including not only the working faces, but also all headings in advance thereof, up to the time of the abandonment;

"(*b*) The pillars of coal or other mineral remaining unworked;

"(*c*) The position, direction, and extent of every known fault or dislocation of the seam, with its vertical throw;

"(*d*) The position of the workings with regard to the surface boundary;

"(*e*) The general direction and dip of the strata; and

"(*f*) A statement of the depth of the shaft from the surface to the seam abandoned; and

"(ii.) A section of the strata sunk through; or, if that is not reasonably practicable, a statement of the depth of the shaft, with a section of the seam.

"Every such plan must be on a scale of not less than that of the Ordnance Survey of 25 inches to the mile, or on the same scale as the plan used at the mine at the time of its abandonment; and its accuracy must be certified, so far as is reasonably practicable, by a surveyor or other person approved in that behalf by an inspector of mines.

"(2) The plan and section shall be preserved under the care of the Secretary of State; but no person, except an inspector under this Act, shall be entitled, without the consent of the owner of the mine or seam, or the licence of a Secretary of State, to see the plan when so sent until after the expiration of ten years from the time of the abandonment. Provided that such licence shall not be granted unless the Secretary of State is satisfied that the inspection of such plan is necessary in the interests of safety."

(2) The High Court, or, in Scotland, the Court of Session, may, on application by or on behalf of the Secretary of State, make an order requiring any person who has for the time being the custody or possession of any plan or section of an abandoned mine or seam, to produce it to the Secretary of State for the purpose of inspection or copying.

ATTRACTION OF THE MAGNETIC NEEDLE BY IRON.

It is well known that many substances attract the needle, especially magnetic iron ore (called magnetite, or magnetic oxide of iron, Fe_3O_4), whilst other more or less magnetic substances include hematite iron ore, nickel, cobalt, manganese, and some kinds of platinum.

The chief sources of attraction against which the surveyor must guard are iron rails, girders, safety-lamps, or iron in any form. It must be borne in mind that the magnetic attraction is not interrupted by the presence of rocks, and therefore the iron in one road might affect the compass needle in another road.

Dialling lamps supposed to be non-magnetic can be obtained; but before being relied upon they should be carefully tested, as the author has frequently found that such lamps affect the needle to a certain extent.

The author has made a number of experiments to ascertain the effect of iron rails, etc., upon the needle, some of which are given below:—

Old iron rails, about 30 lbs. to the yard, 5 yards long—

At 5 feet 10 inches 1 pair of rails deflected the needle ⅛°.
At 7 „ 8 „ 1 „ „ „ *nil.*
At 7 „ 8 „ 2 „ „ „ ½°.
At 7 „ 8 „ 3 „ „ „ 1°.
At 7 „ 8 „ 1 „ „ „ 1⅛° (when raised up level with needle).

At 7 feet 8 inches three rails (not three pairs) on each side of the dial deflected the needle 1⅔°.

By altering one side, so that three rails on one side were 7 feet 8 inches away, and on the other side 17 feet away, 1⅛° deflection was given.

With three rails on each side 17 feet distant, ⅛° deflection.

At 18 feet away, disturbance only just perceptible, even with five rails on each side of dial. The weight of each rail was 150 lbs. (5 yards), so that in this experiment there was over a quarter of a ton of metal on each side of the dial at 18 feet distance.

At 21 feet away there was no disturbance.

When the rails were laid down again, without disturbing the dial, 1¼° deflection was caused; but after disturbing the needle it would settle anywhere with up to 3° deflection.

Substituting *new steel* rails, 22 lbs. to the yard, 4-yard rails, the results were as follows:—

After setting the needle, 528 lbs. of rails were gradually advanced towards the dial in distances of 1 yard at a time, starting at 14 yards distance. No deflection was noticed until a distance of 6 yards was reached, when there seemed to be a very slight disturbance, hardly measurable, but probably $\frac{1}{32}$°.

At 5 yards the disturbance was clearly perceptible, and would be about $\frac{1}{16}$°.

At 4 yards the deflection was ½°.

At 4 yards, but instead of the ends of the rails being towards the dial, they were placed broadside on, the deflection was 1°.

The disturbance of small articles which might be accidentally left near a dial was noted.

A pocket-knife exerted no influence until brought within 12 inches of the needle.

An iron locker 8 lbs. in weight caused disturbance at 2 yards distant; 6 lbs. of fish-plates, at 1½ yard.

A pick, an adze, and several ordinary iron safety-lamps caused no disturbance when 1 yard away, even if brought level with the needle.

The conclusions the author has arrived at from these and similar experiments are as follows:—

1. That provided that the only iron to be guarded against is the rails, then at 8 yards on either side of the dial it is absolutely safe.

2. That dialling "over the rails," under the impression that the "pull" on one side will balance that on the other, is erroneous, and liable to serious error.

3. That provided 8 yards of rail are taken up, it does not seem to matter

whether all the rails taken up go all on one side or one-half one way and one-half another.

4. That small iron articles weighing not more than 2 or 3 lbs., *e.g.* pick, wedge, lamp, etc., will not disturb the needle if more than 1 yard away.

5. That no *rule* can be deduced based on weight of metal and distance, because in one case a rail may be brought to within a few feet without causing disturbance, whilst another rail will deflect the needle twice the distance away, this being due, no doubt, to the rail having acquired some permanent magnetism.

MATHEMATICAL TABLES.

Logarithms.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

These Tables enable the logarithm to be found of numbers 1 to 10000. Example: To find the logarithm of 5779. Looking down the first column on p. 397, we find the figure 57, and in the same line, under the figure 7, we find the figures 7612, which is the mantissa portion of

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9213	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9810	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

the logarithm of 5770. Still further along the same line are the difference columns, and under the figure 9 we find the figure 7, which, added to 7612, gives 0·7619 as the mantissa portion of the logarithm 5779.

ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2506	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

These Tables enable the numbers to be found corresponding to logarithms ·0000 to ·9999. Example: To find the number of which 3·0978 is the logarithm. Looking down the first column on p. 398, we find the figures ·09, and in the same line under the figure 7 we find

ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6965	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	12	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

the figures 1250. Still further along the same line we get the difference for 8, (2), which added to 1250, gives 1252, the number required.

SQUARES OF NUMBERS FROM 1 TO 10000, CORRECT TO FOUR SIGNIFICANT FIGURES.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	1000	1020	1040	1061	1082	1102	1124	1145	1166	1188	3	5	7	9	11	13	15	17	19
11	1210	1232	1254	1277	1300	1322	1346	1369	1392	1416	3	5	7	10	12	14	17	19	21
12	1440	1464	1488	1513	1538	1562	1588	1613	1638	1664	3	5	8	10	13	15	18	20	23
13	1690	1716	1742	1769	1796	1822	1850	1877	1904	1932	3	6	9	11	14	17	19	22	25
14	1960	1988	2016	2045	2074	2102	2132	2161	2190	2220	3	6	9	12	15	18	21	24	27
15	2250	2280	2310	2341	2372	2402	2434	2465	2496	2528	4	7	10	13	16	19	22	25	28
16	2560	2592	2624	2657	2690	2722	2756	2789	2822	2856	4	7	10	14	17	20	24	27	30
17	2890	2924	2958	2993	3028	3062	3098	3133	3168	3204	4	7	11	14	18	21	25	28	32
18	3240	3276	3312	3349	3386	3422	3460	3497	3534	3572	4	8	12	15	19	23	26	30	34
19	3610	3648	3686	3725	3764	3802	3842	3881	3920	3960	4	8	12	16	20	24	28	32	36
20	4000	4040	4080	4121	4162	4202	4244	4285	4326	4368	5	9	13	17	21	25	29	33	37
21	4410	4452	4494	4537	4580	4622	4666	4709	4752	4796	5	9	13	18	22	26	31	35	39
22	4840	4884	4928	4973	5018	5062	5108	5153	5198	5244	5	9	14	18	23	27	32	36	41
23	5290	5336	5382	5429	5476	5522	5570	5617	5664	5712	5	10	15	19	24	27	33	38	43
24	5760	5808	5856	5905	5954	6002	6052	6101	6150	6200	5	10	15	20	25	30	35	40	45
25	6250	6300	6350	6401	6452	6502	6554	6605	6656	6708	6	11	16	21	26	31	36	41	46
26	6760	6812	6864	6917	6970	7022	7076	7129	7182	7236	6	11	16	22	27	32	38	43	48
27	7290	7344	7398	7453	7508	7562	7618	7673	7728	7784	6	11	17	22	28	33	39	44	50
28	7840	7896	7952	8009	8066	8122	8180	8237	8294	8352	6	12	18	23	29	35	40	46	52
29	8410	8468	8526	8585	8644	8702	8761	8821	8880	8940	6	12	18	24	30	36	42	48	54
30	9000	9060	9120	9181	9242	9302	9364	9425	9486	9548	7	13	19	25	31	37	43	49	55
31	9610	9672	9734	9797	9860	9922	9986	1005*	1011*	1018*	7	13	19	26	32	38	45	51	57
32	1024	1030	1037	1043	1050	1056	1063	1069	1076	1082	1	1	2	3	3	4	5	5	6
33	1089	1096	1102	1109	1115	1122	1129	1136	1142	1149	1	1	2	3	4	4	5	6	6
34	1156	1163	1170	1176	1183	1190	1197	1204	1211	1218	1	2	2	3	4	4	5	6	6
35	1225	1232	1239	1246	1253	1260	1267	1274	1282	1289	1	2	2	3	4	4	5	6	7
36	1296	1303	1310	1318	1325	1332	1339	1347	1354	1362	1	2	2	3	4	5	5	6	7
37	1369	1376	1384	1391	1399	1406	1414	1421	1429	1436	1	2	2	3	4	5	5	6	7
38	1444	1452	1459	1467	1474	1482	1490	1498	1505	1513	1	2	2	3	4	5	6	6	7
39	1521	1529	1537	1544	1552	1560	1568	1576	1584	1592	1	2	3	3	4	5	6	6	7
40	1600	1608	1616	1624	1632	1640	1648	1656	1665	1673	1	2	3	3	4	5	6	7	7
41	1681	1689	1697	1706	1714	1722	1730	1739	1747	1756	1	2	3	3	4	5	6	7	8
42	1764	1772	1781	1789	1798	1806	1815	1823	1832	1840	1	2	3	4	4	5	6	7	8
43	1849	1858	1866	1875	1883	1892	1901	1910	1918	1927	1	2	3	4	5	5	6	7	8
44	1931	1945	1954	1962	1971	1980	1989	1998	2007	2016	1	2	3	4	5	5	6	7	8
45	2025	2034	2043	2052	2061	2070	2079	2088	2098	2107	1	2	3	4	5	6	7	7	8
46	2116	2125	2134	2144	2153	2162	2171	2181	2190	2200	1	2	3	4	5	6	7	8	9
47	2209	2218	2228	2237	2247	2256	2266	2275	2285	2294	1	2	3	4	5	6	7	8	9
48	2304	2314	2323	2333	2342	2352	2362	2372	2381	2391	1	2	3	4	5	6	7	8	9
49	2401	2411	2421	2430	2440	2450	2460	2470	2480	2490	1	2	3	4	5	6	7	8	9
50	2500	2510	2520	2530	2540	2550	2560	2570	2581	2591	1	2	3	4	5	6	7	8	9
51	2601	2611	2621	2632	2642	2652	2662	2673	2683	2694	1	2	3	4	5	6	7	8	9
52	2704	2714	2725	2735	2746	2756	2767	2777	2788	2798	1	2	3	4	5	6	8	9	10
53	2809	2820	2830	2841	2852	2862	2873	2884	2894	2905	1	2	3	4	6	7	8	9	10
54	2916	2927	2938	2948	2959	2970	2981	2992	3003	3014	1	2	3	5	6	7	8	9	10

Squares from	1 to 3	contain	1 figure.	
,, ,,	4 to 9	,,	2 figures.	
,, ,,	10 to 31	,,	3 ,,	
,, ,,	32 to 99	,,	4 ,,	
Squares from	100 to 316	contain	5 figures.	
,, ,,	317 to 999	,,	6 ,,	
,, ,,	1000 to 3163	,,	7 ,,	
,, ,,	3163 to 10000	,,	8 ,,	

* The differences for squares from 3171 to 3199 are 1, 1, 2, 3, 3, 4, 5, 5, 6.

SQUARES OF NUMBERS FROM 1 TO 10000, CORRECT TO FOUR SIGNIFICANT FIGURES.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	3025	3036	3047	3058	3069	3080	3091	3102	3114	3125	1	2	3	5	6	7	8	9	10
56	3136	3147	3158	3170	3181	3192	3204	3215	3226	3238	1	2	4	5	6	7	8	9	10
57	3249	3260	3272	3283	3295	3306	3318	3329	3341	3352	1	2	4	5	6	7	8	9	11
58	3364	3376	3387	3399	3411	3422	3434	3446	3457	3469	1	2	4	5	6	7	8	10	11
59	3481	3493	3505	3516	3528	3540	3552	3564	3576	3588	1	3	4	5	6	7	8	10	11
60	3600	3612	3624	3636	3648	3660	3672	3684	3697	3709	1	3	4	5	6	7	9	10	11
61	3721	3733	3745	3758	3770	3782	3795	3807	3819	3832	1	3	4	5	6	8	9	10	11
62	3844	3856	3869	3881	3894	3906	3919	3931	3944	3956	1	3	4	5	6	8	9	10	11
63	3969	3982	3994	4007	4020	4032	4045	4058	4070	4083	1	3	4	5	7	8	9	10	12
64	4096	4109	4122	4134	4147	4160	4173	4186	4199	4212	1	3	4	5	7	8	9	10	12
65	4225	4238	4251	4264	4277	4290	4303	4316	4330	4343	1	3	4	5	7	8	9	11	12
66	4356	4369	4382	4396	4409	4422	4436	4449	4462	4476	1	3	4	5	7	8	9	11	12
67	4489	4502	4516	4529	4543	4556	4570	4583	4597	4610	2	3	4	6	7	8	10	11	12
68	4624	4638	4651	4665	4679	4692	4706	4720	4733	4747	2	3	4	6	7	8	10	11	12
69	4761	4775	4789	4802	4816	4830	4844	4858	4872	4886	2	3	4	6	7	8	10	11	13
70	4900	4914	4928	4942	4956	4970	4984	4998	5013	5027	2	3	4	6	7	9	10	11	13
71	5041	5055	5069	5084	5098	5112	5127	5141	5155	5170	2	3	4	6	7	9	10	12	13
72	5184	5198	5213	5227	5242	5256	5271	5285	5300	5314	2	3	5	6	7	9	10	12	13
73	5329	5344	5358	5373	5388	5402	5417	5432	5446	5461	2	3	5	6	8	9	10	12	13
74	5476	5491	5506	5520	5535	5550	5565	5580	5595	5610	2	3	5	6	8	9	11	12	14
75	5625	5640	5655	5670	5685	5700	5715	5730	5746	5761	2	3	5	6	8	9	11	12	14
76	5776	5791	5806	5822	5837	5852	5868	5883	5898	5914	2	3	5	6	8	9	11	12	14
77	5929	5944	5960	5975	5991	6006	6022	6037	6053	6068	2	3	5	6	8	9	11	13	14
78	6084	6100	6115	6131	6147	6162	6178	6194	6209	6225	2	3	5	6	8	10	11	13	14
79	6241	6257	6273	6288	6304	6320	6336	6352	6368	6384	2	3	5	7	8	10	11	13	14
80	6400	6416	6432	6448	6464	6480	6496	6512	6529	6545	2	3	5	7	8	10	11	13	15
81	6561	6577	6593	6610	6626	6642	6659	6675	6691	6708	2	3	5	7	8	10	12	13	15
82	6724	6740	6757	6773	6790	6806	6823	6839	6856	6872	2	3	5	7	8	10	12	13	15
83	6889	6906	6922	6939	6956	6972	6989	7006	7022	7039	2	3	5	7	9	10	12	14	15
84	7056	7073	7090	7106	7123	7140	7157	7174	7191	7208	2	4	5	7	9	10	12	14	15
85	7225	7242	7259	7276	7293	7310	7327	7344	7362	7379	2	4	5	7	9	10	12	14	16
86	7396	7413	7430	7448	7465	7482	7500	7517	7534	7552	2	4	5	7	9	11	12	14	16
87	7569	7586	7604	7621	7639	7656	7674	7691	7709	7726	2	4	5	7	9	11	12	14	16
88	7744	7762	7779	7797	7815	7832	7850	7868	7885	7903	2	4	5	7	9	11	13	14	16
89	7921	7939	7957	7974	7992	8010	8028	8046	8064	8082	2	4	6	7	9	11	13	14	16
90	8100	8118	8136	8154	8172	8190	8208	8226	8245	8263	2	4	6	7	9	11	13	15	16
91	8281	8299	8317	8336	8354	8372	8391	8409	8427	8446	2	4	6	7	9	11	13	15	17
92	8464	8482	8501	8519	8538	8556	8575	8593	8612	8630	2	4	6	8	9	11	13	15	17
93	8649	8668	8686	8705	8724	8742	8761	8780	8798	8817	2	4	6	8	10	11	13	15	17
94	8836	8855	8874	8892	8911	8930	8949	8968	8987	9006	2	4	6	8	10	11	13	15	17
95	9025	9044	9063	9082	9101	9120	9139	9158	9178	9197	2	4	6	8	10	12	14	15	17
96	9216	9235	9254	9274	9293	9312	9331	9351	9370	9390	2	4	6	8	10	12	14	16	18
97	9409	9428	9448	9467	9487	9506	9526	9545	9565	9584	2	4	6	8	10	12	14	16	18
98	9604	9624	9643	9663	9683	9702	9722	9742	9761	9781	2	4	6	8	10	12	14	16	18
99	9801	9821	9841	9860	9880	9900	9920	9940	9960	9980	2	4	6	8	10	12	14	16	18

Squares from 1 to 3 contain 1 figure.
" " 4 to 9 " 2 figures.
" " 10 to 31 " 3 "
" " 32 to 99 " 4 "

Squares from 100 to 316 contain 5 figures.
" " 317 to 999 " 6 "
" " 1000 to 3162 " 7 "
" " 3163 to 10000 " 8 "

The differences for squares from 3171 to 3199 are 1, 1, 2, 3, 3, 4, 5, 5, 6.

RECIPROCALS OF NUMBERS FROM 1 TO 10000.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0100	9901	9804	9709	9615	9524	9434	9346	9259	9174	9	18	27	36	45	54	63	72	81
11	9091	9009	8928	8849	8772	8696	8621	8547	8474	8403	8	16	23	31	38	46	53	61	68
12	8333	8264	8197	8130	8064	8000	7936	7874	7812	7752	6	13	19	26	32	38	45	51	57
13	7692	7633	7576	7519	7463	7407	7353	7299	7246	7194	5	11	16	22	27	32	38	43	49
14	7143	7092	7042	6993	6944	6896	6849	6803	6757	6711	5	9	14	19	23	28	33	37	42
15	6667	6622	6579	6536	6493	6452	6410	6369	6329	6289	5	9	13	17	21	25	29	34	38
16	6250	6211	6173	6135	6097	6061	6024	5988	5952	5917	4	8	11	15	19	22	26	30	33
17	5882	5848	5814	5780	5747	5714	5682	5650	5618	5586	3	6	10	13	16	19	23	26	29
18	5555	5529	5494	5464	5435	5405	5376	5347	5319	5291	3	6	8	11	14	17	20	23	26
19	5263	5236	5208	5181	5155	5128	5102	5076	5050	5025	3	5	8	10	13	16	18	21	23
20	5000	4975	4950	4926	4902	4878	4854	4831	4808	4785	2	5	7	10	12	14	17	19	21
21	4762	4739	4717	4695	4673	4651	4630	4608	4587	4566	2	4	6	9	11	13	15	17	19
22	4545	4525	4504	4484	4464	4444	4425	4405	4386	4367	2	4	6	8	10	11	13	15	17
23	4348	4329	4310	4292	4273	4255	4237	4219	4202	4184	2	3	5	7	9	11	12	14	16
24	4167	4149	4132	4115	4098	4082	4065	4048	4032	4016	2	4	5	7	9	10	12	14	15
25	4000	3984	3968	3952	3937	3921	3906	3891	3876	3861	1	3	4	6	7	9	10	12	13
26	3846	3831	3817	3802	3788	3773	3759	3745	3731	3717	1	2	4	5	7	8	9	11	12
27	3704	3690	3676	3663	3650	3636	3623	3610	3597	3584	1	2	4	5	6	8	9	10	12
28	3571	3559	3546	3533	3521	3509	3496	3484	3472	3460	2	3	4	5	6	8	9	10	11
29	3448	3436	3424	3413	3401	3390	3378	3367	3356	3344	1	3	4	5	6	7	8	9	11
30	3333	3322	3311	3300	3289	3279	3268	3257	3247	3237	1	3	4	5	6	7	8	9	10
31	3226	3215	3205	3195	3185	3175	3164	3154	3145	3135	2	3	4	5	6	7	8	9	10
32	3125	3115	3105	3096	3086	3077	3067	3058	3049	3039	1	2	3	4	5	6	7	8	9
33	3030	3021	3012	3003	2994	2985	2976	2967	2958	2950	1	2	3	4	4	5	6	7	8
34	2941	2932	2924	2915	2907	2898	2890	2882	2873	2865	0	1	2	3	4	5	5	6	7
35	2857	2849	2841	2839	2825	2817	2809	2801	2793	2785	1	2	3	3	4	5	6	7	7
36	2778	2770	2762	2755	2747	2740	2732	2725	2717	2710	1	2	3	3	4	5	6	6	7
37	2703	2695	2688	2681	2674	2667	2659	2652	2645	2638	1	2	3	3	4	5	5	6	7
38	2631	2625	2618	2611	2604	2597	2591	2584	2577	2571	0	1	2	2	3	4	4	5	6
39	2564	2557	2551	2544	2538	2532	2525	2519	2512	2506	1	2	2	3	4	4	5	6	6
40	2500	2494	2487	2481	2475	2469	2463	2457	2451	2445	1	1	2	2	3	4	4	5	5
41	2439	2433	2427	2421	2415	2410	2404	2398	2392	2387	1	2	2	3	3	4	5	5	6
42	2381	2375	2370	2364	2358	2353	2347	2342	2336	2331	1	1	2	2	3	3	4	5	5
43	2325	2320	2315	2309	2304	2299	2293	2288	2283	2278	1	1	2	2	3	3	4	4	5
44	2273	2267	2262	2257	2252	2247	2242	2237	2232	2227	0	1	1	2	2	3	3	4	4
45	2222	2217	2212	2207	2203	2198	2193	2188	2183	2179	1	1	2	2	3	3	4	4	5
46	2174	2169	2164	2160	2155	2150	2146	2141	2137	2132	0	0	1	1	2	2	3	3	4
47	2128	2123	2119	2114	2110	2105	2101	2096	2092	2088	0	1	1	2	2	2	3	3	4
48	2083	2079	2075	2070	2066	2062	2058	2053	2049	2045	1	1	2	2	2	3	3	4	4
49	2041	2037	2032	2028	2024	2020	2016	2012	2008	2004	0	1	1	2	2	2	3	3	4
50	2000	1996	1992	1988	1984	1980	1976	1972	1968	1965	0	1	1	1	2	2	3	3	3
51	1961	1957	1953	1949	1945	1942	1938	1934	1930	1927	1	1	1	2	2	3	3	3	4
52	1923	1919	1916	1912	1908	1905	1901	1897	1894	1890	1	1	1	2	2	3	3	3	4
53	1887	1883	1880	1876	1873	1869	1866	1862	1859	1855	0	1	1	1	2	2	2	3	3
54	1852	1848	1845	1842	1838	1835	1831	1828	1825	1821	1	1	1	2	2	2	3	3	3

Reciprocals from 2 to 10 = 0·
„ „ 11 to 100 = 0·0
Reciprocals from 101 to 1000 = 0·00
„ „ 1001 to 10000 = 0·000

Numbers in difference columns to be subtracted, not added.

The reciprocal of a number is obtained by dividing it into 1. Example: To find the value of $\frac{1}{883}$. Looking down the first column on p. 403, we find the figures 88, and in the same

RECIPROCALS OF NUMBERS FROM 1 TO 10000.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	1818	1815	1811	1808	1805	1802	1798	1795	1792	1789	1	1	1	2	2	2	3	3	3
56	1786	1782	1779	1776	1773	1770	1767	1764	1760	1757	1	1	1	1	2	2	2	3	3
57	1754	1751	1748	1745	1742	1739	1736	1733	1730	1727	0	1	1	1	1	2	2	2	3
58	1724	1721	1718	1715	1712	1709	1706	1703	1701	1698	0	0	1	1	1	1	2	2	2
59	1695	1692	1689	1686	1683	1681	1678	1675	1672	1669	1	1	1	2	2	2	2	3	3
60	1667	1664	1661	1658	1656	1653	1650	1647	1645	1642	0	1	1	1	2	2	2	2	3
61	1639	1637	1634	1631	1629	1626	1623	1621	1618	1615	0	1	1	1	1	2	2	2	2
62	1613	1610	1608	1605	1602	1600	1597	1595	1592	1590	0	1	1	1	1	2	2	2	2
63	1587	1585	1582	1580	1577	1575	1572	1570	1567	1565	1	1	1	1	2	2	2	2	3
64	1562	1560	1558	1555	1553	1550	1548	1545	1543	1541	0	0	0	1	1	1	1	2	2
65	1538	1536	1534	1531	1529	1527	1524	1522	1520	1517	1	1	1	1	2	2	2	2	2
66	1515	1513	1510	1508	1506	1504	1501	1500	1497	1495	1	1	1	1	1	2	2	2	2
67	1492	1490	1488	1486	1484	1481	1479	1477	1475	1473	0	0	0	0	1	1	1	1	2
68	1470	1468	1466	1464	1462	1460	1458	1456	1453	1451	0	1	1	1	1	2	2	2	2
69	1449	1447	1445	1443	1441	1439	1437	1435	1433	1431	0	1	1	1	1	1	2	2	2
70	1428	1426	1424	1422	1420	1418	1416	1414	1412	1410	0	0	0	0	1	1	1	1	1
71	1408	1406	1404	1402	1400	1399	1397	1395	1393	1391	1	1	1	1	1	2	2	2	2
72	1389	1387	1385	1383	1381	1379	1377	1375	1374	1372	0	0	0	1	1	1	1	1	1
73	1370	1368	1366	1364	1362	1360	1359	1357	1355	1353	0	0	0	0	0	1	1	1	1
74	1351	1349	1348	1346	1344	1342	1340	1339	1337	1335	0	0	0	1	1	1	1	1	1
75	1333	1331	1330	1328	1326	1324	1323	1321	1319	1317	0	0	0	0	0	1	1	1	1
76	1316	1314	1312	1311	1309	1307	1305	1304	1302	1300	0	0	0	1	1	1	1	1	1
77	1299	1297	1295	1294	1292	1290	1289	1287	1285	1284	0	0	0	0	1	1	1	1	1
78	1282	1280	1279	1277	1275	1274	1272	1271	1269	1267	0	1	1	1	1	1	1	2	2
79	1266	1264	1263	1261	1259	1258	1256	1255	1253	1251	0	1	1	1	1	1	1	2	2
80	1250	1248	1247	1245	1244	1242	1241	1239	1238	1236	0	0	0	0	1	1	1	1	1
81	1234	1233	1231	1230	1228	1227	1225	1224	1222	1221	0	0	1	1	1	1	1	1	1
82	1219	1218	1216	1215	1213	1212	1211	1209	1208	1206	0	0	0	1	1	1	1	1	1
83	1205	1203	1202	1200	1199	1198	1196	1195	1193	1192	1	1	1	1	1	1	1	2	2
84	1190	1189	1188	1186	1185	1183	1182	1181	1179	1178	0	0	0	0	0	1	1	1	1
85	1176	1175	1174	1172	1171	1169	1168	1167	1165	1164	0	0	0	0	0	0	0	1	1
86	1163	1161	1160	1159	1157	1156	1155	1153	1152	1151	0	0	0	1	1	1	1	1	1
87	1149	1148	1147	1145	1144	1143	1141	1140	1139	1138	0	1	1	1	1	1	1	1	1
88	1136	1135	1134	1132	1131	1130	1129	1127	1126	1125	0	0	1	1	1	1	1	1	1
89	1123	1122	1121	1120	1118	1117	1116	1115	1113	1112	0	0	0	0	0	1	1	1	1
90	1111	1110	1109	1107	1106	1105	1104	1102	1101	1100	0	0	0	1	1	1	1	1	1
91	1099	1098	1096	1095	1094	1093	1092	1090	1089	1088	0	0	1	1	1	1	1	1	1
92	1087	1086	1084	1083	1082	1081	1080	1079	1077	1076	0	0	0	0	1	1	1	1	1
93	1075	1074	1073	1072	1071	1069	1068	1067	1066	1065	0	0	0	0	0	0	0	0	1
94	1064	1063	1061	1060	1059	1058	1057	1056	1055	1054	0	0	0	0	0	1	1	1	1
95	1053	1051	1050	1049	1048	1047	1046	1045	1044	1043	0	0	0	0	1	1	1	1	1
96	1042	1040	1039	1038	1037	1036	1035	1034	1033	1032	0	0	0	0	0	0	1	1	1
97	1031	1030	1029	1028	1027	1026	1024	1023	1022	1021	1	1	1	1	1	1	1	1	1
98	1020	1019	1018	1017	1016	1015	1014	1013	1012	1011	0	0	0	0	0	0	1	1	1
99	1010	1009	1008	1007	1006	1005	1004	1003	1002	1001	0	0	0	0	1	1	1	1	1

Reciprocals from 2 to 10 = 0·
" " 11 to 100 = 0·0
Reciprocals from 101 to 1000 = 0·00
" " 1001 to 10000 = 0·000

Numbers in difference columns to be subtracted, not added.

line under the figure 8 we find the figures 1126; the reciprocal required is therefore 0·001126.

NATURAL TANGENTS.

	·0°	·1°	·2°	·3°	·4°	·5°	·6°	·7°	·8°	·9°
0°	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965

From 0° to 45° the natural tangent increases from 0 to 1.

NATURAL TANGENTS.

	·0°	·1°	·2°	·3°	·4°	·5°	·6°	·7°	·8°	·9°
45°	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5880	5941
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542
63	1·9626	9711	9797	9883	9970	$\overline{0057}$	$\overline{0145}$	$\overline{0233}$	$\overline{0323}$	$\overline{0413}$
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878
71	2·9042	9208	9375	9544	9714	9887	$\overline{0061}$	$\overline{0237}$	$\overline{0415}$	$\overline{0595}$
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646
78	4·7046	7453	7867	8288	8716	9152	9594	$\overline{0045}$	$\overline{0504}$	$\overline{0970}$
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140
80	5·6713	7297	7894	8502	9124	9758	$\overline{0405}$	$\overline{1066}$	$\overline{1742}$	$\overline{2432}$
81	6·3138	3859	4596	5350	6122	6912	7920	8548	9395	$\overline{0264}$
82	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	$\overline{0285}$
83	8·1443	2636	3863	5126	6427	7769	9152	$\overline{0579}$	$\overline{2052}$	$\overline{3572}$
84	9·5144	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0

From 45° to 90° the natural tangent incteases from 1 to infinity. A dash over the number indicates that the whole number part is increased by 1.

NATURAL SINES.

	·0°	·1°	·2°	·3°	·4°	·5°	·6°	·7°	·8°	·9°
0°	0000	0017	0035	0052	0070	0087	0105	0122	0140	0157
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332
2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854
5	0872	0889	0906	0924	0941	0958	0976	0993	1011	1028
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374
8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719
10	1736	1754	1771	1788	1805	1822	1840	1857	1874	1891
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2232
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	2402
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571
15	2588	2605	2622	2639	2656	2672	2689	2706	2723	2740
16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907
17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239
19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404
20	3420	3437	3453	3469	3486	3502	3518	3535	3551	3567
21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730
22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891
23	3907	3923	3939	3955	3971	3987	4003	4019	4035	4051
24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210
25	4226	4242	4258	4274	4289	4305	4321	4337	4352	4368
26	4384	4399	4415	4431	4446	4462	4478	4493	4509	4524
27	4540	4555	4571	4586	4602	4617	4633	4648	4664	4679
28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833
29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985
30	5000	5015	5030	5045	5060	5075	5090	5105	5120	5135
31	5150	5165	5180	5195	5210	5225	5240	5255	5270	5284
32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432
33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577
34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721
35	5736	5750	5764	5779	5793	5807	5821	5835	5850	5864
36	5878	5892	5906	5920	5934	5948	5962	5976	5990	6004
37	6018	6032	6046	6060	6074	6088	6101	6115	6129	6143
38	6157	6170	6184	6198	6211	6225	6239	6252	6266	6280
39	6293	6307	6320	6334	6347	6361	6374	6388	6401	6414
40	6428	6441	6455	6468	6481	6494	6508	6521	6534	6547
41	6561	6574	6587	6600	6613	6626	6639	6652	6665	6678
42	6691	6704	6717	6730	6743	6756	6769	6782	6794	6807
43	6820	6833	6845	6858	6871	6884	6896	6909	6921	6934
44	6947	6959	6972	6984	6997	7009	7022	7034	7046	7059

NATURAL SINES.

	·0°	·1°	·2°	·3°	·4°	·5°	·6°	·7°	·8°	·9°
45	7071	7083	7096	7108	7120	7133	7145	7157	7169	7181
46	7193	7206	7218	7230	7242	7254	7266	7278	7290	7302
47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420
48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536
49	7547	7558	7570	7581	7593	7604	7615	7627	7638	7649
50	7660	7672	7683	7694	7705	7716	7727	7738	7749	7760
51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869
52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976
53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080
54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181
55	8192	8202	8211	8221	8231	8241	8251	8261	8271	8281
56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377
57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471
58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563
59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652
60	8660	8669	8678	8686	8695	8704	8712	8721	8729	8738
61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821
62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902
63	8910	8918	8926	8934	8942	8949	8957	8965	8973	8980
64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056
65	9063	9070	9078	9085	9092	9100	9107	9114	9121	9128
66	9135	9143	9150	9157	9164	9171	9178	9184	9191	9198
67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265
68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330
69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391
70	9397	9403	9409	9415	9421	9426	9432	9438	9414	9449
71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505
72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558
73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608
74	9613	9617	9622	9627	9632	9636	9641	9646	9650	9655
75	9659	9664	9668	9673	9677	9681	9686	9690	9694	9699
76	9703	9707	9711	9715	9720	9724	9728	9732	9736	9740
77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778
78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813
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